Chapter 5

SM-MIMO Detection and EVS Problem

5.1 Coherent Detection Requirement II: SM-MIMO DATA Detection

5.1.1 Background and Previous Methods

The next generation communication systems demand for transmission speed from 100Mbps (in vehicular) to 1Gbps (in pedestrian) to meet the future communication needs. Since many investigations in the past few years have shown that multiple antenna systems provide an enormous increase in capacity compared to Single Input Single Output (SISO) systems [59], a Multiple Input Multiple Output (MIMO) system is considered as one of the possible techniques. A MIMO technique is classified into two categories: Spatial Diversity (SD) and Spatial Multiplexing (SM). Using MIMO systems [60],[17] enables us to achieve high data rates or reduce the effective error rate on a rich scattering wireless channel. In general, SD technique is simple at the receiver. However, it significantly reduces capacity [53],[12]. In order to meet the requirement of the high data rate, SM method which transmits the independent data streams at each transmit antennas is more appropriate than the SD technique.

In this thesis, the work focus on the SM MIMO systems. In SM MIMO system, since independent data streams are transmitted at each transmit antennas without increasing transmit power and spectral bandwidth, it is considered as one of the possible ways to achieve the high data rate. However, spatial de-multiplexing at the receiver is a challenging task. There are many detection methods which have been actively developed. Among these, exhaustive full search of ML method is considered as the optimum detection method [10]. However, its computational complexity increases exponentially with the number of transmitted antenna and constellation points. The linear
detection methods such as Zero Forcing (ZF) and Minimum Mean Square Error (MMSE) methods have low complexity. However, they undergo a severe performance degradation due to noise enhancement. The OSIC [11],[43] which is known as V-BLAST offers a good trade-off between performance and complexity. Many OSIC-series detectors attempted to reach ML performance with feasible complexity. The OSIC-series detectors such as detectors in [30],[15],[24],[54] are good examples.

The performance improvement in the conventional OSIC is limited due to error propagation, so a successful detection of the first layer is crucial. In an effort to reduce the error propagation, a number of spatial streams are jointly detected in the ML-DFE detection method [5] and then the remaining streams are decoded as in OSIC method. This method mitigates the error propagation. However, the computational complexity due to joint detection is quite high and performance improvement is quite limited. In [25], by assigning all the constellation points to the Most Reliable Layer (MRL), a performance improvement was observed over the conventional OSIC. A sub-optimum QRM-MLD detection method is proposed in [30],[15], where the channel matrix is decomposed into a unitary matrix Q and an upper triangular matrix R, and the M-best nodes are selected at each layer or spatial stream. At the last stage, the ML metrics for the M candidate vectors are compared to decide the best estimate of the transmitted vector.

In [54], the authors asserted that the selection of the Least Reliable Layer (LRL) is a more effective way to mitigate the error propagation, when all the constellation points are assigned to the first layer symbol. The hard-output performance of QR-LRL approaches the ML performance with simple hardware complexity. They also provided soft-output solution for channel decoder [16][54]. However, their solution for soft-output generation is not good enough to deal with empty vector set problem, especially if the modulation order is low. Since as the modulation order increases, the probability of a candidate vector set having all constellation point at each layer increases, which in turn lessen/soothe EVS[26]. Several remedies [16][65][54] are used to find out soft values in the presence of empty vector set, but are not very performance effective. In [26] the author provided a solution to remove empty vector set problem by taking multiple QR decomposition of ordered channel matrix and trying every constellation point at each layer. However, such a solution comes with increased complexity. In [28] the author provided a solution to mitigate empty vector set problem by assigning every constellation point at each layer, where he uses QR matrix transformation while avoiding computation of multiple QR decomposition.
5.1.2 MIMO System

In this section, the MIMO system is described. After that various SM-MIMO detectors, EVS problem and it’s remedies are described.

MIMO System Description

A MIMO system with \( m \) transmit antennas and \( n \) receive antennas is considered. The conventional MIMO system is showed in Fig.(5.1), and received signal \( y \) can be expressed as follows:

\[
y = Hx + z
\]  

\[
\begin{pmatrix}
y_1 \\
y_2 \\
\vdots \\
y_m \\
\end{pmatrix} = \begin{pmatrix}
h_{1,1} & h_{1,2} & \ldots & h_{1,m} \\
h_{2,1} & h_{2,2} & \ldots & h_{2,m} \\
\vdots & \vdots & \ddots & \vdots \\
h_{n,1} & h_{n,2} & \ldots & h_{n,m} \\
\end{pmatrix} \begin{pmatrix}
x_1 \\
x_2 \\
\vdots \\
x_m \\
\end{pmatrix} + \begin{pmatrix}
z_1 \\
z_2 \\
\vdots \\
z_m \\
\end{pmatrix}
\]

where, \( x_i, i = 1, 2, \ldots, m \) is the transmitted signal from the \( i^{th} \) transmit antenna, \( y_j, j = 1, 2, \ldots, n \) is the received signal at the \( j^{th} \) receive antenna, \( z_j \sim CN(0, \sigma^2), j = 1, 2, \ldots, n \) is the circularly symmetric white Gaussian noise, \( h_{ji}, j = 1, 2, \ldots, n, i = 1, 2, \ldots, m \) is the uncorrelated channel coefficient between the \( j^{th} \) receive antenna and the \( i^{th} \) transmit antenna. In this thesis, transmitted symbols are the QAM modulated symbols.
5.2 QR-LRL Based SM-MIMO Detector

Hard-Output QR-LRL Method [54]

In an effort to exactly detect the first layer symbol compared to conventional QR-OSIC [Appendix 2], the author suggested QRMRL [25]. In QR-MRL, all constellation points are assigned to the first layer symbol instead of deciding it. Also, the channel matrix is ordered by selecting MRL as the first detection layer. However, when all the constellation points are considered for first layer symbol, selection of the MRL in ordering is not an effective way to mitigate the error propagation. In QR-LRL [54], by assigning the LRL to the first layer, the error propagation is more reduced than the QR-MRL. Moreover, QR-LRL detection method can achieve near ML performance. The ordering to find the LRL is important in QR-LRL, so that the post-detection SNR $\zeta$ is used for the layer ordering. The detailed ordering process in QR-LRL is summarized in Fig. (5.2).

\[
\mathbf{y} = \mathbf{H}_{\text{ordered}} \cdot \mathbf{x}_{\text{ordered}} + \mathbf{z}
\]

\[
\mathbf{H}_{\text{ordered}} = [h^{(m-1)\text{th RL}}; \ldots ; h^{3\text{rd RL}}; h^{2\text{nd RL}}; h^{\text{MRL}}; h^{\text{LRL}}]
\]

\[
\mathbf{x}_{\text{ordered}} = [x^{(m-1)\text{th RL}}; \ldots ; x^{3\text{rd RL}}; x^{2\text{nd RL}}; x^{\text{MRL}}; x^{\text{LRL}}]
\]

After obtaining the system equation and applying every constellation point at each layer, the Euclidean metric is calculated and minimum distance vector is chosen as the optimum detected vector to achieve hard ML performance. The detection process in QR-LRL is equal to QR-MRL [Appendix 2][25] except detection layer ordering. The LRL is chosen based on post detection SNR $\zeta$, which is $\| g_i \|^2$, $i = 1, 2, 3, \ldots, m$ as in [54], where $G = ((H^T H)^{-1} H^T)$. The whole method is described in Table (5.1).
Table 5.1: Pseudo code for QR-LRL.

<table>
<thead>
<tr>
<th>QRLRL - SM MIMO - Detector</th>
</tr>
</thead>
<tbody>
<tr>
<td>function $[V] = QR - LRL(y, H_{\text{ordered}})$</td>
</tr>
<tr>
<td>$[Q, R] = qr(H_{\text{ordered}})$</td>
</tr>
<tr>
<td>for $i = 1 :</td>
</tr>
<tr>
<td>$x_{(4)} = C(i)$</td>
</tr>
<tr>
<td>$x_{(3)} = Q\left(\frac{y_1 - r_{34}x_{(4)}}{r_{41}}\right)$</td>
</tr>
<tr>
<td>$x_{(2)} = Q\left(\frac{y_2 - r_{23}x_{(3)} - r_{24}x_{(4)}}{r_{42}}\right)$</td>
</tr>
<tr>
<td>$x_{(1)} = Q\left(\frac{y_1 - r_{12}x_{(2)} - r_{13}x_{(3)} - r_{14}x_{(4)}}{r_{11}}\right)$</td>
</tr>
<tr>
<td>$V(:, i) = [x_{(1)}, x_{(2)}, x_{(3)}, x_{(4)}]^T$</td>
</tr>
<tr>
<td>$\text{Dist}(; : i) =</td>
</tr>
</tbody>
</table>

5.3 Empty Vector Set Problem and Previous Remedies

5.3.1 Soft-output Generation

It is well known that Bit-Interleaved Coded Modulation (BICM) can achieve the open loop capacity when proper soft-output values can be provided to the channel decoder at the receiver. Consequently, the capability of soft-output generation is very important for good overall system performance. BICM allows a different Maximum A Posteriori (MAP) based decoding scheme in which the received QAM signals are first demodulated by a soft-output demapper and de-interleaved, and then passed to a standard binary soft-input soft-output viterbi or turbo decoder. The idea is to demap the received signal into soft bits, which has the same sign as provided by a hard detector and whose absolute value indicates the reliability of the decision.

5.3.2 Empty Vector Set

In SM MIMO systems, Log Likelihood Ratio (LLR) or soft output values generally achieve open loop capacity. Generally LLR for the $k^{th}$ bit in $i^{th}$ symbol of received vector $y$ is given by

$$L(b_{k,i}|y) = \ln \frac{Pr[b_{i,k} = 1|y]}{Pr[b_{i,k} = 0|y]}$$

(5.3)

Where $Pr[b_{i,k} = 1|y]$ is the probability for $k^{th}$ $(k \text{ ranging } 0 \rightarrow \log_2|C|)$ bit in $i^{th}$ $(i \text{ ranging } 1 \rightarrow N_T)$ symbol. Since it is very cumbersome to calculate Eq. (5.3), the LLR value for the $k^{th}$ bit
in $l^{th}$ symbol, $k = 1, 2...log_2|C|$, $l = 1, 2,...m$ can be approximated as

$$L(b_{k,l}|y) \approx \min_{x \in S(k,l)^- \subset V} D(x) - \min_{x \in S(k,l)^+ \subset V} D(x)$$ (5.4)

Where $S(k,l)^-$ is the candidate vector set corresponding to $b_{k,l} = -1$, $S(k,l)^+$ is the candidate vector set corresponding to $b_{k,l} = +1$, $C$ denotes constellation point set and $|C|$ is its size. The distance metric is given by

$$D(x) = \frac{1}{\sigma^2} \| y - Hx \|^2$$

$V$ is the candidate vector set obtained after all layer symbol detection. It is possible that either $S(k,l)^-$ or $S(k,l)^+$ is an empty set as shown in Fig. (5.3), which causes hindrance in generating LLR.

### 5.3.3 Previous Remedies

**Threshold in QRM-MLD Detector [16]**

QRM-MLD [30][Appendix2] signal detection method performs the generation of LLRs in the last stage, so it always suffers from the empty vector set problem. Therefore, in [16], the authors proposed the likelihood function generation method for non-existing bit, which can mitigate the empty vector set problem. In the proposed scheme, LLR generation method is described in the following process:

- The symbol replica candidates among the remaining symbol replica candidates in the last stage, which bit '1' or '-1' remains, are selected independently.
- For the symbol replica candidates selected for each bit '1' and '-1', the likelihood function of each bit for each symbol replica is obtained using the Euclidean distance for selected symbol replicas.
- For each bit in the data symbols, the likelihood function of bit '1' is derived as the minimum Euclidean distance among candidates of the Euclidean distance obtained in the second step. The likelihood function for bit '-1' is obtained in the same manner.
- For a bit, which has both bits '1' and '-1' in the remaining symbol replica candidates in the last stage (let NB be the number of bits which has both bits '1' and '-1' in the remaining symbol replica candidates in the last stage), the larger likelihood function of two bits, '1' or '-1', is selected.
Figure 5.3: Empty-Vector-Set-Problem

- Among the bits, which have both bits ‘1’ and ‘-1’ in the remaining symbol replica candidates in the last stage, the selected larger likelihood function of each bit in the data symbols are summed (note there are NB bits) and its summation is normalized by NB.

- Finally, the likelihood function calculated in the previous step is multiplied by a factor of ‘X’. This value is used for the symbol candidates suffering from Empty Vector Set problem. Author [16] uses value of ‘X = 1.5’ based on simulation environment.

Threshold in QR-LRL Detector [54]

Since each constellation point is tried at LRL, $S(k, l)^-$ and $S(k, l)^+$ will not be an empty set at LRL. Using this property threshold value $Th$ is derived at LRL[54], which is used for calculating LLR for the bits suffering from EVS at higher layers. At first, LLR at LRL corresponding to $x(m)$ (where m varies from 1, 2...$|C|$) is calculated as

$$L_D(b_{k,(m)} | y) \approx \min_{x \in S(k,(m))^+} D(x) - \min_{x \in S(k,(m))^+} D(x)$$  \hspace{1cm} (5.5)$$

$$T^+(k) = \min_{x \in S^+(k,(m))} D(x), 1 < k \leq \log_2 |C|$$  \hspace{1cm} (5.6)$$

$$T^-(k) = \min_{x \in S^+(k,(m))} D(x), 1 < k \leq \log_2 |C|$$  \hspace{1cm} (5.7)$$

$$T(k) = \max(T^+(k), T^-(k)), 1 < k \leq \log_2 |C|$$  \hspace{1cm} (5.8)$$
From above Eq. Threshold value 'Th' is derived as

\[ Th = \frac{1}{\log_2|C|} \sum_{k=1}^{\log_2|C|} T(k) \] (5.9)

This threshold is used for calculating LLR for the bits suffering from EVS at higher layers.

**Enlarged QR-DD Detector [26]**

In QR-DD method, every constellation point is tried at each layer and LLR is calculated using enlarged candidate vector set. For example, consider a 4 × 4 MIMO system.

- At first, H is ordered according to the minimum error propagation ordering suggested in [9] with LRL as first detection layer. At the same time, since all constellation points are tried at each layer, the ordering criteria is used with loose ordering method based on SNR instead of post detection SNR [26]. Thus H is ordered \( \| h_i \|^2, i = 1, 2, 3, 4 \) as shown below.

\[
H_{ordered,1} = [h_{3rdRL}, h_{2ndRL}, h_{MRL}, h_{LRL}]
\]

Where \( \| h_{MRL} \|^2 > \| h_{2ndRL} \|^2 > \| h_{3rdRL} \|^2 > \| h_{LRL} \|^2 \). Where \( h_{2ndRL}, h_{3rdRL} \) represents the 2\(^{nd}\) and 3\(^{rd}\) reliable layer of \( h \) respectively.

- Now using upper QR structure, candidate vector set \( V_{S,12} \) is generated for LRL and MRL as per Table 1.

\[
V_{S,12} = [V_{cand,1}, V_{cand,2}]
\]

Where \( V_{S,12} \) represents enlarged candidate vector set consist of vectors \( V_{cand,1} \) and \( V_{cand,2} \). Here \( V_{cand,1} \) and \( V_{cand,2} \) are vector set corresponding to vectors generated while trying each constellation point at LRL and MRL respectively.

- Again, to apply each constellation point at the remaining two layers i.e. 2\(^{nd}\) and 3\(^{rd}\) detection layer, H is reordered as shown below.

\[
H_{ordered,2} = [h_{LRL}, h_{MRL}, h_{3rdRL}, h_{2ndRL}]
\]

Applying QR decomposition on reordered H, \( V_{S,12} \) is generated for the remaining two layers, 2\(^{nd}\) and 3\(^{rd}\) reliable layer using Table (5.2), where \( V_{S,12} = [V_{3rdRL}, V_{2ndRL}] \). Here \( V_{S,12} \) represents enlarged candidate vector set which consists of vectors \( V_{2ndRL} \) and \( V_{3rdRL} \), where \( V_{2ndRL} \) and \( V_{3rdRL} \) are vector sets corresponding to vectors generated while trying each
Table 5.2: Candidate Vector Set at LRL,MRL layer.

\[
\begin{align*}
\text{function} &\quad [V_{S,12}] = \text{Candidate}V_{12}(\mathbf{y}, \mathbf{H}_{\text{ordered}}) \\
&\quad [Q, R] = \text{qr}(\mathbf{H}_{\text{ordered}}) \\
&\quad \text{for } i = 1 : |C| \\
&\quad x(4) = C(i) \\
&\quad x(3) = Q\left(\frac{\bar{y}_3 - r_{33}x(4)}{r_{11}}\right) \\
&\quad x(2) = Q\left(\frac{\bar{y}_2 - r_{23}x(3) - r_{24}x(4)}{r_{11}}\right) \\
&\quad x(1) = Q\left(\frac{\bar{y}_1 - r_{12}x(2) - r_{13}x(3) - r_{14}x(4)}{r_{11}}\right) \\
&\quad V_{\text{cand},1}(\cdot, i) = [x(1), x(2), x(3), x(4)]^T \\
&\quad \text{Dist}_{\text{cand},1}(\cdot, i) = ||\mathbf{y} - \mathbf{H}_{\text{ordered}} \cdot (V_{\text{cand},1}(\cdot, i))||^2 \\
&\quad \text{end} \\
&\quad \text{for } i = 1 : |C| \\
&\quad x(3) = C(i) \\
&\quad x(4) = Q\left(\frac{\bar{y}_3 - r_{33}x(3)}{r_{11}}\right) \\
&\quad x(2) = Q\left(\frac{\bar{y}_2 - r_{23}x(3) - r_{24}x(4)}{r_{11}}\right) \\
&\quad x(1) = Q\left(\frac{\bar{y}_1 - r_{12}x(2) - r_{13}x(3) - r_{14}x(4)}{r_{11}}\right) \\
&\quad V_{\text{cand},2}(\cdot, j) = [x(1), x(2), x(3), x(4)]^T \\
&\quad \text{Dist}_{\text{cand},2}(\cdot, i) = ||\mathbf{y} - \mathbf{H}_{\text{ordered}} \cdot (V_{\text{cand},2}(\cdot, i))||^2 \\
&\quad \text{end} \\
&\quad V_{S,12} = [V_{\text{cand},1}, V_{\text{cand},2}] \\
&\quad \text{Dist}_{S,12} = [\text{Dist}_{\text{cand},1}, \text{Dist}_{\text{cand},2}] \\
&\quad \text{endfunction}
\end{align*}
\]

constellation point at \(i^{th}\) RL respectively.

Overall, \(V = [V_{LRL}, V_{MRL}, V_{3rdRL}, V_{2ndRL}]\)

Now \(V\) consists of vectors when every constellation point is applied at each layer. EVS problem is effectively removed by this method at the cost of multiple QR decomposition \(\approx \lceil N^2 \rceil\) for enlarging \(V\). Complexity increases exponentially as \(N = \min(N_t, N_r)\) increases, where \(N_t, N_r\) are number of transmit and receive antennas respectively.