PUBLICATIONS

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T 247
Ranges of electrons

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Abstract. A simple method has been developed to evaluate theoretically the ranges of electrons in absorbers of any \(Z\). The calculations have been done by adding the intrinsic penetration depth as defined by Bethe with the remaining range after the diffusion sets in, using the age equation of diffusion theory. These ranges are calculated for electrons of energies up to 5.0 MeV in carbon, aluminium, copper, silver and lead absorbers. The agreement between the present calculated ranges and practical ranges was found to be very good for carbon and aluminium and satisfactory for copper, silver and lead.

1. Introduction

The knowledge of the ranges of electrons in matter is of great importance for those engaged in utilisation and measurements of electrons. Several empirical formulae (Katz and Penfold 1952, Tabata et al. 1972, and references cited in both) are listed in the literature to this effect, as the theoretical evaluation of these ranges becomes very difficult owing to simultaneous presence of multiple scattering (MS) with the slowing down process. The predominance of the straggling phenomenon near the end of electron ranges where the energy of electron is small, further makes this evaluation difficult.

So far no method has been developed to determine theoretically the practical ranges of the electrons in a satisfactory manner. Rohrlich and Carlson (1954) tried to determine these ranges in Al and Pb for electrons of energies up to \(\approx 2.0\) MeV. They calculated these ranges by adding the average penetration depth \(\langle z_d \rangle\) to the remaining range \(r_{av}\) after the diffusion sets in. Even by using the more realistic theory of electron penetration given by Lewis (1950), for determination of \(z_d\) in unison with the age diffusion theory for the determination of \(r_{av}\), disagreement was found between their calculated and practical ranges. This disagreement was present for Al as well as Pb, and their calculated ranges disagreed by as much as 50% with the corresponding practical ranges in the case of Pb.

Since the said disagreement was present for electrons of energies \(< m_e c^2\) for Al also, the inclusion of radiation losses (neglected by them) cannot be expected to improve matters. Also the inclusion of a correction amounting to a 20% increase in their \(z_d\) values (Metchnik and Tomlin 1963) due to the use of Lewis's theory in the context of a semi-infinite target would not provide the said agreement for Pb (since \(z_d < R_{\text{Pb}}\)). Rohrlich and Carlson (1954) themselves attributed the disagreement to the use of the age equation of diffusion theory in calculating remaining range.

In the present paper the intrinsic penetration depths \(S_D\) as defined by Bethe et al. (1938) have been used instead of \(z_d\). At these depths diffusion may be considered iso-
tropically uniform. The remaining range to be added to $S_D$ may then be calculated by using the age equation of diffusion theory. The present calculated ranges were found to be in better agreement with the practical ranges than those calculated by Rohrlich and Carlson (1954). The agreement was almost 100% for C and Al and 90% to 100% for Cu, Ag and Pb in the energy region of interest.

2. Theory

According to Bethe (1940) the electrons are supposed to travel linearly into the target up to a specified distance, the intrinsic penetration depth ($S_0$), and they then diffuse isotropically. This theory acknowledges the fact that owing to MS the electron progress eventually becomes random and therefore at the intrinsic penetration depth $S_0$, the average cosine of the angle between the actual direction of motion and direction of primary beam becomes $1/e$. This corresponds to a critical kinetic energy $T_0$ such that

$$\int_{\gamma_0}^{\gamma_1} \chi_1(\gamma) \frac{d\gamma}{\gamma} = 1 \quad (1)$$

where $\gamma$ is the total electron energy in its mass units, $\gamma_0$ and $\gamma_1$ are the quantities corresponding to initial and critical kinetic energies $T_0$ and $T_a$ respectively, and

$$\chi_1(\gamma) = 2\pi N \int_{\theta_m}^{\theta_M} \sigma_{\text{Mott}}(\theta, \gamma)(1 - \cos \theta) \sin \theta d\theta$$

where $N$ is the number of atoms cm$^{-3}$ of the scatterer,

$$\theta_m = \frac{aZ^{1/3}}{(y^2 - 1)^{1/2}}$$

is the non-zero lower limit of the integral due to use of Mott's single scattering cross-section $\sigma_{\text{Mott}}(\theta, \gamma)$ (Mott 1929, 1932) and $\theta_M = \pi$ is the upper limit of the integral. In the present case Mott's single scattering cross-section expansion up to the first two terms has been used, which is valid for low Z only, but it makes the determination of $S_0$ and $r_{sv}$ simple. The trend of error in $S_0$ and $r_{sv}$ due to use of $\sigma_{\text{Mott}}(\theta, \gamma)$ in this form is that sought quantitatively for medium and high Z absorbers by including higher-order terms in the $\sigma_{\text{Mott}}(\theta, \gamma)$ expansion. Substituting $\sigma_{\text{Mott}}(\theta, \gamma)$ in the formerly mentioned form and assuming sin $(\theta_m/2) = \theta_m/2$ owing to the very small value of $\theta_m$, one gets,

$$\chi_1(\gamma) = 4\pi N r^2 Z(Z + 1) \left[ \ln \left( \frac{2(\gamma - 1)^{1/3}}{aZ^{1/3}} \right) - \frac{\gamma^2 - 1}{2\gamma^2} - \frac{\alpha Z^{1/3}}{(y^2 - 1)^{1/2}} \right]$$

where $r_0$ is the classical radius of the electron and $Z(Z + 1)$ instead of the usual $Z^2$ takes account of the scattering of incident electrons by the orbital electrons also. Batra and Sehgal's (1970, 1973) total stopping power relations in continuous slowing down approximation (CSDA) are then introduced. These relations take account of radiation losses with the collision losses. From equations (1) and (3) using these relations one gets

$$F_1(\gamma_0) = F_1(\gamma_0) + D_1^{-1} \quad T_0 < 0.5 \text{ MeV} \quad (4a)$$

$$F_2(\gamma_0) = F_2(\gamma_0) + D_2^{-1} \quad 0.2 \text{ MeV} < T_0 < 5.0 \text{ MeV} \quad (4b)$$
Ranges of electrons

where

\[ D_1 = 4\pi N e^2 Z (Z + 1) m_e \rho (m_1 Z + C_1) \]  

\[ D_2 = 4\pi N e^2 Z (Z + 1) m_e \rho (m_2 Z + C_2). \]  

Here \( \rho \) is the density of scatteror, \( m_1, m_2, C_1 \) and \( C_2 \) are the constants listed in the papers of Batra and Sehgal (1970, 1973) and

\[ F_1(\gamma) = \left[ I_1 \ln \left( \frac{2}{\alpha Z^1/2} \right) + \frac{1}{2} (I_3 - I_9) + \frac{\pi \alpha Z}{2} (I_4 - \frac{\pi \alpha^2 Z^{4/3}}{2} I_8) \right] \]  

\[ F_2(\gamma) = \left[ (I_1' - I_3') \ln \left( \frac{2}{\alpha Z^{1/2}} \right) + \frac{1}{2} (I_3' - I_9) - \frac{1}{2} (I_3' - I_9) \right] \]  

\[ \frac{\pi \alpha Z}{2} (I_4' - I_4') - \frac{\pi \alpha^2 Z^{4/3}}{2} (I_8' - I_8') \]  

where \( I_1, I_3, \ldots, I_9, I_1', I_3', \ldots, I_9' \) are the integrals. Out of these \( I_9, I_1', I_3', \ldots, I_9' \) can be integrated directly and the integrand for rest of them can be expanded in the form of convergent series. In the present case these series are integrated up to 200 terms with the help of an IBM 1130.

Corresponding to an initial kinetic energy \( T_0 \), the critical kinetic energy \( T_c \) of the electron can then be estimated from the curves between \( F_1(\gamma_0) \) versus \( T_0 \) and \( F_2(\gamma_0) \) versus \( T_0 \). The intrinsic penetration depth \( S_0 \) can then be estimated from

\[ S_0 = \int_{\gamma_0}^{\gamma_e} \left( \frac{1}{\rho} \right)^{1/2} dy. \]  

The remaining range \( r_{av} \) after diffusion sets in can then be estimated from the age equation of diffusion theory used by Bethe et al (1938) and later by Rohrlich and Carlson (1954) e.g.

\[ r_{av} = 1.05 \int_1^{\gamma_e} \frac{1}{X(\gamma)} \left| \frac{d\gamma}{dy} \right| \]  

This expression can be integrated numerically by using Simpson’s rule with the help of the IBM 1130. The present calculated ranges of the electron are then \( R_{cal} = S_0 + \sqrt{r_{av}} \).

3. Results and discussion

The ratio of the present calculated ranges \( (R_{cal}) \) with the practical ranges \( (R_p) \) of electrons extracted from Tabata et al’s (1972) empirical formula are plotted (figure 1) against the kinetic energies of electrons up to 5.0 MeV, for C, Al, Cu, Ag and Pb. We estimate an error of 5% in our calculated values of ranges. Out of this 2% is due to graphical estimation of \( r_i \), and the remaining 3% is on account of the inherent errors involved in the Batra and Sehgal total stopping power relations. The latter error was found to be very large for medium and high Z absorbers below 0.25 MeV electron energy For example, at 0.1 MeV electron energy it was 6.8%, 7.3% and 15.0% for Cu, Ag and Pb respectively. Therefore, the comparison between the present calculated and the practical ranges has not been done below 0.25 MeV electron energies, in the case of these absorbers. The practical range values by Tabata et al (1972) include their respective
The effect of including higher order terms in the expansion of $\sigma_{\text{Mott}}$ on $S_D$ and $r_{\text{av}}$ is then sought quantitatively. This is estimated for a few selected kinetic energies of electrons to understand the trend of error to be introduced. It has been measured graphically in the same manner as estimated by Singh (1976) using Curr's (1955) expansion of $\sigma_{\text{Mott}}(\theta, \gamma)$ up to 5th power in $aZ$. The values of $S_D$ thus calculated for electrons of energies 0.5, 2.0 and 5.0 MeV respectively are 47.3, 361.0 and 1293.1 mg cm\(^2\) for Cu, 32.0, 247.7 and 962.6 mg cm\(^2\) for Ag and 20.5, 165.4 and 654.3 mg cm\(^2\) for Pb. The corresponding $S_D$ values with $\sigma_{\text{Mott}}(\theta, \gamma)$ expansion up to first two terms used in the present case are 47.5, 363.6 and 1306.0 mg cm\(^2\) for Cu, 32.2, 250.1 and 976.2 mg cm\(^2\) for Ag and 20.7, 167.8 and 667.1 mg cm\(^2\) for Pb. The values of $r_{\text{av}}$ for these energies after including higher order terms in $\sigma_{\text{Mott}}(\theta, \gamma)$ expansion are respectively 66.1, 363.0 and 838.4 mg cm\(^2\) for Cu, 62.0, 345.6 and 831.5 mg cm\(^2\) for Ag and 54.8, 307.2 and 756.9 mg cm\(^2\) for Pb. These values of $r_{\text{av}}$ are almost the same as those calculated in the present case. This seems to have occurred due to an approximate balance between the increase in $\gamma$, the upper limit of integration, and the decrease in integrand due to inclusion of higher order terms in $\sigma_{\text{Mott}}(\theta, \gamma)$ expansion in the equation (8). Thus the overall effect of including higher order terms on the present calculated ranges amounts to a decrease by less than 1% in the case of Pb, for electrons of energies 5.0 MeV for which this effect ought to be maximum. It is therefore neglected and $\sigma_{\text{Mott}}$ expansion used in the present case may be considered sufficient so far as the range calculations are concerned for the present region of electron energies. This also simplifies the calculations.

The agreement between the present calculated and practical ranges might be considered a direct consequence of using Bethe's theory for the present calculations, since Bethe's theory seems to be nearer to the experimental technique, where the transmitted
intensity is measured along, or in the very near vicinity of, the direction of incidence by using a very narrow (well collimated) incident beam of electrons.

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THEORETICAL RANGES OF POSITRONS IN METALS AND COMPOUNDS

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Practical ranges of positrons are calculated for several absorbers by adding the intrinsic depth as defined by Bethe with the remaining range after diffusion sets in, obtained by using age equation of diffusion theory. These calculations are performed for several metals, rare earths and compounds for several energies in the region 0.03 MeV to 5.0 MeV. In general, there is good agreement between the present calculated and experimentally obtained practical ranges.

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Theoretical Ranges of Positrons in Metals and Compounds

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The knowledge of practical ranges of positrons is of prime importance for evaluating the sensitive depth of a solid state detector to ensure its suitability for counting + particles of a particular energy. Unfortunately the experimental data /1/ to /4/ for positron ranges is very scarce. Theoretically Ebrahimi and Sehgal /5/ calculated ratio of 1.86 MeV positron-to 1.77 MeV electrons ranges following Wilson /6/ model, neglecting any contribution by the diffused particles towards their range. Recently, we have noticed /7/ for electrons that the remaining range after the diffusion sets in must be taken into account to obtain practical ranges of these particles.

Practical ranges (R_p^+) of positrons can be calculated similarly as we reported /7/ for electrons, viz. by adding the intrinsic depth (S_p^+) with their remaining range (R_e^+) after diffusion sets in, simply by using different elastic single scattering cross section /8/ and empirical total stopping power relation /9/ for positrons. Further, to calculate R_p^+ in the energy region 50 keV ≤ E ≤ 350 keV new empirical total stopping power relation /10/ suggested by us. The error in the present calculations due to factors discussed earlier /7/ has been estimated as 5%.

From tables I and II a good agreement can be noticed between present calculated (R_p^+) and the practical ranges (R_e^+) extracted from experimental /1/ and /2/ transmission curves. We have also noticed good agreement between the calculated ratio R_p^+/R_e^- and the experimental /3/ ratio R_e^+/R_e^- for 324 keV positrons and 312 keV electrons. Such agreement for several absorbers except that for rare earths and liquid compounds can be noticed from table III in the case of 1.86 MeV positrons and 1.77 MeV electrons. While discrepancy for rare earths may be due to effect of their different crystal structure, it remains unexplained in the case of liquids.

Table I: Comparison of present calculated (R_p^+) and practical /1/ ranges (R_e^+) of positrons in Al and Cu

<table>
<thead>
<tr>
<th>Kinetic Energy (KeV)</th>
<th>Practical ranges of positrons in Al (cm)</th>
<th>Practical ranges of positrons in Cu (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R_p^+ (cm^-1cm^-1)</td>
<td>R_e^+ (cm^-1cm^-1)</td>
</tr>
<tr>
<td>57.4</td>
<td>4.1±0.2</td>
<td>4.2±0.2</td>
</tr>
<tr>
<td>84.5</td>
<td>8.7±0.4</td>
<td>8.6±0.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.8±0.3</td>
</tr>
</tbody>
</table>
### Table II: Comparison of present calculated (R\textsuperscript{p}\textsuperscript{+}) and practical /2/ ranges (R\textsubscript{ex}\textsuperscript{+}) of positrons in Al, Ag, Sn, Au and Pb.

<table>
<thead>
<tr>
<th>Element</th>
<th>Kinetic Energy (keV)</th>
<th>Practical ranges of positrons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R\textsuperscript{p}\textsuperscript{+} )</td>
<td>( R\textsubscript{ex}\textsuperscript{+} )</td>
</tr>
<tr>
<td>Al</td>
<td>159</td>
<td>25.8±1.6</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>54.0±2.2</td>
</tr>
<tr>
<td></td>
<td>356</td>
<td>85.4±3.4</td>
</tr>
<tr>
<td>Ag</td>
<td>159</td>
<td>17.8±0.9</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>37.5±2.1</td>
</tr>
<tr>
<td></td>
<td>356</td>
<td>59.0±2.6</td>
</tr>
<tr>
<td>Sn</td>
<td>180</td>
<td>21.8±1.1</td>
</tr>
<tr>
<td></td>
<td>279</td>
<td>45.9±2.2</td>
</tr>
<tr>
<td>Au</td>
<td>293</td>
<td>40.9±2.0</td>
</tr>
<tr>
<td>Pb</td>
<td>199</td>
<td>16.5±0.8</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>32.7±1.6</td>
</tr>
<tr>
<td></td>
<td>356</td>
<td>51.7±2.6</td>
</tr>
</tbody>
</table>

### Table III: Comparison of present calculated ratio \( R\textsuperscript{p}\textsuperscript{+}/R\textsuperscript{p}\textsuperscript{-} \) with the experimental ratio /4/ \( R\textsubscript{ex}\textsuperscript{+}/R\textsubscript{ex}\textsuperscript{-} \) of 1.88 MeV positrons and 1.77 MeV electrons practical ranges.

<table>
<thead>
<tr>
<th>Material</th>
<th>( R\textsuperscript{p}\textsuperscript{+}/R\textsuperscript{p}\textsuperscript{-} )</th>
<th>( R\textsubscript{ex}\textsuperscript{+}/R\textsubscript{ex}\textsuperscript{-} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al</td>
<td>1.11±0.08 1.12±0.06</td>
<td>Toluene 1.09±0.09 1.89±0.19</td>
</tr>
<tr>
<td>Cu</td>
<td>1.17±0.08 1.19±0.06</td>
<td>Water 1.09±0.08 1.78±0.16</td>
</tr>
<tr>
<td>Sn</td>
<td>1.22±0.09 1.29±0.07</td>
<td>Benzene 1.08±0.08 1.72±0.17</td>
</tr>
<tr>
<td>Pb</td>
<td>1.47±0.10 1.55±0.07</td>
<td>Plasken 1.08±0.08 0.98±0.10</td>
</tr>
<tr>
<td>T</td>
<td>1.19±0.08 1.44±0.04</td>
<td>Lumarith 1.08±0.08 1.17±0.12</td>
</tr>
<tr>
<td>Nd</td>
<td>1.28±0.09 1.67±0.03</td>
<td>Teflon 1.10±0.06 1.42±0.11</td>
</tr>
<tr>
<td>Ho</td>
<td>1.30±0.10 1.89±0.06</td>
<td>Tb 1.31±0.09 1.47±0.04</td>
</tr>
</tbody>
</table>

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EMPIRICAL TOTAL STOPPING POWER RELATIONS
FOR LOW ENERGY ELECTRONS AND POSITRONS

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ABSTRACT

Some simple convenient empirical relations for total stopping power of electrons and positrons of kinetic energies from 30 keV to 350 keV in materials of all atomic numbers have been developed. The deviation in the total stopping power values calculated by using these relations has been found generally upto 5.0% from the corresponding continuous slowing down approximation data in case of elements and compounds.

INTRODUCTION

The exact knowledge of total stopping power of electrons and positrons is required to understand their interaction mechanism with matter. Thus their knowledge is of great practical interest for several applications in nuclear physics, semiconductor detector fabrication, radiation technology and other related fields. The complicated nature of the expression for total stopping power hinder the development of a realistic

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transport theory. To get rid of this difficulty, Batra and Sehgal\textsuperscript{2-4} found analytical empirical relations for total stopping power of electrons and positrons with kinetic energies up to 5.0 MeV valid for all materials. However, these relations\textsuperscript{2-4} are not accurate below 100 keV as the results obtained from these are off from the tabulated theoretical values\textsuperscript{1}. For example, the disagreement between the theoretical and empirical total stopping values in the case of 50 keV electrons varies from 6.0\% to 17.0\% as the atomic number increases from 13 to 82. The said discrepancy increases further for lower energies and for higher values of atomic number. The same also holds good in the case of positrons.

Therefore, it is highly desirable to look for convenient empirical relations which can also be used to find total stopping power of electrons and positrons, in the energy region below 100 keV for materials of all 2. Such relations would further be of great use for the following purposes.

1. For analytical calculations of 'cada' ranges and their difference for electrons and positrons in the energy region below 100 keV.

2. For simple calculations of practical ranges of these particles in different materials for the said energy region\textsuperscript{5}.

3. For more accurate calculations of practical ranges at higher energies (upto 5.0 MeV) by using diffusion model\textsuperscript{6,7}. This
is because evaluation of the part of range contributed by
diffused electrons or positrons requires a total stopping
power law to hold good, down to nearly zero kinetic energy.

NEW LOW ENERGY EMPIRICAL RELATIONS

We have been able to develop analytically convenient
empirical relations for total stopping power of electrons and
positrons in two regions of energies up to 350 keV. The present
relations are applicable for materials of all Z through a linear
Z dependence in three different regions of atomic number Z.
These relations are given as,

\[- \frac{1}{\rho} \frac{d\mathcal{E}}{ds}_{_{\text{Tot}}} = (N_1 \cdot z + C_1) (\gamma^K - \gamma^K_{\gamma} = 1) \]

\[\text{... (1)}\]

and

\[- \frac{1}{\rho} \frac{d\mathcal{E}}{ds}_{_{\text{Tot}}} = (N_2 \cdot z + C_2) (\gamma^{A+Z+B} + \gamma^{B^+} = 1) \]

\[\text{... (2)}\]

where \(\rho\) denotes the density of the material and \(\gamma\) represents
the total energy of \(e^+\) or \(e^-\) in units of electron rest
mass, and the superscripts (+) and (-) are used for positrons
and electrons respectively. The empirical relation given in
eq. (1) is valid for kinetic energies \((T_0)\) from 30 keV to 100
keV for all \(Z\) values. However, for \(Z \geq 79\), eq. (1) becomes
valid up to an enhanced upper limit of 150 keV. Relation given
by eq. (2) is valid for kinetic energies from 50 keV to 350 keV
for all materials.
The values of constants $K_1^+$, $K_2^+$, $A^+$ and $B^+$ occurring in eqs. (1) and (2) are given below:

\begin{align*}
K_1^+ &= 4.8 \\
K_2^+ &= 3.7 \\
A^+ &= 0.00393 \\
B^+ &= 3.70852
\end{align*}

\begin{align*}
K_1^- &= 5.1 \\
K_2^- &= 3.9 \\
A^- &= 0.0044 \\
B^- &= 3.9324
\end{align*}

The constants $K_1$, $C_1$ and $K_2$, $C_2$ for materials of low, medium and high atomic numbers are listed in tables 1 and 2 respectively. Further, for $Z \leq 10$, eqs. (1) and (2) are to be multiplied\(^3\) by a factor $22/A$, where 'A' is the atomic weight of the material.

RESULTS AND DISCUSSION

To compare the present calculated values of total stopping power of electrons from eqs. (1) and (2) with the corresponding theoretical values\(^1\), plots (solid curves) of their percentage difference 'D' vs. kinetic energy ($E_0$) for some representative cases, vis. $C$, $Al$, $Cu$, $Ag$ and $Au$ has been given in Figs. 1 to 5. In order to make such comparison in the case of positrons, the following procedure has been adopted. In the first instance, values of collision stopping power of positrons have been obtained using tables 9 and 1 of Ref\(^1\). Due to lack of experimental data for bremsstrahlung losses these values of collision stopping power of positrons for several energies from 10 keV to 500 keV and for the values of 'Z' listed in table 9 of Ref\(^1\) are added.
with the same percentage of bremsstrahlung losses as those tabulated\(^1\) for corresponding energy of electrons. These sums may be treated approximately as the theoretical values of total stopping power of positrons. The values thus obtained are plotted against kinetic energies. Theoretical total stopping power of positrons for any energy between 10 keV and 500 keV may then be obtained from these plots. To compare the present empirical values with these theoretical values their percentage difference 'D' has also been plotted (dashed curves) in the same Figs. (from 1 to 5). It can be noticed that the empirical values of total stopping power of electrons as well as positrons are in agreement with the corresponding theoretical values within 5.0\(^{2}\).

It can further be noticed that the percentage difference 'D' between the empirical and theoretical values of total stopping power of positrons and electrons is in the same direction. Therefore, the difference between empirical values of total stopping power of these particles would nearly be the same as their actual difference. Therefore the present empirical relation can also be used to study the positron-electron difference in their total stopping power.

Using Bragg's additivity theorem\(^8,9\) in conjunction with the present empirical relations we have calculated total stopping power of electrons in several compounds, viz. methane, carbon dioxide, ethylene, polyethylene, xylene, toluene, stilbene, polystyrene, lucite, water, etc. Comparison of present calculated values for some of the above mentioned compounds, with the corresponding theoretical\(^1\) total stopping power values has been
shown in tables 3 and 4. It can be noticed that the two are also in agreement in general, within 5.0%.

We have also calculated the total stopping power of electrons in compounds by another approach. This has been done directly from eqs. (1) and (2) using effective values of atomic number ($Z_{\text{eff}}$) and atomic weights ($A_{\text{eff}}$), given as

$$Z_{\text{eff}} = \frac{\sum n_i Z_i}{\sum n_i} \quad \text{and} \quad A_{\text{eff}} = \frac{\sum n_i A_i}{\sum n_i}$$

where $n_i$ denotes the number of atoms per molecule of the $i$th kind of constituent. The values of total stopping power calculated by using this approach were also found almost the same as those obtained from the former method (using additivity theorem). From this we conclude that both of these approaches are valid for total stopping power of electrons in compounds, at least in the energy region of present interest ($30 \text{ keV} \leq T_0 \leq 350 \text{ keV}$).

Similar calculations have been performed for positrons in compounds, using any one of the two approaches mentioned for electrons. The calculated values for the compounds having their constituents listed in table 9 of Ref. can be compared with the approximate theoretical values obtained by using additivity theorem in conjunction with tables 9 and 1 of Ref. The present calculated values for positrons in several compounds have also been found in agreement with the theoretical values within 5.0%. However, such comparison cannot be made by using additivity theorem for compounds having oxygen and other elements as one of
the constituent, not given in table 9 of Ref¹. This is because the extrapolation or interpolation for these values of Z would introduce an inaccuracy in theoretical values of total stopping power of positrons, rendering them of little use for comparison with the present empirical values.

It is, therefore, concluded that the present empirical relations are able to provide a good account of the total stopping power of electrons and positrons in case of elements as well as compounds, even for energies as low as 30 keV. The computational labour involved in the present approach is highly reduced as compared to numerical method.

ACKNOWLEDGEMENT

The authors are thankful to Prof. M. Z. Rehman Khan, Chairman, Department of Physics, ANU, Aligarh for his interest in the present work. One of the authors, SKR is also thankful to Mr. R. P. Gautam and Mr. D. C. Tomar for assistance in computational work.
References

Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5
### Table 1

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**CAPTIONS FOR FIGURES**

**Fig. 1.** Percentage difference 'D' between the theoretical and empirical values of total stopping power vs kinetic energy ($T_o$) of the positrons (Dashed curves) and electrons (Solid curves) for Carbon and Aluminium in the energy region $30 \text{ keV} \leq T_o \leq 100 \text{ keV}$.

**Fig. 2.** Percentage difference 'D' between the theoretical and empirical values of total stopping power vs kinetic energy ($T_o$) of the positrons (Dashed curves) and electrons (Solid curves) for Copper and Silver in the energy region $30 \text{ keV} \leq T_o \leq 100 \text{ keV}$.

**Fig. 3.** Percentage difference 'D' between the theoretical and empirical values of total stopping power vs kinetic energy ($T_o$) of the positrons (Dashed curves) and electrons (Solid curves) for Gold in the energy region $30 \text{ keV} \leq T_o \leq 150 \text{ keV}$.

**Fig. 4.** Percentage difference 'D' between the theoretical and empirical values of total stopping power vs kinetic energy ($T_o$) of the positrons (Dashed curves) and electrons (Solid curves) for Aluminium and Copper in the energy region $50 \text{ keV} \leq T_o \leq 350 \text{ keV}$.

**Fig. 5.** Percentage difference 'D' between the theoretical and empirical values of total stopping power vs kinetic energy ($T_o$) of the positrons (Dashed curves) and electrons (Solid curves) for Silver and Gold in the energy region $50 \text{ keV} \leq T_o \leq 350 \text{ keV}$.
### CAPTIONS FOR TABLES

**Table 1**  
Numerical values of constants $M_1$ and $C_1$.

**Table 2**  
Numerical values of constants $M_2$ and $C_2$.

**Table 3**  
Comparison of Empirical and Theoretical values of Total Stopping Power of Electrons in some Compounds ($30 \text{ keV} \leq T_0 \leq 100 \text{ keV}$).

**Table 4**  
Comparison of Empirical and Theoretical values of Total Stopping Power of Electrons in some Compounds ($90 \text{ keV} \leq T_0 \leq 350 \text{ keV}$).
P-wave neutron strength functions

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Abstract. P-wave neutron strength functions for 20 nuclei have been extracted from the average radiative capture cross sections \( \sigma(n, \gamma) \) at 24 keV, using the latest available values of \( \langle \Gamma_{\gamma o} \rangle, \langle D_0 \rangle, \langle \Gamma_{\gamma p} \rangle \) and \( \langle D_1 \rangle \) in contrast to the earlier analysis where certain assumptions were made about the P wave resonance parameters. Our values for the P wave strength functions are compared on the one hand with previous experimental 'local' values and on the other hand with the theoretical predictions of the optical model of Buck and Perey.

NUCLEAR REACTIONS S- and P wave resonance parameters, \( \sigma(n, \gamma) \), \( E_a = 24 \text{ keV} \), Bilpuch expression: deduced P wave neutron strength functions.

1. Introduction

Neutron strength functions, originally introduced as 'sticking probabilities' (Bethe 1937), are defined as the ratio of the average reduced neutron width to the average level spacing. They are recognised to be properties of the nuclear surface, and are a measure of the probability of formation of a compound nucleus. The data on neutron strength functions are due either to the analysis of average capture cross sections or of neutron resonance parameters.

P-wave neutron strength functions obtained so far (Gibbon et al 1961, Chaubey and Sehgal 1965, 1966, Bilpuch et al 1960, Murty et al 1973, Siddappa et al 1974) from the analysis of average capture cross sections are dependent on certain assumptions made about the P-wave gamma-ray strength function and the P-wave neutron reduced width distribution in the absence of experimental data on P-wave resonance parameters. S- and P-wave gamma-ray strength functions had been assumed to be equal (Gibbon et al 1961, Chaubey and Sehgal 1965, 1966, Bilpuch et al 1960), i.e. \( \langle \Gamma_{\gamma s} \rangle/\langle D_1 \rangle = \langle \Gamma_{\gamma p} \rangle/\langle D_0 \rangle \). Murty et al (1973) and Siddappa et al (1974) used \( \langle \Gamma_{\gamma p} \rangle/\langle D_1 \rangle = 3(\langle \Gamma_{\gamma s} \rangle/\langle D_0 \rangle) \) by assuming that \( \langle \Gamma_{\gamma s} \rangle = \langle \Gamma_{\gamma p} \rangle \) and \( \langle D_1 \rangle = \langle D_0 \rangle \). It is also worth pointing out that previous workers had tacitly assumed the single-channel Porter–Thomas distribution of S- and P-wave neutron reduced widths while extracting P-wave strength functions from average capture cross sections within the framework of the Bilpuch formula. These assumptions may lead to large inconsistencies in the resulting P-wave neutron strength functions. On the other hand, P-wave neutron strength functions deduced from resonance parameters represent 'local' values obtained from resonances within a limited energy interval. It has recently been pointed out (Weighmann et al 1979) that a short-range energy dependence of strength functions is not uncommon. It is, therefore, desirable to extract P-wave neutron strength functions.
strength functions from average capture cross sections using actual P wave resonance parameters.

2. Method of extraction of P-wave neutron strength functions

Using the Breit-Wigner theory for nuclear resonances, an expression for the average capture cross section was derived (Bilpuch et al 1960) as a sum of partial cross sections for different angular momenta $l$ of the incident neutron:

$$
\sigma_\text{av} = \frac{1}{2} \pi (2l + 1) \frac{2 \times 10^6 \langle I_{\ell} \rangle}{E_n} \frac{F_\text{av}}{\langle D_\ell \rangle} \left[ 1 - \sqrt{b - \langle I_{\ell} \rangle/2v_0} \left( 1 - \frac{2}{\sqrt{\pi} v_0} e^{-\frac{v_0^2}{4}} \right) \right]
$$

(1)

where $b - \langle I_{\ell} \rangle/2v_0 = 1$ and $v_0 = k^2 R^2/(1 + k^2 R^2)$; here $R$ is the radius of the target nucleus and $k = 1/\lambda$, where $\lambda$ is the Dirac wavelength. The energy $E_n$ of the neutron is in units of eV, $S_\ell$ is the neutron strength function, $\langle D_\ell \rangle$ is the average level spacing and $\langle I_{\ell} \rangle$ is the average radiation width for a particular orbital angular momentum $l$.

The S wave resonance parameters $\langle I_{\ell=0} \rangle$, $\langle D_0 \rangle$ and $S_0$ are used to compute S wave neutron contributions to average radiative capture cross sections $\sigma_\text{av}^S$ using expression (1). They are then subtracted from the corresponding total average radiative cross sections $\sigma(n, y)$ to yield P-wave contributions to the latter (in accordance with the tacit assumption that only S and P-wave neutrons contribute to $\sigma(n, y)$ at neutron energies $E_n \sim 24$ keV). The P-wave neutron strength functions are then extracted from the same expression (1) using the experimental values of $\langle I_{\ell=0} \rangle$ and $\langle D_0 \rangle$.

The errors assigned to these P-wave neutron strength function values, listed in the next section, are the compounded errors due to the error present in the average radiative cross section values on the one hand and the errors in each of the quantities $\langle I_{\ell=0} \rangle$, $\langle D_0 \rangle$, $S_0$, $\langle I_{\ell=1} \rangle$ and $\langle D_1 \rangle$ reported in literature on the other hand.

3. Results and discussion

The results of the present analysis are summarised in table 1. In the first column the nuclei for which P-wave strength functions have been extracted are listed. Columns 2-4 give the experimental S wave resonance parameters while column 5 gives the S wave contributions to the average radiative capture cross sections. Columns 6 and 7 incorporate the P wave resonance parameters $\langle I_{\ell=1} \rangle$ and $\langle D_1 \rangle$. Column 8 presents the experimentally measured average neutron radiative cross sections $\sigma(n, y)$, while column 9 shows the P wave contributions to $\sigma(n, y)$. The values of our present calculated P wave neutron strength functions are shown in column 10, with their corresponding literature values (local) in column 11. It may be noticed from columns 5 and 9 that P wave contributions to $\sigma(n, y)$ are either larger or comparable with the corresponding S wave contributions, thereby justifying the subtraction technique employed in the extraction of P wave strength functions.

Figure 1 gives a comparison of our present P wave neutron strength functions (represented by $\Delta$) with the experimental local values (references a-i, shown as O for $^4$Ca, $^{52}$Cr, $^{54}$Fe, $^{94}$Mo, $^{96}$Mo, $^{160}$Ce and $^{205}$Tl) on the one hand and the theoretically predicted values of Buck and Perey (1962; full curve) on the other hand.

It is clear from table 1 that for $^{44}$Ca, $^{52}$Cr, $^{54}$Fe, $^{94}$Mo, $^{96}$Mo, $^{160}$Ce and $^{205}$Tl our present $S_1$ values are in reasonable agreement with the values reported earlier. The 'local' P wave
### Table 1

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<th>(\langle D_1\rangle) (keV) (6)</th>
<th>(\langle T_\nu\rangle) (meV) (7)</th>
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\(a2, n\) \(\sigma_2^e\) \(S_i \times 10^4\) \(S_i \times 10^4\) \(S_i \times 10^4\)

*Musgrove et al. (1977), †BNL-325 (1973), ‡Siegelz et al. (1971), §Allen et al. (1971); Beer and Spencer (1975), Allen et al. (1976), †BNL-325 (1976), ‡Boldeman et al. (1976b), §Jonson (1977), §Camarda (1977), †Boldeman et al. (1975), ‡Boldeman et al. (1976a), *Musgrove et al. (1976), *Musgrove et al. (1975), *Murty et al. (1974), ‡Camarda (1978), §Bird et al. (1976); Liou et al. (1975)
neutron strength function values are larger than our calculated values for $^{88}\text{Sr}$, $^{90}\text{Y}$, $^{90}\text{Zr}$, $^{94}\text{Zr}$, $^{93}\text{Nb}$, $^{95}\text{Mo}$, $^{96}\text{Mo}$, $^{97}\text{Mo}$, $^{125}\text{Ba}$ and $^{139}\text{La}$. This tendency reveals some energy dependence of the $S_1(E)$ values. However, to draw a definite conclusion about the energy dependence of $S_1(E)$, much more data are required for analysis.

It is obvious from figure 1 that the present $P$ wave neutron strength function values lie, in general, below theoretical optical model predictions (Buck and Perey 1962). However, they reproduce the maximum around $A = 94$. The optical model parameters employed in these calculations are $V_0 = 5.5 \text{ MeV}$, $V_\infty = 8 \text{ MeV}$, $W_D = 5.4 \text{ MeV}$, $r_p = 1.35 \text{ fm}$, $a = 0.65 \text{ fm}$ and $b = 0.47 \text{ fm}$.

Finally, we would like to point out that the values of $S_1$ obtained from the analysis of average neutron capture cross sections within the framework of the Bilpuch formula are prone to uncertainty in the absence of accurate $P$ wave resonance parameters. For example, the value of $S_1$ for $^{108}\text{Mo}$ calculated from $\sigma^2 = 114 \pm 20 \text{ mb}$ (Murty et al. 1973) by using $P$ wave resonance parameters comes out to be $1.13 \pm 0.08$. This value is about 30% smaller than that reported by Murty et al. (1973), which incorporated certain assumptions regarding $\langle \Gamma_{\text{in}} \rangle$ and $\langle D_4 \rangle$. It is, therefore, necessary to have much more accurate information about $\langle \Gamma_{\text{in}} \rangle$, $\langle D_4 \rangle$ and $\sigma(n, \gamma)$ over a range of a few tens of keV neutron energy for a larger number of nuclei to obtain more reliable data on $P$ wave neutron strength functions.
Acknowledgments

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CALCULATIONS OF NEUTRON CAPTURE CROSS-SECTIONS FOR SOME NUCLEI USING BILPUCH FORMULA
S. K. RATHI*, V. P. VARSHNEY**, R. N. AGRAWAL***

ABSTRACT

Neutron capture cross-sections $\sigma(n,\gamma)$ at 24 keV are calculated for twenty four nuclei by using Bilpuch formula. The latest available values of resonance parameters $<\Gamma_{\gamma 0}>$, $<D_0>$ and $\beta_0$, i.e. the average gamma level width, average level spacing and strength function respectively for $S$-wave and the corresponding quantities $<\Gamma_{\gamma 1}>$, $<D_1>$ and $\beta_1$ for $P$-wave are used to this effect. These calculations are also done by employing some earlier assumptions made about $P$-wave resonance parameters in literature. It is found that the agreement between the theoretical and experimental neutron capture cross-sections for the cases enumerated is quite satisfactory when actual $P$-wave resonance parameters are used. In the absence of these parameters the assumption $<\Gamma_{\gamma 1}>/<D_1>$ $\approx <\Gamma_{\gamma 0}>/<D_0>$ seems to be reasonably good in predicting $\sigma(n,\gamma)$ in the few tens of keV energy region.

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Short Running Title :
1. Introduction

Neutron capture cross-sections data in the keV energy region are useful in the design of fast reactors as well as in the study of cosmological theory of element formation in the universe. To understand clearly the element formation, neutron capture cross-section data are required in the few tens of keV energy region. Unfortunately, capture cross-sections in the keV region are known at isolated energies; and these are also not known for all isotopes because most of these cross-sections have been measured using activation technique, which is based on the determination of the activity measured in the sample after neutron bombardment and is, therefore, limited to nuclides which leave a suitably active residual isotope. In order to understand nucleosynthesis theory, Allen et al. used empirical expressions for finding the neutron capture cross-sections at 30 keV where they were not known experimentally. For this reason, the need may arise for calculating the cross-sections in those cases when they are not known from experiment.

In this energy region the neutron capture cross-sections can be calculated on the basis of Bilpuch formalism. The only problem in the calculation of average capture cross-section is the number and uncertainty of the parameters involved in the Bilpuch expression. While \( \langle \gamma N_0 \rangle \), \( \langle E_0 \rangle \) and \( E_0 \) are often well known from low energy resonance parameters, \( \langle \gamma N_1 \rangle \), \( \langle E_1 \rangle \) are only poorly known and, as a result, calculated
cross-sections are subject to large errors.

Because of the lack of $P$-wave resonance data, previous workers\textsuperscript{4-8} made certain assumptions about the $P$-wave resonance parameters in order to extract out $P$-wave neutron strength functions from average radiative capture cross-sections within the framework of Bilpuch formula. So far no attempt has been made to calculate $\sigma(n,\gamma)$ on the basis of Bilpuch formula in the few tens of keV region by employing actual $P$-wave resonance parameters ($\langle \Gamma_{\gamma_1} \rangle \cdot \langle D_4 \rangle \cdot \langle D_3 \rangle$). The present work is motivated to show that the Bilpuch formula (an extension of Breit–Wigner single level expression) is fairly good to provide experimental $(n,\gamma)$ cross-sections in few tens of keV region provided $S$ and $P$-wave neutron resonance parameters are known. In the absence of these $P$-wave resonance parameters the assumption $\langle \Gamma_{\gamma_1} \rangle / \langle D_4 \rangle \approx \langle \Gamma_{\gamma_0} \rangle / \langle D_3 \rangle$ seems to be reasonably good in predicting $\sigma(n,\gamma)$ in those cases for which there are no experimental data in the few tens of keV region.

2. Calculations Based on Bilpuch Formula

Using the Breit–Wigner theory for nuclear resonance, an expression for the average capture cross-section was derived by Bilpuch\textsuperscript{4} as a sum of partial cross-sections for different angular momenta of the incident neutron:

$$\frac{1}{c} = \left\{ 1.3 \times 10^6 \frac{(2\lambda+1)/B_{\lambda}}{\langle D_3 \rangle} \right\} \left( \frac{\langle \Gamma_{\gamma_1} \rangle / \langle D_3 \rangle}{\langle \Gamma_{\gamma_0} \rangle / \langle D_3 \rangle} \right) \left[ 1 - (\gamma D) \right]^{\gamma_2} \left[ 1 - 2 \gamma_2 \int_0^B \exp\left(-B^2 d\lambda\right) \exp(B) \right]$$
where \( I = \langle r_{y1} \rangle / 2 x_{1} \cdot B_{2} \cdot E_{2} \cdot V_{2} / \Lambda \), \( V_0 = 1 \) and \( \Lambda = \frac{x_{1}^{2}}{(1+x_{1}^{2})} \); here \( \Lambda \) is the radius of the target nucleus and \( \chi = 1 / \hbar \), where \( \hbar \) is the Dirac wavelength. The energy \( E_{2} \) of the neutron is in units of eV. \( B_{2} \) is the neutron strength function, \( \langle B_{1} \rangle \) is the average level spacing and \( \langle \gamma_{y1} \rangle \) is the average radiation width for a particular orbital angular momentum \( l \).

The S-wave contribution to capture cross-section (\( \sigma_{2} \)) has been calculated using S-wave resonance parameters \( (\langle \gamma_{y0} \rangle, \langle B_{0} \rangle, \langle L_{0} \rangle) \) available in the literature\(^{8,23} \). The P-wave contribution has been calculated following three different approaches:

a. \( \langle \gamma_{y1} \rangle / \langle B_{1} \rangle \) is same\(^{4-6} \) for S and P-wave neutrons, \[ \text{i.e. } \langle \gamma_{y1} \rangle / \langle B_{1} \rangle = \langle \gamma_{y0} \rangle / \langle B_{0} \rangle \] [denoted as \( \sigma_{2} (1) \)].

b. \( \langle \gamma_{y1} \rangle / \langle B_{1} \rangle \) is \( (2J + 1) \) times\(^{7,8} \) of that for the S-wave neutrons, assuming that \( \langle \gamma_{y1} \rangle \approx \langle \gamma_{y0} \rangle \) and \( \langle B_{1} \rangle \approx \langle B_{0} \rangle / 3 \) [denoted as \( \sigma_{2} (2) \)].

c. P-wave resonance parameters \( \langle \gamma_{y1} \rangle, \langle B_{1} \rangle \) and \( L_{1} \) are taken from literature [denoted as \( \sigma_{2} (3) \)].

The P-wave contributions were added to S-wave contributions to capture cross-section to obtain total capture cross-sections in accordance with the tacit assumption that only S and P-wave neutrons contribute to \( \sigma (n, \gamma) \) at neutron energy \( \approx 24 \text{ keV} \).

\( \sigma_{1}, \sigma_{2} \) and \( \sigma_{3} \) are defined as \( \sigma_{2} + \sigma_{2} (1), \sigma_{2} + \sigma_{2} (2) \) and \( \sigma_{2} + \sigma_{2} (3) \), respectively and their ratio with experimentally measured cross-sections (\( \sigma_{\text{exp}} \)) are plotted against atomic
mass number $A$ of the target nuclei in Fig. 1.

3. Results and Discussion

The present calculations are limited to the nuclei for which the $P$-wave resonance parameters are available in the literature. The input parameters ($\langle \Gamma_{y_0} \rangle$, $\langle D_0 \rangle$, $S_0$, $\langle \Gamma_{y_1} \rangle$, $\langle D_1 \rangle$, and $S_1$) for these nuclei required for calculations are listed in Tab. 1. The experimentally measured average cross-sections ($\sigma_{\text{exp}}$) are also given. The $S$-wave contribution to the capture cross-section ($\sigma_S$) and the $P$-wave contribution to it $\sigma_P(1)$, $\sigma_P(2)$ and $\sigma_P(3)$ following approaches (a), (b) and (c) respectively, are calculated by using these input parameters and are included in Table 1.

The behaviour of $\sigma_1/\sigma_{\text{exp}}$, $\sigma_2/\sigma_{\text{exp}}$, and $\sigma_3/\sigma_{\text{exp}}$ versus the atomic mass number of the target nuclei has been illustrated in Fig. 1.

It is evident from the data on points corresponding to ratio $\sigma_3/\sigma_{\text{exp}}$ in Fig. 1 that the Bilpuch expression reproduces experimental values of capture cross-sections fairly well for 80% of these nuclei, when actual $P$-wave resonance parameters are employed. It can also be noticed from data on points corresponding to ratios $\sigma_1/\sigma_{\text{exp}}$ and $\sigma_2/\sigma_{\text{exp}}$ that the reproduction of experimental values is possible only for $\approx 50\%$ of these nuclei, when the assumption either $\langle \Gamma_{y_1} \rangle / \langle D_1 \rangle = \langle \Gamma_{y_0} \rangle / \langle D_0 \rangle$ or $\langle \Gamma_{y_1} \rangle / \langle D_1 \rangle = 3 \langle \Gamma_{y_0} \rangle / \langle D_0 \rangle$ is taken into account. It can further be noticed from Fig. 1 that the
scattering of points corresponding to $\sigma_2 / \sigma_{\text{exp}}$ is larger than those for $\sigma_1 / \sigma_{\text{exp}}$ around the unit values of the ratio. This justifies the assumption $\langle \Gamma'_{Y_1} \rangle / \langle \mathcal{B}_1 \rangle \approx \langle \Gamma'_{Y_0} \rangle / \langle \mathcal{B}_0 \rangle$ in predicting unknown $\sigma(n, \gamma)$ in the few tens of keV region. Finally, we would like to point out that in the absence of P-wave data, the assumption $\langle \Gamma'_{Y_1} \rangle / \langle \mathcal{B}_1 \rangle = \langle \Gamma'_{Y_0} \rangle / \langle \mathcal{B}_0 \rangle$, which has been verified experimentally for several nuclei, is appropriate in predicting the experimental cross-sections in the few tens of keV region.

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a) Ref. 9  b) Ref. 10  c) Ref. 12  d) Ref. 20  e) Ref. 3  f) Ref. 13  g) Ref. 25  h) Ref. 14  i) Ref. 15  j) Ref. 16  k) Ref. 17  l) Ref. 18  m) Ref. 19  n) Ref. 11  o) Ref. 21  p) Ref. 8  q) Ref. 22  r) Ref. 24  s) Ref. 23