CHAPTER IV

CALCULATIONS OF PRACTICAL RANGES OF ELECTRONS AND POSITRONS
4.1 MULTIPLE SCATTERING EFFECTS ON TRANSPORT OF ELECTRONS AND POSITRONS IN MATTER

The electrons and positrons during their passage through matter interact inelastically as well as elastically with the atoms of the absorber. This fact is to be utilized for calculations of the practical ranges of these particles in the target material. The path of the electron and positron gets distorted during slowing down due to occurrence of multiple scattering. This distortion in the path of these particles increases with decreasing energy. This is because of enhanced significance of multiple scattering at low energies. Until at some stage when the energy of electrons and positrons is decreased to the extent when the motion of these particles becomes random. This random motion results in diffusion of these particles in the absorber. The energy at which the diffusion sets in, depends upon the incident kinetic energy of these particles as well as on the nature of the absorber material. However, in any case, the randomly moving electrons and positrons are left with substantial fraction of their incident kinetic energy. Therefore even after the diffusion sets in, the electrons and positrons can travel on an average, some distance which is a substantial fraction of their practical range in the absorber. Thus, for calculation of practical
ranges of these particle in an absorber, both parts of their journey, viz, before and after the diffusion sets in, must be taken into account. At present there is no simple method for calculations of practical ranges which could satisfy the following requirements:
1. The method must be applicable to all elements and if possible for compounds also.
2. It should take into account total energy losses i.e. bremsstrahlung losses along with collision energy losses, so that the method becomes applicable to all energies, low as well as high.
3. The method must be applicable for electrons as well as that for positrons.

It has been pointed out in Chapter II that the inclusion of bremsstrahlung losses could not bring down the discrepancy in the calculated ranges of Kohrlich and Carlson\textsuperscript{1}). Therefore, in addition to above mentioned requirements, our approach for calculating practical ranges must be such that it is much nearer to experimental results on transmission of electrons and positrons. The experimental situation is such that the transmitted intensity of electrons and positrons is measured along or in very near vicinity of the direction of incidence, by using a well collimated beam of these particles incident normally on the target.
4.2 Present Method for Theoretical Calculations of Practical Ranges

The theory of electron penetration due to Bethe, Rose and Smith\(^2\) which takes into account the diffusion of these particles also had been used to study transmission of electrons. The following assumptions used in this theory not only bring this theory much nearer to experimental situation but also make the present approach for calculating the practical ranges much simpler. These assumptions can be extended for positrons also and are given as under:

1) The electrons and positrons penetrating the matter suffer energy loss but very little multiple scattering in the beginning of their journey. Thus they travel almost along a straight path.

2) As the energy of electrons and positrons decreases the multiple scattering gains significance until finally a stage of diffusion is reached, when the motion of these particles becomes almost random.

3) The multiple scattering during the slowing down of electrons and positrons in the absorber is taken into account for finding out \(T_0\), the critical kinetic energy at which the diffusion starts. However to treat both these phenomenon viz. multiple scattering
and slowing down simultaneously is not easy. Therefore, to simplify, this situation is approximated by assuming a direct transition from a straight motion into diffusion.

Using definition of transport mean free path this transition was visualized by Bethe, Rose and Smith\(^2\) to occur when the average cosine of the angle \(\theta\) between the direction of motion of these particles in the absorber and the direction of primary beam, becomes \(1/e\), viz.

\[
< \cos \theta > = 1/e \quad [4.1]
\]

Thus by knowing \(< \cos \theta >\) both the straight motion and diffusion of electrons and positrons can be treated to determine their practical ranges.

According to Bethe, Rose and Smith\(^2\) after traversing a path length, \(s\), average cosine of angle \(\theta\) is given by,

\[
< \cos \theta > = \exp \left[ - \int_0^s \chi(s) \, ds \right] \quad [4.2]
\]

If the initial kinetic energy \(T_0\) of the electron or positron is degraded to \(T\) in traversing this path length \(s\), eqn.\(4.2\) extended for positrons also can be written in terms of energy as,

\[
< \cos \theta >^2 = \exp \left[ - \int_{\gamma_0}^{\gamma} \chi^2(\gamma) \left[ \frac{d\gamma}{ds} \right]^{-1} \, d\gamma \right] \quad [4.3]
\]

where, \(\gamma_0\) and \(\gamma\) are the initial and final total energy of
electron or positron respectively, in terms of electron rest mass units. Also \( \chi^\pm(\gamma) \) is the reciprocal of transport mean free path \( \lambda^\pm(\gamma) \), is given for positrons and electrons as,

\[
\chi^\pm(\gamma) = 1/\chi^\pm(\gamma) = 2\pi N \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \chi^\pm(\theta, \gamma)(1-\cos \theta) \sin \theta \, d\theta \tag{4.4}
\]

Where 'N' and superscripts (+) and (-) have their usual meaning. In eqn (4.4) \( \theta_{\text{min}} \) is the non-zero lower limit of the integral which takes into account the atomic screening. In the present calculations, we have taken this limit as given by Williams\(^3\) viz, \( \theta_{\text{min}} = a^2 1/3 / (\gamma^2-1)^{1/2} \), where 'a' is the fine structure constant. The upper limit \( \theta_{\text{max}} \) may safely be taken as '\( \pi \)', its natural value for a point nucleus as long as the energy remains below 5.0 MeV\(^4\) which is also the upper limit of energy to which the present calculations are restricted. Further \( \sigma^\pm(\theta, \gamma) \) in eqn. (4.4) is the Mott's\(^5\) elastic single scattering cross section expansion taken upto first two terms for electrons and \( \sigma^\pm(\theta, \gamma) \) is the corresponding quantity extended for positrons, by Massey\(^6\).

They are given as,

\[
\sigma^\pm(\theta, \gamma) = \frac{r_0^2 Z(Z+1)\gamma^2}{(\gamma^2-1)^{3/2}} \sin^2 \theta \left[ \frac{1 - \gamma^2}{\gamma} \sin^2 \theta/2 + \frac{\gamma (\gamma^2 - 1)^{1/2}}{\gamma} \sin^2 \theta/2 (1 - \sin^2 \theta/2) \right] \tag{4.5}
\]

where \( r_0 \) is the classical radius of an electron and \( Z(Z+1) \) has been used instead of usual \( Z^2 \) to take account of scattering.
of incident electrons or positrons by orbital electrons also. The choice of elastic single scattering cross section in this form (eqn. 4.5) extends our calculations for positrons also. At the same time it takes account of the relativistic effects. These effects have to be taken into account for electrons and positrons even at low energies owing to their small mass.

4.21 Calculations in the Energy Region 50 keV ≤ T₀ ≤ 350 keV

Using Eq. (3.3) for total stopping power of positrons and electrons in the energy region 50 keV ≤ T₀ ≤ 350 keV one gets,

\[
(\frac{d\gamma}{ds})^{-1} = \frac{\alpha e^2}{\rho (M_2Z + C_2)} \left[ \gamma (A^+Z + B^+ - k_{II}^2) - \gamma (A^+Z + B^+) \right] \quad [4.6]
\]

Substituting value of \((\frac{d\gamma}{ds})^{-1}\) from eqn (4.6) in eqn (4.3) and integrating over \(\gamma\) the average cosine of multiple scattering angle \(\theta\) for positrons and electrons is given by

\[
< \cos \theta > = \exp[-D_2 \left\{ F\gamma(\gamma) - F\gamma(\gamma) \right\}] \quad [4.7]
\]

where,

\[
D_2 = \frac{\hbar w M e^2}{Z(Z+1) e^2/ \rho (M_2Z + C_2)} \quad [4.8]
\]

is a constant for a particular material. Also where
\[ F_{II}^{\pm} \left( \gamma \right) = \left[ (I_1^\pm - I_2^\pm) \ln(2/a^2 1/3) + (I_3^\pm - I_4^\pm) / 2 - \right. \\
( I_5^\pm - I_6^\pm) / 2 + \mu a^2 (I_7^\pm - I_8^\pm) / 2 \pm \mu a^2 z^4/3 x \\
\left. (I_9^\pm - I_{10}^\pm) / 2 \right] \]  

\[ [4.9] \]

where, \( I_1^\pm, I_2^\pm, \ldots, I_{10}^\pm \) are the integrals given as under:

\[ I_1^\pm = \int \frac{\gamma^{-(A^\pm Z + B^\pm - K_{II}^\pm + 2)}}{(\gamma^2 - 1)^2} \, d\gamma \]

\[ I_2^\pm = \int \frac{\gamma^{-(A^\pm Z + B^\pm - 2)}}{(\gamma^2 - 1)^2} \, d\gamma \]

\[ I_3^\pm = \int \frac{\gamma^{-(A^\pm Z + B^\pm - K_{II}^\pm - 2)} \times \ln(\gamma^2 - 1)}{(\gamma^2 - 1)^2} \, d\gamma \]

\[ I_4^\pm = \int \frac{\gamma^{-(A^\pm Z + B^\pm - 2)} \times \ln(\gamma^2 - 1)}{(\gamma^2 - 1)^2} \, d\gamma \]

\[ I_5^\pm = \int \frac{\gamma^{-(A^\pm Z + B^\pm)} \times \ln(\gamma^2 - 1)}{(\gamma^2 - 1)^2} \, d\gamma \]

\[ I_6^\pm = \int \frac{\gamma^{-(A^\pm Z + B^\pm - K_{II}^\pm)}}{(\gamma^2 - 1)} \, d\gamma \]

\[ I_7^\pm = \int \frac{\gamma^{-(A^\pm Z + B^\pm - K_{II}^\pm - 1)}}{(\gamma^2 - 1)^{3/2}} \, d\gamma \]
\[ I_0^+ = \int \frac{\gamma^-(A^2Z + B^2 - 1)}{(\gamma^2 - 1)^{3/2}} \, d\gamma \]
\[ I_9^+ = \int \frac{\gamma^-(A^2Z + B^2 - K_{II}^2 - 1)}{(\gamma^2 - 1)^2} \, d\gamma \]
\[ I_{10}^+ = \int \frac{\gamma^-(A^2Z + B^2 - 1)}{(\gamma^2 - 1)^2} \, d\gamma \]

All the above-mentioned integrals for positrons as well as electrons involve fractional powers of \( \gamma \). These integrals can be evaluated as follows. The integrands are first expanded by using binomial theorem. Each term of the binomial series is integrated. Then all the integrated terms pertaining to a particular integral are summed up. The series are terminated such that the magnitude of the next possible term has no significant effect on the final value. As the energy decreases the binomial series converges slowly. Therefore, around 50 keV kinetic energy of positrons or electrons these integrals are evaluated by summing over nearly one hundred terms. Thus the functions \( F_{II}^+(\gamma) \) and \( F_{II}^-(\gamma) \) have been computed for different absorbers with the help of VAX-11/780 electronic computer. These calculations have been done for several kinetic energies. The values of \( F_{II}^+(\gamma) \) and
Fig. 4.1 Values of $F_{II}(Y)$ for Aluminium vs Kinetic energy ($T_o$)
Fig. 4.2 Values of $F_{II}(V)$ for Copper vs. Kinetic energy ($T_0$)
Fig. 4.3 Values of $F_{ll}(Y)$ for Silver vs. kinetic energy ($T_0$)
Fig. 4.4 Values of $F_{ii}(Y)$ for $T_{in}$ vs kinetic energy ($T_0$)
Fig. 4.5 Values of $F_{II} (\gamma)\,\text{for Gold vs. kinetic energy} \,(T_0)$
Fig. 4.6 Values of $F_H(\gamma)$ for Lead vs kinetic energy ($T_0$)
R(\gamma) thus computed are plotted against kinetic energy of positron and electron for different absorbers. Some of these plots for positrons and electrons in the case of Aluminium, Copper, Silver, Tin, Gold and Lead are shown in figs. 4.1 to 4.6. It can be noticed from these figures that the difference between $R^+(\gamma)$ and $R^-(\gamma)$ increases with increasing value of $\gamma$. This is due to the third term in eqn (4.5) for Mott and Massey's elastic single scattering cross sections, because of which the difference between these cross sections increases with increasing Z-value.

Now one can find out in the following manner the critical kinetic energy ($T^\pm_0$) i.e. the kinetic energy left with the positron or electron at the instant when the straight motion of these particles comes to an end and the diffusion starts. Using condition given in eqn (4.1) for the situation of direct transition from straight motion to diffusion, in conjunction with eqn (4.7) one gets,

$$R^\pm_{II}(\gamma^\pm_0) = R^\pm_{II}(\gamma_0) + D^{-1}_2 \tag{4.10}$$

where $\gamma_0'$ and $\gamma_0''$ are the total incident and critical kinetic energies respectively, expressed in electron rest mass units. Values of $D^{-1}_2$ for different absorbers are listed in table 4.1. $R^\pm_{II}(\gamma^\pm_0)$ for a particular incident kinetic energy can be
Fig. 4.7  Estimated values of $T_0$ for positrons and electrons vs. kinetic energy ($T_0$) in the energy region 50 keV ≤ $T_0$ ≤ 350 keV.
calculated from eqn (4.10) by just adding $D_2^{-1}$ for that absorbers with the computed values of $F_{II}^+ (\gamma')$. Knowing thus $F_{II}^+ (\gamma')$, the value of critical energy ($T_0^+$) corresponding to a particular incident kinetic energy ($T_0$) can be extracted from the plot $F_{II}^+ (\gamma')$ vs kinetic energy of positron and electron for a particular material.

<table>
<thead>
<tr>
<th>Element</th>
<th>$D_2^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon</td>
<td>1.63017</td>
</tr>
<tr>
<td>Aluminium</td>
<td>0.73092</td>
</tr>
<tr>
<td>Copper</td>
<td>0.30302</td>
</tr>
<tr>
<td>Silver</td>
<td>0.17518</td>
</tr>
<tr>
<td>Tin</td>
<td>0.16836</td>
</tr>
<tr>
<td>Gold</td>
<td>0.09352</td>
</tr>
<tr>
<td>Lead</td>
<td>0.08940</td>
</tr>
</tbody>
</table>

The values of critical kinetic energy for positrons and electrons, $T_0^+$ and $T_0^-$ thus obtained are plotted against incident kinetic energy $T_0$, for different absorbers. These plots in the case of Aluminium, Copper, Tin and Lead are shown in fig 4.7. It can be noticed from this figure that with decreasing $T_0$ and increasing $Z$, the fraction
$T_c/T_0$ of energy left with the electron or positron increases rapidly. Also for any absorber $T_0^+$ is smaller than $T_0^-$ except when $T_0 < 100$ keV, when $T_0^+ \approx T_0^-$. Further the difference between $T_0^+$ and $T_0^-$ increases with increasing value of $T_0$.

Once the critical kinetic energy $T_0^{\pm}$ of a positron or electron for a particular incident kinetic energy becomes known for some absorber, the calculations of the part of range before and after diffusion can be done and their sum is equal to the practical range.

The straight distance traversed by a positron or electron vis. intrinsic depth ($s_D^{\pm}$) can be obtained in continuous slowing down approximation as,

$$s_D^{\pm} = R_{csda}^\pm (T_0) - R_{csda}^\pm (T_0)$$

where $R_{csda}^\pm (T_0)$ and $R_{csda}^\pm (T_0)$ are the 'csda' ranges of positrons and electrons for $T_0$, and $T_0^{\pm}$; these ranges can be obtained by using the Berger and Seltzer7) tables.

Then by assuming a uniform source of positrons and electrons at $s = s_D^{\pm}$, the age equation of diffusion theory vis.

$$\frac{\partial F}{\partial s} + \frac{1}{2} \lambda^2 F = 0$$

[4.12]

can be used to evaluate the distance traversed after onset of diffusion by a positron or electron along the direction
of incidence. Where \( F \) is the total density of positrons or electrons. As before\(^{1,2}\) the expression for mean square of the distance \( \left( \gamma_{av}^2 \right) \) traversed by these particles after the diffusion sets in can be obtained from eqn(4.12), viz.

\[
\left( \gamma_{av}^2 \right)^2 = 1.05 \int \frac{1}{\chi^2(\gamma)} \left( \frac{d\gamma}{ds} \right)^2 -1 \ d\gamma
\]

Using eqns (4.4) to (4.6), one can obtain the root mean square distance \( \gamma_{av}^+ \) \( \{ = \sqrt{\left( \gamma_{av}^2 \right)^2} \} \) from eq. (4.13). This distance will be same along any direction including direction of incidence as per assumption of uniform diffusion taken in the present approach.

The calculations of \( \left( \gamma_{av}^2 \right)^2 \) for a particular absorbers for different \( T_c^z = \left( \gamma_c^z -1 \right) m_e c^2 \) pertaining to different incident kinetic energies can be done from eqn (4.13) by numerical integration method. This has been done with the help of VAX-11/780® electronic computer by using standard Simpson one third rule programme. The upper limit of this integral is \( \gamma_c^z \) itself, while the lower limit is set slightly more than one viz 1.001, in the present case for such calculations. This has been done to overcome the difficulty of becoming the integrand of eqn (4.13), indeterminant at \( \gamma=1 \). The effect of setting the lower limit slightly higher than one has been assumed insignificant in evaluating \( \left( \gamma_{av}^2 \right)^2 \) for the energy region of our interest i.e.
50 keV ≤ T₀ ≤ 350 keV, for any absorber.

Having thus calculated both the parts vis. before and after diffusion sets in, the practical range \( R_p^\pm \), of these particles can be obtained as,

\[
R_p^\pm = s_p^\pm \sqrt{\left( \frac{\gamma_{av}}{\gamma_{av}} \right)^2}
\]  

As a sample we have done calculations for practical range of electrons of kinetic energy 159 keV in Aluminium. The value of \( F_{\Pi}^- (\gamma) \) at this kinetic energy has been computed as 5.97621. This value is then added with \( D_2^{-1} = 0.73092 \) from table 4.1. Thus the value of \( F_{\Pi}^- (\gamma_{av}) \) from eqn (4.10) is obtained as 6.70713. The critical energy \( T_c^- = 124.5 \) keV has been obtained from fig. 4.1, the plot of \( F_{\Pi}^- (\gamma) \) vs kinetic energy in case of Aluminium. The values of \( s_p^- \) and \( \gamma_{av}^- \) in this case have been computed as 13.69 mgm/cm² and 12.52 mgm/cm² respectively. Using eqn (4.14) the practical range of electron of 159 keV kinetic energy in Aluminium will then be equal to 26.21 mgm/cm².

4.22 Calculations in the Energy Region 30 keV ≤ T₀ ≤ 100 keV

The calculations of the practical range in the energy region 30 keV ≤ T₀ ≤ 100 keV can also be performed exactly in the same manner as described in section 4.21, except with the difference that now the empirical total stopping power relation given by eqn (3.2) is used instead of that given
by eqn (3.3). Therefore $F(\gamma)$ and $D'$ will be different from the previous region ($50 \text{ keV} \leq T_0 \leq 350 \text{ keV}$) and are denoted as $F_1(\gamma)$ and $D_1$ respectively, where,

$$D_1 = \frac{4\pi N}{Z} \frac{Z}{M_\gamma} \frac{c^2}{\rho (M_\gamma Z + C_1)}$$

and values of $D_1^{-1}$ for several absorbers are listed in Table 4.2.

<table>
<thead>
<tr>
<th>Absorber</th>
<th>$D_1^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon</td>
<td>1.52496</td>
</tr>
<tr>
<td>Aluminium</td>
<td>0.68596</td>
</tr>
<tr>
<td>Copper</td>
<td>0.29105</td>
</tr>
<tr>
<td>Silver</td>
<td>0.16746</td>
</tr>
<tr>
<td>Tin</td>
<td>0.15951</td>
</tr>
<tr>
<td>Gold</td>
<td>0.09008</td>
</tr>
<tr>
<td>Lead</td>
<td>0.08612</td>
</tr>
</tbody>
</table>

and where,

$$F_1^x(\gamma) = \left[ (I_1^x - I_{II}^x) \times \ln(2/\alpha^{1/3}) + \frac{1}{2}(I_{II}^x - I_{IV}^x) \right.$$

$$- \frac{1}{2}(I_V^x - I_{VII}^x) + \frac{2\alpha^2}{3}(I_{VII}^x - I_{VIII}^x) \pm \frac{\alpha^2Z^{1/3}}{2}$$

$$\times (I_{IX}^x - I_{X}^x) \right]$$

[4.16]
where $I_1, I_{II}, \ldots, I_{IX}, \ldots, I_{X}$ are different integrals

\[
I_I = \int \frac{\gamma}{(\gamma^2 - 1)^2} \, d\gamma \\
I_{II} = \int \frac{\gamma - (K_I^+ - 2)}{(\gamma^2 - 1)^2} \, d\gamma \\
I_{III} = \int \frac{\gamma \ln(\gamma^2 - 1)}{(\gamma^2 - 1)^2} \, d\gamma \\
I_{IV} = \int \frac{-(K_I^+ - 2) \ln(\gamma^2 - 1)}{(\gamma^2 - 1)^2} \, d\gamma \\
I_{V} = \int \frac{d\gamma}{\gamma(\gamma^2 - 1)} \\
I_{VI} = \int \frac{\gamma - K_I^+}{(\gamma^2 - 1)} \, d\gamma \\
I_{VII} = \int \frac{d\gamma}{(\gamma^2 - 1)^{3/2}} \\
I_{VIII} = \int \frac{\gamma - (K_I^+ - 1)}{(\gamma^2 - 1)^{3/2}} \, d\gamma \\
I_{IX} = \int \frac{d\gamma}{(\gamma^2 - 1)^{3/2}} \\
I_{X} = \int \frac{\gamma - (K_I^+ - 1)}{(\gamma^2 - 1)^{3/2}} \, d\gamma
Fig. 4.8 Values of $F_1(Y)$ for Carbon vs. kinetic energy ($T_o$)
Fig 4.9 Values of $F_1(Y)$ for Aluminium vs. kinetic energy ($T_0$)
Fig. 4.10 Values of $h_1$ for Lead vs kinetic energy ($T_0$).
Out of these integrals $I_I$, $I_{II}$, $I_V$, $I_{VII}$ and $I_{IX}$ are elementary integrals while $I^{a}_{II}$, $I^{a}_{IV}$, $I^{a}_{VI}$, $I^{a}_{VIII}$ and $I^{a}_{X}$ are the integrals involving fractional powers of $\gamma$. The later integrals can be solved in the same manner as described in section 4.21 except, now the number of terms in the binomial expansion are to be taken quite large, viz. for 30 keV we have taken two hundred terms into account.

As before, the function $F^a_I(\gamma)$ have been computed for several materials with the help of VAX-11/780 electronic computer. The values of $F^a_I(\gamma)$ are plotted against kinetic energy and some of these plots viz. for Carbon, Aluminium and Lead are shown in Figs. 4.8 to 4.10. The behaviour of these plots with respect to kinetic energy and $Z$ is same as that of the corresponding plots in the energy region 50 keV $\leq T_0 \leq 350$ keV. After extracting $r^+_0$ corresponding to an incident kinetic energy $T_0$ from these plots one can find out intrinsic depth $s^+_D$ in the manner described in sec. 4.21.

As before the value of $r^+_av$ for this energy region can be obtained from eqn (4.13) after substituting for $(\frac{d\gamma}{ds})^{-1}$ from eqn(3.2) for total stopping power in this case. Thus the practical range for this energy region can be obtained by using eqn (4.14).

4.23 Calculations for the Energy Region 0.35 MeV $\leq T_0 \leq 5.0$ MeV

Calculations of practical ranges for the energy region
Fig. 4.11 Values of $F(Y)$ for Carbon vs. kinetic energy $(T_o)$
Fig. 4.12 Values of $F(\gamma)$ for Aluminium vs. kinetic energy ($T_0$)
Fig. 4.13

Values of $F(y)$ for copper vs kinetic energy ($T_o$)

- $\bigcirc$ POSITRON
- $\bullet$ ELECTRON
Values of $F(\gamma)$ for Silver vs kinetic energy ($T_0$)

Fig. 4.16
Fig 4.15

Values of $F(Y)$ for Lead vs. kinetic energy ($T_0$)
Fig 4.16 Estimated values of $T_c$ for positrons vs. kinetic energy ($T_0$) in the energy region $0.03$ MeV $\leq T_0 \leq 6$ MeV.
Fig. 4.17 Estimated values of $T_{C}$ for electrons vs. kinetic energy ($T_0$) in the energy region $0.03 \text{ MeV} \leq T_0 \leq 5.0 \text{ MeV}$.
0.35 MeV < T_O < 5.0 MeV can be performed by using empirical total stopping power relation due to Batra and Sehgal (eqn 2.15) as input parameter for computation of F^±(γ), D, T_O, s_D^± and r_{AV}^±. Some of the plots of F^±(γ) vs kinetic energy for Carbon, Aluminium, Copper, Silver and Lead are shown in Figs. 4.11 to 4.15. The values of T_O^+ and T_O^- for different incident kinetic energies T_O, upto 5.0 MeV in case of these materials can be extracted from these plots. These values of T_O^+ and T_O^- are plotted against T_O and such plots for Carbon, Aluminium, Copper, Silver and Lead are shown in Figs. 4.16 and 4.17 respectively. These plots also include the values of T_O^+ and T_O^- corresponding to incident kinetic energies T_O, in the energy region described in section 4.21 and 4.22. Also the behaviour of T_O^+ and T_O^- with respect to T_O is similar as described in section 4.21. The intrinsic depth s_D^± can be obtained as described earlier in secs. 4.21 and 4.22, and the practical ranges can be computed by adding to it r_{AV}^± from eqn (4.13) after substituting for \( (\frac{d\gamma}{ds})^{-1} \) from eqn (2.15).

Thus from the simple method used in secs. 4.21 to 4.23 the calculation for practical ranges of positrons and electrons with kinetic energies between 0.35 MeV and 5.0 MeV can be performed in materials of any atomic number.
REFERENCES


