CHAPTER-IV

POLARIZATION

In the preceding chapter it was shown that the nuclear quadrupole moment interacts with the field gradient at the nucleus and it leads to the admixture of m-projection quantum number states. The Mössbauer radiation measurements of angular and polarization distribution of a particular energy should give hyperfine field parameters. The measurement of the degree of polarization in the radiation emitted from a single crystal has some unique features; complications due to anisotropic Debye-Waller factors which always affect the angular distribution of the Mössbauer radiation in single crystals, will be absent. It is almost impossible to separate the hyperfine field parameters from unavoidable crystal orientation dependent Debye-Waller factors if one is to measure only the angular distributions. These complications do not exist in polarization measurements. Therefore, the polarization determination has several advantages over the angular distribution measurements. The results obtained with polarization measurements also pave the way for the determination of mean square displacement tensors in single crystals.

An attempt has been made by Houseley et al. to understand the polarization effects in Mössbauer absorption or emission of M1 radiation in single crystals. These authors have considered the special case in which the principal axis system of the EFG tensor coincides with crystal fixed axis system.
The restriction severely limits the application of the results to a particular class of substances and limited only to the M1 radiation. However, the principal axis system of the EFG tensor has no a priority relation to the crystal axis system. Therefore, it is imperative that proper calculations become essential to understand the polarization measurements.

4.1 Characterization of Polarization

There are two well established methods for calculating polarization of the electromagnetic radiation. The Stoke's method is ideally suited when the measurements allow the determination of the relative phase of the electric field vector, which are orthogonal at a given point, specifying the electromagnetic radiation. This method is applicable in the optical spectroscopic region but certainly is not so for the gamma radiation. A suitable method, therefore, would be the determination of the degree of polarization using density matrix formalism. In the determination of polarization it is essential that we should consider the coherent properties of the radiation; the coherence in this context is to be interpreted as that property which retains degree of polarization of the electromagnetic wave as it is propagated in space from the source to the detector. The Mössbauer radiation, in several cases, is highly coherent and the coherent time is of the order of nuclear life time and hence the radiation is propagated from source to detector without the change in polarization. For example, the coherent length
would be ~30 meters for Fe$^{57}$ 14.4keV Mössbauer radiation and it is ~3 cm for 100keV of W$^{183}$.

In the following we give the calculation of polarization observable in the single crystal Mössbauer resonance.

4.2 Polarization of the Magnetic Dipole Radiation

In the presence of known degeneracy due to nuclear quadrupole interaction the degree of polarization of the Mössbauer gamma rays is no longer unity. The electric field of the electromagnetic radiation emitted in the M1 transition for a given nuclear spin sequence can readily be calculated for each of the transitions of the Mössbauer radiation of Fe$^{57}$ in the presence of the electric quadrupole interaction which gives a doublet spectrum. The transition amplitudes for the radiation as calculated in the preceding chapter are given as

(i) higher energy transition:
\[ G(\frac{a_1 A_{+1}^{-1} - a_2 A_{-1}^{-1}}{6}) \text{ for } \Delta m = 1, \]
\[ G(- \frac{a_2 A_{+1}^{0}}{\sqrt{3}}) \text{ for } \Delta m = 0; \]
also
\[ G(- \frac{a_2 A_{+1}^{+1} - a_1 A_{-1}^{+1}}{\sqrt{6} A_{+1}^{+1}}), \Delta m = 1, \]
\[ G(- \frac{a_2 A_{+1}^{0}}{\sqrt{3} A_{+1}^{0}}) \text{ for } \Delta m = 0; \]
\[ ... (4.1) \]

and

(ii) lower energy transition
\[ G(- \frac{a_2 A_{+1}^{0} + a_1 A_{-1}^{0}}{\sqrt{6} A_{+1}^{0}}), \Delta m = 0 \]
\( C(\frac{a_1}{\sqrt{2}}A_1) \) for \( m = -1; \)

also

\( C(A_0 \frac{a_1}{\sqrt{2}}) \) for \( m = 0, \)

\( C(- \frac{a_2}{\sqrt{6}}A_1 + \frac{a_1}{\sqrt{6}}A_1) \) for \( m = -1 \) ... (4.2).

Where \( C \) is a constant, depending on the reduced matrix element.

Explicit expressions can be obtained by expressing \( A_{\pm}^1 \) and \( A_0^1 \) in terms of solutions already obtained in chapter-III (see Eq. 3.8'). Thus we get, for the first line

\[
C(\sqrt{3}/16\pi) \left[ - \frac{a_1}{2} \left\{ \cos((\hat{x}+i\hat{y})-\sin \theta \ e^{-i\phi} \ \hat{z}) - \frac{a_2}{n} \left\{ \cos((\hat{x}-i\hat{y}) - \sin \theta \ e^{-i\phi} \ \hat{z}) \right\} \right]\right],
\]

\[
C[-i\sqrt{3}/8\pi] \left[ - \frac{a_2}{n} (\sin \theta \ \hat{x} - \cos \theta \ \hat{y}) \right];
\]

also

\[
C[-i\sqrt{3}/8\pi] \left[ - \frac{a_2}{n} (\sin \theta \ \hat{x} - \cos \theta \ \hat{y}) \right],
\]

\[
C(\sqrt{3}/16\pi) \left[ - \frac{a_2}{n} \left\{ \cos((\hat{x}+i\hat{y}) - \sin \theta \ e^{-i\phi} \ \hat{z}) \right\} \right] \quad ... (4.3)
\]

and for the second line

\[
C(\sqrt{3}/16\pi) \left[ - \frac{a_2}{n} \left\{ \cos((\hat{x}+i\hat{y}) - \sin \theta \ e^{-i\phi} \ \hat{z}) \right\} + \frac{a_1}{n} \left\{ \cos((\hat{x}-i\hat{y}) - \sin \theta \ e^{-i\phi} \ \hat{z}) \right\} \right],
\]

\[
C[-i\sqrt{3}/8\pi] \left[ \frac{a_1}{n} (\sin \theta \ \hat{x} - \cos \theta \ \hat{y}) \right];
\]

also

\[
C[-i\sqrt{3}/8\pi] \left[ \frac{a_1}{n} (\sin \theta \ \hat{x} - \cos \theta \ \hat{y}) \right] \quad ... (4.4)
\]

where \( \hat{x}, \hat{y} \) and \( \hat{z} \) are the unit vectors in the direction of \( x, y \) and \( z \) respectively. The coefficients of \((\hat{x} + i\hat{y})\) and \((\hat{x} - i\hat{y})\)
Fig. 4.1. unit vectors \( \hat{e}_r, \hat{e}_\theta \) and \( \hat{e}_\phi \) are mutually orthogonal and \( \hat{e}_\phi \) is \( \perp \) to the plane defined by \( OZ, ON \) or \( OM \).
will give information of the right handed circular polarization and left handed circular polarization respectively. The unit vectors \( \hat{x}, \hat{y}, \hat{z} \) and \( \hat{e}_\theta \), \( \hat{e}_\phi \) and \( \hat{e}_0 \) as shown in the Fig-4.1 are related as

\[
\begin{align*}
\hat{x} &= \hat{e}_r \sin\theta \cos\phi + \hat{e}_0 \cos\phi \sin\phi - \hat{e}_0 \sin\theta \\
\hat{y} &= \hat{e}_r \sin\theta \sin\phi + \hat{e}_0 \cos\phi \sin\phi + \hat{e}_0 \cos\phi \\
\hat{z} &= \hat{e}_r \cos\theta - \hat{e}_0 \sin\theta
\end{align*}
\]

... (4.5)

The radial part of matrix elements is not of importance for the polarization. Therefore only the coefficients of \( \hat{e}_\theta \) and \( \hat{e}_\phi \) are retained and are designated as \( E_\theta \) and \( E_\phi \) respectively. Since \( \hat{e}_\theta \) and \( \hat{e}_\phi \) are unit vectors in the direction of \( \theta \) and \( \phi \) and \( \hat{E} = \hat{E}_\theta \hat{e}_\theta + \hat{E}_\phi \hat{e}_\phi \), therefore the explicit field amplitudes obtained with the help of Eqs. 4.3, 4.4, and 4.5 are

\[
\begin{align*}
-\sqrt{1/32\pi} \left( \sqrt{3} a_1 e^{i\phi} + a_2 e^{-i\phi} \right) \hat{e}_\theta - i \cos\theta \left( \sqrt{3} a_1 e^{i\phi} - a_2 e^{-i\phi} \right) \hat{e}_\phi \\
\text{ia}_2 \sqrt{1/8\pi} \sin\theta \hat{e}_\phi
\end{align*}
\]

also

\[
\text{ia}_2 \sqrt{1/8\pi} \sin\theta \hat{e}_\phi
\]

\[-\sqrt{1/32\pi} (a_2 e^{i\phi} - \sqrt{3} a_1 e^{-i\phi}) \hat{e}_\theta + i \cos\theta \sqrt{1/32\pi} (\sqrt{3} a_1 e^{i\phi} - a_2 e^{-i\phi}) \hat{e}_\phi.
\]

And for the second line

\[
\text{i} \sqrt{1/8\pi} a_1 \sin\theta \hat{e}_\phi
\]

also

\[
\sqrt{1/32\pi} (a_1 e^{i\phi} - \sqrt{3} a_2 e^{-i\phi}) \hat{e}_\theta + \text{i} \sqrt{1/32\pi} \cos\theta (a_1 e^{i\phi} + \sqrt{3} a_2 e^{-i\phi}) \hat{e}_\phi.
\]

The coefficients of \( \hat{e}_\theta \) and \( \hat{e}_\phi \) can be expressed in the following way
\[ E_\theta = \sqrt{\frac{1}{16}} (\sqrt{3}a_1 e^{i\theta} + a_2 e^{-i\theta}) \]

and

\[ E_\phi = -\sqrt{\frac{1}{16}} \cos(\sqrt{3}a_1 e^{i\theta} - a_2 e^{-i\theta}) \quad \text{for} \quad |3/2\rangle \rightarrow |1/2\rangle \]

From \( E_\theta \) and \( E_\phi \) which are coefficients of \( \hat{e}_\theta \) and \( \hat{e}_\phi \), the polarization matrix for any particular transition amplitude is

\[ \mathbf{f} = \begin{bmatrix} E_{\theta}\ast & E_{\phi}\ast \\ E_{\theta} & E_{\phi} \end{bmatrix} \quad \ldots (4.6) \]

The degree of polarization \( P \) is defined as the ratio of the intensity of the polarized portion to the total intensity, i.e.,

\[ P = \frac{I_{\text{pol}}}{I_{\text{tot}}} = \frac{\sqrt{1 - 4|f|^2 / (f_{11} + f_{22})^2}} \quad \ldots (4.7) \]

where \( P = 1 \) for the monochromatic radiation since \( |f| = 0 \) and the wave is said to be completely polarized, for \( P = 0 \) the wave is said to be completely unpolarized. In all other cases \( (0 < P < 1) \) we say that the radiation is partially polarized.

The coherence matrix due to these amplitudes is equal to the sum of the coherence matrices of the individual waves. The coherence matrix by definition is

\[ \mathbf{f} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \]

In our particular case of \( M1 \) radiation \( f_{11}, f_{12} \ldots \) etc., come out to be

\[ f_{11}^{3/2} \rightarrow |1/2\rangle = \frac{1}{32\pi} (3a_1^2 + a_2^2 + 2\sqrt{3}a_1a_2 \cos 2\theta) \]
\[ \rho_{12}^{3/2} \rightarrow |1/2\rangle = -i\cos\theta \frac{3a_1^2 - a_2^2}{32\pi} - \frac{\cos^2\theta}{16\pi} a_1 a_2 \sin 2\theta, \]

\[ \rho_{21}^{3/2} \rightarrow |1/2\rangle = (\rho_{12}^{3/2} \rightarrow |1/2\rangle)^* \]

\[ \rho_{22}^{3/2} \rightarrow |1/2\rangle = \frac{\cos^2\theta}{32\pi}(3a_1^2 + a_2^2 - 2\sqrt{3}a_1 a_2 \cos 2\theta) \]

\[ \rho_{11} = \frac{1}{16\pi} (3a_1^2 + a_2^2 - 2\sqrt{3}a_1 a_2 \cos 2\theta) \]

Similarly other matrix elements \( \rho_{11}, \rho_{12} \) etc., can be obtained. The matrix element for the first line can be written as the sum of all \( \rho_{11} \)'s, i.e.

\[ \rho_{11}^{\text{ist}} = \rho_{11}^{\text{ist}} + \rho_{11}^{\text{ist}} + \rho_{11}^{\text{ist}} + \rho_{11}^{\text{ist}} \]

\[ = \frac{1}{16\pi} (3a_1^2 + a_2^2 - 2\sqrt{3}a_1 a_2 \cos 2\theta) \]

and similarly

\[ \rho_{12}^{\text{ist}} = -(\sqrt{3}/8\pi)a_1 a_2 \cos 6\sin 2\phi \]

\[ \rho_{21}^{\text{ist}} = -(\sqrt{3}/8\pi)a_1 a_2 \cos 6\sin 2\phi \]

\[ \rho_{22}^{\text{ist}} = \frac{\cos^2\theta}{16\pi}(3a_1^2 + a_2^2 - 2\sqrt{3}a_1 a_2 \cos 2\theta) + \frac{a_2^2}{4\pi} \sin^2 2\theta \]

For the second line

\[ \rho_{11}^{\text{II}} = \frac{1}{16\pi} (3a_1^2 + a_2^2 - 2\sqrt{3}a_1 a_2 \cos 2\theta) \]

\[ \rho_{12}^{\text{II}} = (\sqrt{3}/8\pi)a_1 a_2 \cos 6\sin 2\phi \]

\[ \rho_{21}^{\text{II}} = \rho_{12}^{\text{II}} \]
Using expressions 4.7, 4.5 and 4.10 one can calculate degree of polarization for each line in the principal axis system.

4.3 Transformation of the Degree of Polarization from the Principal Axis System to the Crystal Field Axis System

In the preceding section we attempted to calculate the degree of polarization in the PAS. Since measurements are made in the laboratory system (body fixed axis system), therefore, it becomes essential to make transformation from the body fixed axis system to the principal axis system and vice-versa. Thus, if $A_{1}^{m}$ is a vector potential then under a general rotation $R$ of $A_{1}^{m}$, $R A_{1}^{m} R^{-1}$ which is

$$R A_{1}^{m} R^{-1} = \sum_{m=-1}^{1} D_{m,m}^{l'}(\alpha \beta \gamma) A_{1}^{m'}$$

where $R$ is the rotation operator defined by Euler rotation, $D_{m,m}^{l'}(\alpha \beta \gamma)$ are rotation matrices and $\alpha, \beta, \gamma$ are the Euler angles

In a similar manner the relevant amplitudes transform in the following way

$$R(- \frac{a_{1}}{2} A_{1}^{1} - \frac{a_{2}}{6} A_{1}^{1}) R^{-1}$$

for the transition $|3/2\rangle \rightarrow |1/2\rangle$. Substituting the value of $\hat{x}, \hat{y}$ and $\hat{z}$ in terms of $\hat{e}_{\theta}$ and $\hat{e}_{\phi}$ and then collecting the coefficients of $\hat{e}_{\phi}$ and $\hat{e}_{\theta}$ we obtain
\[ E_{0} = -\frac{3}{\sqrt{16\pi}} \left[ \frac{a_{1}^{D}}{\sqrt{2} - 1,1} + \frac{a_{2}^{D}}{\sqrt{6} - 1,1} \right] e^{i\phi_{c}} + \frac{a_{1}^{D}}{\sqrt{2} - 1,1} + \frac{a_{2}^{D}}{\sqrt{6} - 1,1} e^{-i\phi_{c}} \]

\[ E_{0} = -\frac{3}{\sqrt{16\pi}} \cos\theta \left[ \frac{a_{1}^{D}}{\sqrt{2} - 1,1} + \frac{a_{2}^{D}}{\sqrt{6} - 1,1} \right] e^{i\phi_{c}} + \frac{a_{1}^{D}}{\sqrt{2} - 1,1} + \frac{a_{2}^{D}}{\sqrt{6} - 1,1} e^{-i\phi_{c}} \]
The matrix elements of the coherency matrices are obtained with the help of Appendix-I and the results are

\[ |1/2\rangle \rightarrow |1/2\rangle: \]

\[ E_{11} = -\frac{3}{16\pi} \left( \frac{a_1}{\sqrt{3}} D_{1,0} e^{i\theta_0} + \frac{a_1}{\sqrt{3}} -1,0 e^{-i\theta_0} \right), \]

\[ E_{12} = -i \frac{3}{16\pi} \cos\theta_0 \left( \frac{a_1}{\sqrt{3}} D_{1,0} e^{i\theta_0} - \frac{a_1}{\sqrt{3}} -1,0 e^{-i\theta_0} \right) \]

\[ -i \frac{a_1}{\sqrt{3}} 0,0 \sin\theta_0 \frac{3}{8\pi}, \]

\[ |1/2\rangle \rightarrow |1/2\rangle: \]

\[ E_{21} = -\frac{3}{16\pi} \left[ \left( \frac{a_1}{\sqrt{6}} D_{1,1} + \frac{a_2}{\sqrt{2}} D_{1,-1} \right) e^{i\theta_0} + \left( -\frac{a_2}{\sqrt{6}} D_{1,-1} + \frac{a_2}{\sqrt{2}} D_{1,-1} \right) e^{-i\theta_0} \right], \]

\[ E_{22} = -i \frac{3}{16\pi} \cos\theta_0 \left( \frac{a_1}{\sqrt{6}} D_{1,1} e^{i\theta_0} - \frac{a_1}{\sqrt{6}} D_{1,-1} e^{-i\theta_0} \right) \]

\[ + \frac{a_2}{\sqrt{2}} D_{1,-1} e^{-i\theta_0} \right] - i \frac{3}{8\pi} \sin\theta_0 \left( \frac{a_1}{\sqrt{6}} D_{1,0} + \frac{a_2}{\sqrt{2}} D_{1,0} \right), \]

\[ |1/2\rangle \rightarrow |1/2\rangle: \]

\[ E_{31} = -\frac{3}{16\pi} \left( \frac{a_1}{\sqrt{3}} D_{1,0} e^{i\theta_0} + \frac{a_1}{\sqrt{3}} D_{1,-1,0} e^{-i\theta_0} \right), \]

\[ E_{32} = -i \frac{3}{16\pi} \cos\theta_0 \left( \frac{a_1}{\sqrt{3}} D_{1,0} e^{i\theta_0} - \frac{a_1}{\sqrt{3}} D_{1,-1,0} e^{-i\theta_0} \right) - i \frac{3}{8\pi} \sin\theta_0 \frac{a_1}{\sqrt{3}} D_{1,0,0}. \]

The matrix elements of the coherency matrices are obtained with the help of Appendix-I and the results are

\[ \rho_{11}^{3/2} \rightarrow |1/2\rangle = E_{0}, E_{0}^* = 0 \left[ \left( \frac{a_1}{2} + \frac{a_2}{6} \right) \frac{1+\cos^2\beta}{2} + \frac{1-\cos^2\beta}{2} \cos 2\alpha - \beta_0 \right] \]

\[ + \frac{a_1 a_2}{12} \left( 1-\cos^2\beta \cos 2\gamma + \frac{1}{2} (1-\cos\beta)^2 \cos 2\alpha - \gamma - \beta_0 \right) \]

\[ + \frac{1}{2} (1+\cos\beta)^2 \cos 2\alpha + \beta_0 \right], \]

\[ \rho_{12}^{3/2} \rightarrow |1/2\rangle = E_{0}, E_{0}^* = 0 \cos\theta_0 \left[ \left( \frac{a_1}{2} + \frac{a_2}{6} \right) \frac{1-\cos^2\beta}{2} \sin 2\alpha - \beta_0 \right] \]

\[ + \frac{2a_1 a_2}{12} \left( \frac{1-\cos^2\beta}{2} \sin 2\alpha - \gamma - \beta_0 \right) \]

\[ + \frac{1}{2} (1+\cos\beta)^2 \sin 2\alpha + \beta_0 \right] - C \sin\theta_0 \sin\beta \]
Thus we can find other matrix elements for each transitions in a similar way. The coherency matrix elements for the first line become
\[ \rho_{11} = \rho_{11}^{\text{3/2}} + \rho_{11}^{\text{-1/2}} + \rho_{11}^{\text{-3/2}} + \rho_{11}^{\text{-3/2}} \]

\[ = \frac{a_1^2}{2} + \frac{a_2^2}{6} + \frac{1}{2} (1 - \cos \beta)^2 \cos 2\alpha - \theta_c \frac{a_1 a_2}{12} \]

\[ \times \left\{ (1 - \cos \beta) \cos 2\gamma + \frac{1}{2} (1 - \cos \beta)^2 \cos 2\alpha - \gamma - \theta_c \right\} \]

\[ + \frac{a_2^2}{2} \sin^2 \beta \left( 1 - \cos 2\alpha - \theta_c \right), \]

\[ \rho_{12} = \cos \theta_c \left[ \frac{a_1^2}{2} + \frac{a_2^2}{6} \right] \left( \frac{1}{2} - \frac{1}{2} \cos^2 \beta \right) \sin 2\alpha - \theta_c + \frac{2a_1 a_2}{12} \left( \frac{1}{2} - \frac{1}{2} \cos^2 \beta \right) \]

\[ \sin \theta_c \sin \beta \left[ \frac{a_1^2}{2} + \frac{a_2^2}{6} \right] \]

\[ \times \left( 1 + \cos \beta \right) \sin 2\gamma - \gamma - \theta_c \left( \frac{1}{2} + \frac{a_2^2}{6} \right) \cos \beta \sin \gamma - \theta_c \]

\[ + \frac{a_2^2}{2} \sin \theta_c \sin \beta \sin 2\alpha - \theta_c - \cos \theta_c \sin^2 \beta \sin 2\alpha - \theta_c, \]

\[ \rho_{21} = (\rho_{12}^*)^\dagger, \]

\[ \rho_{22} = \cos^2 \theta_c \left[ \frac{a_1^2}{2} + \frac{a_2^2}{6} \right] \left( 1 - \cos^2 \beta \right) \cos 2\alpha - \theta_c + \frac{a_1 a_2}{12} \]

\[ \times \left\{ (1 - \cos^2 \beta) \cos 2\gamma - \gamma - \theta_c - \frac{1}{2} (1 + \cos \beta)^2 \right\} \]

\[ \times \cos 2\alpha - \gamma - \theta_c \left( \frac{1}{2} + \frac{a_2^2}{6} \right) \cos \beta \sin 2\alpha - \gamma - \theta_c \]

\[ + \frac{a_2^2}{2} \sin^2 \theta_c \cos^2 \beta \left[ \frac{a_1^2}{2} + \frac{a_2^2}{6} \right] - \frac{2a_1 a_2}{12} \cos^2 \theta_c \sin^2 \beta \sin 2\alpha - \theta_c \]

\[ + \frac{a_2^2}{2} \sin^2 \theta_c \cos^2 \beta - \frac{a_2^2}{2} \sin^2 \theta_c \sin 2\theta_c \cos 2\alpha - \theta_c \]
FIG. 4.4 Degree of Polarization In the Crystal Fixed Axis System (Ni-radiation).
FIG. 4.2  
Degree of Polarization a) 1st line and b) 11 line in the Principal Axis System (M1-radiation).
**FIG. 4.7** Illustration of the degree of polarization as a function of the EFG asymmetry parameter (it refers to the higher-energy transition and II to the lower-energy one; curves I(1), II(1), and I(2), II(2) have been drawn for \( \phi_c = 0^\circ \) and \( \phi_c = 90^\circ \) respectively): a) \( \alpha = \beta = \gamma = 0^\circ, \theta_c = 30^\circ \), b) \( \alpha = \beta = \theta_c = 30^\circ, \gamma = 45^\circ \).
and for the second line we obtain

\[
\begin{align*}
\mathcal{P}_{11}^{2nd} &= \frac{a_2^2 + a_1^2}{2} \left( 1 + \cos^2 \beta \right) - \frac{1}{2} \left( 1 - \cos^2 \beta \right) - \frac{a_1 a_2}{\sqrt{12}} \left( 1 - \cos^2 \beta \right) \cos 2\gamma - \frac{1}{2} \left( 1 + \cos \beta \right)^2 \\
&\times \cos 2\alpha - \theta_0 + \frac{1}{2} \left( 1 + \cos \beta \right)^2 \cos 2\alpha - \gamma - \theta_0 - \frac{1}{2} \left( 1 + \cos \beta \right)^2 \\
&\times \left( \frac{a_2^2}{2} + \frac{a_1^2}{6} \right) \cos 2\alpha - \gamma - \theta_0 + \frac{1}{2} \left( 1 + \cos \beta \right)^2 \cos 2\alpha - \gamma - \theta_0 \\
&\times \left( \frac{a_2^2}{2} + \frac{a_1^2}{6} \right) \cos 2\alpha - \gamma - \theta_0 + \frac{1}{2} \left( 1 + \cos \beta \right)^2 \cos 2\alpha - \gamma - \theta_0 \\
&\times \sin^2 \theta_0 \sin^2 \beta \sin 2\alpha - \theta_0 \\
&\mathcal{P}_{12}^{2nd} = \cos \theta_0 \left( \frac{a_2^2}{2} + \frac{a_1^2}{6} \right) \left( 1 - \cos^2 \beta \right) \sin 2\alpha - \theta_0 - \frac{2a_1 a_2}{\sqrt{12}} \left( 1 - \cos^2 \beta \right) \\
&\times \sin 2\alpha - \gamma - \theta_0 + \frac{1}{2} \left( 1 + \cos \beta \right)^2 \sin 2\alpha - \gamma - \theta_0 \\
&\times \left( \frac{a_2^2}{2} + \frac{a_1^2}{6} \right) \cos \beta \sin 2\alpha - \theta_0 + \frac{1}{2} \left( 1 + \cos \beta \right)^2 \cos 2\alpha - \gamma - \theta_0 \\
&\times \sin \theta_0 \sin 2\beta \sin \gamma - \theta_0 \\
&\mathcal{P}_{21}^{2nd} = \mathcal{P}_{12}^{2nd} \\
&\mathcal{P}_{22}^{2nd} = \cos \theta_0 \left( \frac{a_2^2}{2} + \frac{a_1^2}{6} \right) \left( 1 + \cos^2 \beta \right) - \frac{1}{2} \left( 1 - \cos^2 \beta \right) \cos 2\alpha - \theta_0 \\
&\times \left( \frac{a_2^2}{2} + \frac{a_1^2}{6} \right) \cos 2\alpha - \gamma - \theta_0 - \frac{1}{2} \left( 1 + \cos \beta \right)^2 \cos 2\alpha - \gamma - \theta_0 \\
&\times \left( \frac{a_2^2}{2} + \frac{a_1^2}{6} \right) \cos 2\alpha - \gamma - \theta_0 - \frac{1}{2} \left( 1 + \cos \beta \right)^2 \cos 2\alpha - \gamma - \theta_0 \\
&\times \sin \theta_0 \sin 2\beta \sin \gamma - \theta_0 + \frac{1}{2} \left( 1 + \cos \beta \right)^2 \cos 2\alpha - \gamma - \theta_0 \\
&\times \sin \theta_0 \sin 2\beta \sin \gamma - \theta_0 + \frac{1}{2} \left( 1 + \cos \beta \right)^2 \cos 2\alpha - \gamma - \theta_0 \\
&\times \sin \theta_0 \sin 2\beta \sin \gamma - \theta_0 + \frac{1}{2} \left( 1 + \cos \beta \right)^2 \cos 2\alpha - \gamma - \theta_0 \\
&\times \sin \theta_0 \sin 2\beta \sin 2\alpha - \theta_0.
\end{align*}
\]

And the degree of polarization for both the lines can be calculated by using the Eq. 4.7. Fig-4.2 to 4.7 represent several measurable properties associated with the polarization phenomenon.
4.4 Polarization of the Electric Quadrupole Radiation

In the case of quadrupole radiation no attempt has been made to study polarization of the Mössbauer gamma rays. In this section we intend to make use of the theory to determine the degree of polarization for each line. However when \( \eta \neq 0 \) the degeneracy is removed for the integer angular momentum nuclear states. Thus the Mössbauer radiation is coherent and will have degree of polarization \( P \) equal to unity. However, when \( \eta = 0 \) the quadrupole degeneracy is not removed and hence the degree of polarization should depend only on the orientation of the PAS w.r.t. CFAS. Therefore calculations are carried out for this case of E2 transition. It is to be noted that removal of the degeneracy is not connected with the E2 transition. We have selected the even integer angular momentum nuclear states since they are the most common involving E2 Mössbauer transition. The transition amplitudes for the \( 2^+ \rightarrow 0^+ \) decay are given as

\[
|+2\rangle \rightarrow |0\rangle, \text{ constt. } (A^2_2 + A^{-2}_2),
\]

\[
|-2\rangle \rightarrow |0\rangle, \text{ constt. } (A^{-2}_2 - A^2_2),
\]

\[
|-1\rangle \rightarrow |0\rangle, \text{ constt. } (A^{-1}_2 + A^1_2),
\]

\[
|+1\rangle \rightarrow |0\rangle, \text{ constt. } (A^{-1}_2 - A^1_2),
\]

\[
|0\rangle \rightarrow |0\rangle, \text{ constt. } A^0_2
\]

where \( A^2_2 \), \( A^1_2 \) and \( A^0_2 \) have already been defined earlier (See Eq. 3.16). Now if we calculate \( E_0 \) and \( E_0' \) for each line then we shall be able to have expressions for the matrix elements for each polarization matrix. The transition considered in
this work has the spin sequence $2^+ \rightarrow 0^+$ and the EFG interaction with $\eta = 0$ does not lift the $m$-sub level degeneracy completely. Thus, there are two incoherent 'beam' amplitudes for the two of three Mössbauer lines. For the $|1^2\rangle \rightarrow |0\rangle$ transition the radiation fields are $(A_2^{-2} + A_2^{2})$ and $(A_2^{-2} - A_2^{2})$ which in the crystal axis system becomes
\[
R(A_2^{-2} + A_2^{2})R^{-1} = \sum_{m} (D_{m,-2}^2 + D_{m,2}^2)\Lambda_{2}^{m}(\theta_0, \phi_0)
\]
and defining
\[
\Theta^\pm (1) = D_{-2,2}^2 \pm D_{-2,-2}^2;
\]
\[
\Theta^\pm (2) = D_{-1,2}^2 \pm D_{-1,-2}^2;
\]
\[
\Theta^\pm (3) = D_{0,2}^2 \pm D_{0,-2}^2,
\]
the calculated electric field components along the $\hat{e}_0$ and $\hat{e}_\phi$ directions for the amplitude $R(A_2^{-2} + A_2^{2})R^{-1}$ are
\[
E_{\hat{e}_0}' = iJ\Theta^+(1) - (C + iD)\Theta^+(2) \quad \text{and}
\]
\[
E_{\hat{e}_\phi}' = F\Theta^+(2) + iK\Theta^+(1) + L\Theta^+(3) - E\Theta^+(2)
\]
where
\[
J = -2 \sin \theta_0 \sin 2\phi_0,
\]
\[
K = -\sin 2\theta_0 \cos 2\phi_0,
\]
\[
L = \sqrt{3/2} \sin 2\theta_0,
\]
\[
C = (3\cos^2 \theta_0 - 1 - \sin^2 \theta_0 \cos 2\phi_0) \cos \theta_0 \sin \phi_0,
\]
\[
D = \sin^2 \theta_0 \sin 2\phi_0 \cos \theta_0 \cos \phi_0 + 2\sin^2 \theta_0 \cos \theta_0 \sin \phi_0,
\]
\[
E = -\sin^2 \theta_0 \sin 2\phi_0 \sin \phi_0 \quad \text{and}
\]
\[
F = (3\cos^2 \theta_0 - 1 - \sin^2 \theta_0 \cos 2\phi_0) \cos \phi_0.
\]
**FIG. 4.8 (a)**

The dependence of the Polarization parameter on the azimuthal angle $\phi_c$ in the CFAS for the transition $1s1 \rightarrow 10$ under the conditions $\gamma = 0$ and $\phi = 30^\circ$.

**FIG. 4.9 (b)**

The dependence of the degree of polarization on the $\phi_c$ in CFAS for the $1s2 \rightarrow 10$ transition when $\gamma = 0$ and $\phi_c = 30^\circ$. 

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**RAW TEXT END**
The electric field components $E_{\phi_c}''$ and $E_{\theta_c}''$ for the amplitude $R(A_2^{-1} - A_2^1)R^{-1}$ are the same as of Eq. 4.11 except $\theta^+$ is replaced by $\phi^-$. Thus the coherence matrices $\mathcal{F}'$ and $\mathcal{G}''$ can be formed with the prescription defined by Eq. 4.6. The degree of polarization as defined by Eq. 4.7 has been calculated for the line under consideration and results are given in Fig-4.8(a). Similar analysis for the transition $|\pm 1\rangle \rightarrow |0\rangle$ with amplitude $R(A_2^{-1} + A_2^1)R^{-1}$ gives

$$E_{\phi_c}' = i J \mathcal{D}^+(4) - (C + i D) \mathcal{D}^+(5)$$
$$E_{\theta_c}' = P \mathcal{D}^+(5) + i K \mathcal{D}^+(4) + L \mathcal{D}^+(6) - E \mathcal{D}^+(5) \quad \ldots \quad (4.12)$$

with

$$\mathcal{D}^+(4) = D_{-2,-1}^2 \pm D_{-2,1}^2,$$
$$\mathcal{D}^+(5) = D_{-2,-1}^2 \pm D_{-1,1}^2,$$
$$\mathcal{D}^+(6) = D_{0,-1}^2 \pm D_{0,1}^2,$$

where $\mathcal{F}^-$ are applicable to the $R(A_2^{-1} - A_2^1)R^{-1}$ radiation amplitude. The degree of polarization of this line as a function of polar angles in the CAFS is given in Fig-4.8(b). The results of Fig-4.8(a) and (b) clearly indicate that the degree of polarization is a very sensitive function of the hyperfine field parameters. Thus the polarization measurement can be an effective tool in the Mössbauer spectroscopy.