PREFACE

The present Thesis entitled 'Contributions to Group Theory' is the research work done since 1st April, 1961, on Subtractive Operation in Group Theory. This work has been executed by me at the Aligarh Muslim University under the close and inspiring supervision and also coordination of M.A. Kazim Esqr. Reader in the Department of Mathematics and Statistics of this University.

A study of inverse operations has already been made by M. Ward, D.G. Rabinow, T. Evans, H. Furstenberg, J.V. Whittaker and several others because of the utility of inverse operation in its applications in various abstract concepts such as Loop Theory, particularly free loops and concepts connected with a group such as congruences, subgroups, cosets etc. and also in other branches of Mathematics. Our approach is mostly confined to subtraction as fundamental operation (both in the generalized and ordinary sense) which, apart from being interesting on its own account, will also be fundamental in study of free loops, in the study of geometrical calculus where vectors are introduced as the differences of points, widening the scope of the study of difference sets, and in other studies where 'addition' is of secondary importance.

This Thesis consists of six Chapters, the material of all the Chapters except the third one has been accepted for publication in standard journals and is either under print or to appear shortly. In Chapter I, a subtractive group is defined, and the consistency and independence of the postulates are displayed. Earlier M. Ward (1930), D.G. Rabinow (1937) and J.V. Whittaker (1955) have given set
of postulates defining a group in terms of subtraction as binary operation and H. Furstenberg (1955) in terms of an inverse binary operation \( a \circ b = ab^{-1} \), but their postulates do not appear to enable them to provide a direct equivalent of the associative law in an additive or multiplicative group whereas we have achieved this by regarding subtraction as a two-sided binary operation and by introducing the concept of a 'right difference' and a 'left difference'. In our case, even the commutativity postulate is just in the natural form. Chapter II deals with the deductions and consequences of postulates given in Chapter I. We have proved several results for the development of the concept introduced and have at the same time established through this approach several those which were found out by previous workers on inverse operations. We have concluded the Chapter with number characterizations of a non-abelian and an abelian group. The material of Chapters I and II is substantially the same as the paper by M.A. Kazim and F. Husain (Bibliography - Chapter I, [5]). Chapter III provides an analysis of three postulates of Chapter I giving rise to various abstract structures and their characterizations in a natural way. The work done in this Chapter has not yet been sent for publication which, however, will be done recently. In Chapter IV, a comprehensive study of all possible analogues to the associative law (Postulate II, Chapter I) has been made by replacing it in the postulates for a subtractive group (Chapter I) by any one of the analogues, and thus investigated several finer characterizations of subtractive abelian groups and loops. This chapter is also
substantially the same as the paper by M.A. Kazim and F. Husain (Bibliography - Chapter IV, [5]) and is shortly to appear in the Journal of Indian Mathematical Society. In Chapter V, improving on characterization of subtractive abelian groups of Chapter IV we have characterized abelian groups by a set of two postulates only. We may note that in characterization of abelian groups we can safely appeal only to right difference or left difference. The material of this chapter is appearing very shortly as a paper in the Nieuw Archief Voor Wiskunde, Netherlands, Vol. XII (1964), 1-4. The sixth and the last chapter which is under publication in the form of paper in the Journal of Indian Mathematical Society is devoted to a further extension of some of the results obtained in Chapter IV to the concept of a field as the Theory of Fields is closely connected with the Theory of Groups.

It will be proper here to thank the referees of the several papers accepted for publication and included in this Thesis, for the interest shown in the subtraction operation and also for their appreciating comments and remarks. I am obliged to the Ministry of Scientific Research and Cultural Affairs which has helped me financially by awarding research scholarship in carrying out my research work. My thanks are also due to Professor J. A. Siddiqi, Head of the Department of Mathematics and Statistics and all others who have at any stage helped me and provided me facilities towards the completion of my objective. My appreciation also goes to the courtesy and help of the office staff of the Department.
Lastly I again take this opportunity to express my deep sense of gratitude to my supervisor for his valuable attention, his sympathetic criticism and his inspiring guidance and affection.

A. M. U. Aligarh
March 11, 1964.

Affusain

(F. Husain)