Chapter 5

Use of Learned Dictionaries and Gabor Prior

In the work of previous chapter, significant PCA components are used to obtain super-resolution of hyperspectral images. Due to large number of atoms in raw dictionaries of LR-HR, the required computational time is very high in order to obtain initial SR estimation. Hence in chapter 4 we restricted our CS based approach to obtain initial estimate on first PCA component only. Besides this, we assumed degradation as an averaging effect which is not true in practice. This chapter presents a novel approach to increase the spatial resolution of HS images using the compressive sensing (CS) and a new prior called “Gabor prior”. The novelty of the proposed approach lies in the use of: (i) jointly learned CS dictionaries, (ii) estimated degradation matrix, and (iii) a new prior called “Gabor prior” in order to super-resolve the significant PCA transformed images. Given the hyperspectral images, we first represent the HS observations as linear combination of small number of basis image planes (BIPs) using principal component analysis (PCA) and the data of reduced dimension is used in our work. In order to obtain SR image for each HS band we first obtain the initial estimates of the super-resolution on this reduced dimension. Since SR is an ill-posed problem, the final solution in PCA domain (SR PCA components) is obtained by using a regularization framework. Similar to the previous work, applying inverse PCA to significant SR PCA components results in super-resolved hyperspectral bands in spatial domain. Experiments are conducted on two different HS data sets namely 31-band natural hyperspectral image (HSI) collected under controlled laboratory environments and 224-band real HS images collected by remote sensing sen-
Previous Work

Super-resolution enhancement refers to an algorithmic approach to overcome the inherent spatial resolution limitation of imaging systems [153]. Tsai and Huang [20] were first to propose the SR technique in frequency domain. They reconstructed HR image from a set of aliased LR images. In the last three decades many researchers have attempted to increase the spatial resolution of the HS images using auxiliary information in the form of (i) multiple LR observations [21, 33, 154, 34, 5], (ii) HR multispectral images [22, 37], and (iii) HR panchromatic image [15, 53, 16, 23, 40, 28, 41, 54]. These algorithms are generally referred to as fusion algorithms.

Wilson et al. [21] used combination of multiple LR HSIs in order to obtain a subset of HR images while maintaining the visual information necessary for human analysis. They assume that LR HSIs are registered and are acquired from a single sensor or multiple sensors. To reduce the computations, [33] modeled HS image acquisition process as weighted linear combinations of a small number of basis image planes. A set-theoretic method is used to combine the information from multiple LR HSIs to obtain HR HSI. Another method proposed by [34] used LR multiangular HSIs that are registered using thin plate spline nonrigid transform to reconstruct HR HSI using Delaunay triangulation-based nonuniform interpolation method. Zhang et al. [5] proposed a maximum a posteriori (MAP) based multi-frame SR algorithm utilizing principal component analysis in order to reduce the complexity. Gomez et al. [22] proposed the fusion between HS and MS images using the wavelet based method. The method reported in [37] employs generalized MAP approach that makes use of stochastic mixing model in order to obtain high-resolution HSI. Explicit spectral relationship between MS and HS image is not required in this method.

Many algorithms use HR panchromatic image to fuse details into the LR HS image to enhance its the spatial resolution. Winter and Winter [15] replaces first PCA component of LR HSI with HR PAN image for resolution enhancement, but performance of this
method decreases when correlation between the spectral response of the PAN and HSI decreases. First PCA component represents the intensity component, hence the resulting HR HSI has only the intensity variations at finer resolution. The main limitation of this method is that the spectral characteristics are not preserved. A different approach presented by Hardie et al. [16] used MAP estimator for enhancing the spatial details using co-registered PAN image to obtain enhanced HSI. The method allows for any number of spectral bands in primary and auxiliary image. On a similar line Eismann and Hardie [23] used MAP estimation framework combined with a stochastic mixing model (SMM) for reconstructing subpixel spatial information. Here SMM is used to provide the constraint in estimation of HR HSI. Capobianco et al. [40] fused PAN data with Hyperion HSI using two different linear injection models, namely single spatial detail (SSD) and the band-dependent spatial detail (BDSD) models. In the SSD model same PAN image is used to enhance all the bands of HSI, while in BDSD model an optimum detail image is extracted from the PAN data and the same is fused with the HSI, thus providing more accurate results than SSD model. Garzelli et al. [28] proposed constraint spectral angle (CSA) fusion to preserve spectral properties in HR HSI while increasing its spatial resolution. A method proposed by Bar et al. in [41] extracts anomalies from LR HSI captured using Compact army Spectral Sensor (COMPASS) and a subregion from the HR PAN image is extracted to match each anomaly resulting in HR HSI. Few researchers have proposed methods that use fusion as preparation stage and uses them in linear mixture model to improve the performance of their algorithm [54, 53]. All these methods based on fusion of HR MS or PAN data require accurate coregistration of LR HSI and HR image acquired over the same area.

Availability of auxiliary information can be very expensive or sometimes impossible, hence indirect approaches based on spectral mixture analysis and learning [24, 26, 60, 62], and compressed sensing [61, 79] were proposed by the researchers. A method proposed by Gu et al. [24] first obtains abundance map using linear spectral mixture analysis (LSMA). Then based on spatial correlation of landcovers, learning based SR mapping is performed by using back propagation neural network (BPNN) to enhance the spatial resolution of HSI. Villa et al. In a different approach Mianji et al. [26] used LR test image and its downsampled version to train the BPNN. They perform learning based SRM after SMA by considering spatial correlation of different materials present in the HS image.
Villa et al. [62] proposed the algorithm in which first the spectral unmixing is performed to determine proportion of endmembers in each pixel, then subpixels are located by SR mapping performed either by simulated annealing or pixel swapping in unsupervised way. However, the limitation of these algorithms is the requirement of high computational load because of large number of spectral bands of HSIs. In a method proposed by Zhao et al. [61] used trained dictionaries created from different PAN images which are rich in edges and textures. By utilizing the sparse representation and spectral regularization based on linear mixing model (LMM) they obtained HR HSI. Here all bands are super-resolved individually without applying dimensionality reduction, which increases computational complexity of the algorithm.

In this chapter, we present a novel approach for super-resolution of HSIs that uses CS theory and Gabor prior. The compressive sensing theory is used for obtaining a close approximation to SR which in turn is used to obtain the final solution by using regularization framework in which we use a new prior called “Gabor prior” which is based on a bank of bandpass filters. Our method makes use of HR and LR registered HS training images to create CS dictionaries corresponding to LR and HR data. This is one time offline process. The training set consists of the HS data of an HR HSI sensor that can also be used for capturing the LR data. One approach to super-resolve HSI is to obtain SR for each spectral band separately. But this results in two major problems (i) Due to hundreds of spectral bands computational load is increased manifold, (ii) HS bands are highly correlated, hence considering each band independently do not exploit the correlation among them explicitly which in turn results in changing the pure spatial colors resulting in spectral distortion. Hence in this chapter, we first used principal component analysis (PCA) to reduce the dimensionality of the HSI. Since most of the information is contained in first few principal components, we apply SR reconstruction to these few PCA components to obtain their HR counterparts, thereby greatly reducing the computational complexity. Using CS based approach on this reduced set we obtain an initial SR image. Since SR is an ill-posed problem, we improve the solution by using regularization that uses a suitable prior. Our prior called “Gabor prior” consists of outputs of bandpass filters which correspond to bandpass features of LR image and initial SR estimate. Use of this prior restricts the solution space of final SR image. While regularizing the solution, degradation matrix entries are not assumed to be fixed rather
estimated from the initial SR image separately for significant PCA bands. Experimental results are validated using 31-band natural HSI captured under controlled laboratory environment and 224-band remotely sensed HSI. Experimental results show that our method improves spatial resolution without introducing considerable spectral distortions. Visual and quantitative comparison validates the effectiveness of proposed algorithm.

5.2 Block Diagram of the Proposed Algorithm

A block diagram showing the procedural flow of the proposed approach of CS based SR of hyperspectral image is illustrated in Figure 5.1. Given LR test image and a training database of LR-HR HS images, the proposed technique is implemented using the following steps:

1. Reduce the dimensionality of the LR test HSI using PCA. We now have PCA transformed LR primary images consisting of most of the information of HSI.

2. Apply CS based approach to primary PCA components to obtain initial SR images in PCA domain.

3. Use the initial SR image to estimate the degradation matrix for each of the primary PCA components obtained in step 2 that represents the observation model.

4. Regularize using Gabor priors and observation model estimated in step 3 to obtain the final super-resolved images in PCA domain.

5. Apply inverse PCA to obtain final SR HSI in spatial domain.

The spectral content of HSIs are inherently low dimensional, hence we exploit it by using PCA, a standard tool for analysis of multivariate data [8]. In our approach, we first represent the HS observations from different wavelengths, as a weighted linear combination of small number of basis image planes using principal component analysis (PCA) transform. The first few principal components referred to as primary components contain most of the information of HS observations and remaining PCA components referred to as secondary components contain very less information. In our work we do not consider secondary components as they represent very small portion of total information.
The proposed super-resolution algorithm is applied on the reduced set of primary PCA images to decrease computational burden of the algorithm.

In the next step, we need trained LR and HR dictionaries in CS based approach to obtain initial SR estimate. LR and HR raw dictionaries of respective PCA components, consisting larger number of atoms are generated using training database of LR and HR hyperspectral images. These raw dictionaries of the PCA components are jointly trained using K-singular value decomposition (K-SVD) algorithm [145], obtaining optimum number of atoms in each dictionary. We train a pair of dictionaries for each of the PCA band. An initial SR is obtained by using these dictionaries in CS based reconstruction.

We assume linear image observation model for the proposed SR algorithm. The LR image is modeled as the aliased and noisy version of the corresponding HR image. Using initial SR PCA image and LR test PCA image, degradation matrix entries are estimated for each PCA component. Estimated degradation matrices are used to further regularize the initial solution.

Our method of obtaining initial SR estimate do not consider the contextual dependencies among pixels as it is patch based. This results in artifacts in initial SR image around the patch boundaries. Hence prior knowledge about HS imagery is utilized in order to obtain better solution. Regularization based on Gabor prior is performed in order to obtain final solution for each of the initial SR PCA components. A simple optimization technique like gradient descent is used to minimize the cost function. This results in final SR HSI in PCA domain. Applying inverse PCA transformation on primary SR PCA components yields final super-resolved HS image.

5.3 Proposed Approach

5.3.1 Use of PCA and CS for Dimensionality Reduction and Sparseness Estimation

Hyperspectral image consists of large number of spectral bands and the spectral content of HS images are inherently low dimensional, hence this must be exploited. In this work, we first use principal component analysis (PCA) on LR test image as well as on LR-HR pairs of training HSIs to construct dictionaries. After learning these dictionaries using
5.3 Proposed Approach

Figure 5.1: Detailed block diagram of proposed approach for HS image super-resolution algorithm. Here blocks are not drawn as per scale. The size of LR HSI is $M \times M \times B$, primary PCA images are of size $M \times M \times K$, secondary PCA images are of size $M \times M \times (B - K)$, initial SR PCA and final SR PCA are of size $rM \times rM \times K$, and SR HSI is of size $rM \times rM \times B$, where $r$ is super-resolution factor.

K-SVD we use CS framework to obtain initial SR of all significant PCA components. In chapter 4, we already discussed on PCA in section 4.2.1. Here it is discusses to maintain continuity in discussion of present work.

Suppose we have a dataset of hyperspectral bands, represented by the matrix $L = [L_1, L_2, \ldots, L_B]_{M^2 \times B}$, where $L_i$, $i = 1, 2, \ldots, B$ is the $i^{th}$ hyperspectral band of size $M \times M$ arranged in lexicographical order and $B$ is the total number of bands of HSI. A set of eigen vectors $E = [e_1, e_2, \ldots, e_B]_{B \times B}$ are computed from the covariance matrix, $\sum = \sum_{i=1}^{B} (L_i - m_L)^T (L_i - m_L)$, where $m_L$ is the average image intensity defined by $m_L = \frac{1}{B} \sum_{i=1}^{B} L_i$. Here the top $K \ll B$ eigen vectors corresponding to maximum eigenvalues contribute to maximum information of HS observations and the remaining $B - K$ eigen
vectors cover very less information. By projecting HS observations on these eigen vectors we obtain $K$ number of primary PCA components and $B - K$ number of secondary PCA components each of dimension $M \times M$. Here we use compressive sensing (CS) framework which is already described in chapter 3 in section 3.1.1. In present work, we use dictionaries constructed from the available empirical data as discussed in chapter 4 sections 4.2 and 4.3.1. The difference lies in the use of joint dictionary learning using K-SVD algorithm on all raw dictionaries of significant PCA components.

### 5.3.2 Generating Trained Dictionaries

Learned dictionary provides more compact representation of the signal compared to raw dictionary which simply samples large amount of patches. This results in substantial reduction in computation while estimating the initial approximation to SR. CS based approach used in our earlier work is further extended here by using joint learning of dictionaries. Here the CS is applied on all the primary PCA bands and the initial estimates are derived for each of the primary bands. We create a set of HR and corresponding set of LR patches using a training database. Here we choose training images of the same class so that basis vectors of PCA better represent the materials of interest. The choice of basis vectors depends on the field of application. Note that dictionaries are not created directly from all bands of HS image, rather we use primary PCA images corresponding to maximum variability of HSI.

We first apply PCA on the training HR and LR HSIs and retain only primary PCA components of HR and LR both, then work on the first three primary components to obtain SR in PCA domain. We construct the joint dictionaries by randomly choosing raw patches from the HR and corresponding LR PCA components. Considering a resolution factor of $r = 4$, we select HR patch of size $8 \times 8$ (i.e. atom of $64 \times 1$ vector) and corresponding LR patch of size $2 \times 2$ (i.e. atom of $4 \times 1$ vector). Appending the LR to HR vector we obtain a joint vector of size $68 \times 1$. This way, we obtain a joint raw dictionary for each PCA component, having large number of patches (100000). We now have three dictionaries of raw patches corresponding to three primary components each of size $68 \times 100000$.

By assigning different weightages to HR and LR patches, two dictionaries in the high
and low-resolution spaces are balanced while training to achieve better initial estimate. These dictionaries were trained using K-SVD algorithm \cite{145}, obtaining optimum number of atoms in each dictionary. During dictionary training, we keep arbitrary number of atoms to represent each signal until a specific representation error is reached. It is to be noted that this kind of dictionary learning reduces the reconstruction error while obtaining the initial estimate. This way the number of atoms of joint dictionary are reduced to 1000 and the size of each dictionary becomes $68 \times 1000$. From the joint dictionary we separate out HR and LR trained dictionaries to obtain two dictionaries each of size $64 \times 1000$ and $4 \times 1000$, respectively. Now we have HR and LR dictionaries $D_{Hm}$ and $D_{Lm}$, $m = 1, ..., K$ corresponding to $K$ number of primary PCA components.

It is to be noted that dictionary training is one time and offline procedure. Using these dictionaries and LR HSI test images as inputs, initial SR PCA estimates are generated using CS based approach. Corresponding to each primary PCA component, we have a pair of LR-HR dictionaries. These PCA dictionaries are used in CS based approach to obtain initial SR estimates in PCA domain.

### 5.3.3 Initial Estimate of Super-resolution

Here we use dictionary based approach on the primary principal components to obtain the initial SR estimates in PCA domain. Due to similar statistical properties, HS test image patches can be represented as a sparse linear combination of LR dictionary elements using equation (3.1). We assume that the same sparsity holds good for its corresponding HR image which is unknown. Hence one may recover HR image using the HR dictionary.

Given LR HS test image cube $L$ of size $M \times M \times B$ and trained dictionaries $D_{Hm}$ and $D_{Lm}$, $m = 1, ..., K$, the proposed algorithm to obtain initial SR images (primary SR PCA) of size $rM \times rM$, where $r$ is the super-resolution factor, is described below.

1. Generate mean subtracted LR HS test image $L_{ms} = [L_{1ms}, L_{2ms}, ..., L_{Bms}]_{M^2 \times B}$, where $B$ is the total number of bands of HSI. Here all mean subtracted bands of LR HS image are arranged lexicographically to convert them in vectors of size $M^2 \times 1$.

2. Obtain basis eigen vectors of the covariance matrix $C = [L_{ms}^T L_{ms}]_{B \times B}$. Retain primary basis eigen vectors $e_{lm}, m = 1, ..., K$, $K \ll B$ corresponding to maximum variability of data (i.e., highest variance), each of size $1 \times B$. The percentage of
information retained in primary PCA components is given by

\[ \text{Information retained} = \frac{\sum_{i=1}^{K} \lambda_i}{\sum_{i=1}^{B} \lambda_i} \times 100. \]  \hspace{1cm} (5.1)

3. Create LR transformed images \( Y_m, m = 1, ..., K \) by projecting \( L_{ms} \) on primary basis eigen vectors generated in step 2

\[ Y_m = e_{lm} \times L_{ms}^T; \]

\( Y_m \) is still a vector representing \( m^{th} \) image. Compute \( Y_m, m = 1, ..., K \) and obtain matrix of size \( M \times M \) for each test image. We now have \( K \) number of primary PCA transformed test images each of size \( M \times M \).

4. Consider a patch of size \( b \times b \) from the transformed \( m^{th} \) LR test image \( Y_m \). Convert it into lexicographic order to obtain a vector \( y \) of size \( b^2 \times 1 \).

5. Solve CS based \( l_1 \)-minimization optimization problem i.e.,

\[ \min \| x \|_1 \text{ such that } y = D_{Lm} x. \]

Here \( x \) gives sparse representation of test patch \( y \) in terms of LR dictionary \( (D_{Lm}) \) atoms.

6. Obtain SR patch using \( z = D_{Hm} x \). This is the SR patch in the PCA transform domain.

7. Repeat steps (4) to (6) for all patches of \( Y_m \) to obtain transform domain HR image \( Z_m \). The same procedure is repeated for other primary images that gives initial SR approximation to all primary LR components. Note that the spatial dependency still exists within the pixels of these transform domain SR images. However, different PCA components of HS image are uncorrelated. Hence there is no spatial dependency of pixels among different PCA components.

Thus we obtain the initial estimate of the SR image for each test image having a size of \( rM \times rM \). We considered patch size of \( b = 2 \) for \( r = 4 \). Note that the inverse PCA using \( Z_m, m = 1, 2, ..., K \) can be used to obtain super-resolved HSIs in the spatial domain.
5.4 Final Solution using Regularization

Since we are estimating the initial SR image using patch based approach, the spatial correlation is not considered, as the sparse representation of patches is done independently. Hence we need to regularize it further to obtain a better solution. We restrict the solution space for the SR image by using our proposed Gabor prior. For regularization purpose one needs to have data fitting term and regularization term. Hence we first model the image formation in order to get the data fitting term.

5.4.1 Observation Model

A linear image observation model is used to relate the desired HR image to the observed LR image for decimation factor of $r$. Continuing with the transformed components the observed LR HSIs are modeled as decimated and noisy versions of the corresponding HR HSIs. Let $Y_m (m = 1, 2, ..., K)$ be the LR PCA image of $m^{th}$ PCA band of size $M \times M$ and and $Z_m$ be the corresponding HR PCA image of size $rM \times rM$, then the model of image formation is represented as:

$$y_m = D_m z_m + n, \quad m = 1, 2, ... K,$$

where $y_m$ and $z_m$ represent the lexicographically ordered vectors of size $M^2 \times 1$ and $r^2M^2 \times 1$, respectively with $z_m$ representing the SR vector to be estimated. Here $n$ is the independent and identically distributed (i.i.d.) noise vector with zero mean and variance $\sigma_n^2$ and has the same size as that of $y$. $D_m$ is the downsampling/decimation matrix taking care of aliasing caused as a result of downsampling. For an integer downsampling factor of $r$, matrix $D_m$ consists of $r^2$ non-zero elements along each row at appropriate locations. It models the integration of light intensity that falls on the HR detectors of corresponding spectral bands.

In most of the earlier research, either implicitly or explicitly the same degradation matrix $D_m$ with fixed entries is considered to construct degradation model for all bands of the multispectral and HSIs [23, 13, 61]. This clearly means that LR pixel of any band is considered as equally weighted sum of corresponding $r^2$ HR pixels for all bands, i.e., the ideal squared response optical point spread function (PSF) is considered. Generally,
the decimation matrix used to model aliased pixel intensities from the corresponding HR pixels for a decimation factor of $r$, has the form \cite{126}

$$D_m = \frac{1}{r^2} \begin{pmatrix} 1 & 1 \ldots & 1 & 0 \\ & 1 & 1 \ldots & 1 \\ & & \ddots & \vdots \\ 0 & \ldots & \ldots & 1 \end{pmatrix}, \tag{5.3}$$

In practice, many factors like diffraction, shape, location, physical construction and electronic response of the detectors, and the electronics of the amplifications contribute to the PSF of any spaceborne radiometer. The effect of diffraction is significant at higher wavelength in the HS imager. This results in spatially and spectrally varying PSF of degradation function. For more details on this readers may refer to the \cite{17}.

In our work, we do not consider LR pixel as sum of equally weighted HR pixels, rather we estimate the alias by estimating the entities of matrix $D_m$. For the estimation of aliasing we need true HR image which is not available. Since we have availability of initial estimate of SR image, we estimate decimation matrix $D_m$ using the the available LR PCA test component and initial SR PCA component. Then the form of decimation matrix for $m^{th}$ LR-HR pair is modified as below

$$D_m = \begin{pmatrix} d_{1,m} & d_{2,m} & \ldots & d_{r_2,m} \\ & d_{1,m} & d_{2,m} & \ldots & d_{r_2,m} \\ & & \ddots & \vdots \\ 0 & \ldots & \ldots & d_{1,m} & d_{2,m} & \ldots & d_{r_2,m} \end{pmatrix}, \tag{5.4}$$

where $0 < d_{i,m} < 1, i = 1, 2, \ldots r^2$ are unknown. Here we use a simple least squares approach to estimate the decimation coefficients $d_i$. It is worth to mention that for each primary PCA component we estimate the decimation matrix $D_m$ i.e., different PSF is considered for each primary spectral basis. Hence the estimated $D_m$ matrix for each PCA component is close to the true degradation of HSI, hence incorporation of this degradation model leads to better solution.
5.4 Final Solution using Regularization

5.4.2 Regularization using Gabor Prior

Hyperspectral images contain various textured regions having different frequency contents. Hence it is necessary that these frequency details are preserved in the SR image. This can be achieved using a prior that incorporates the information about the details at various frequencies. In computer vision community, a linear filter named Gabor filter, is widely used for feature extraction at various bandpass frequencies. Frequencies and orientation representation of this filter are similar to those of the human visual system and they have been found very useful for texture representation and discrimination. This motivates us to use Gabor prior for regularization in our work. The impulse response of Gabor filter is given by [155]

\[ G(p, q, f, \theta, \sigma_p, \sigma_q) = e^{-\frac{1}{2} \left( \frac{p^2}{\sigma_p^2} + \frac{q^2}{\sigma_q^2} \right)} \cos(2\pi fp'), \] (5.5)

where \((p, q)\) represents spatial coordinates, \((p', q') = (pcos\theta + qsin\theta, -psin\theta + qcos\theta)\), \(\sigma_p\) and \(\sigma_q\) are the spatial extent of the filter in \(p\) and \(q\) directions, respectively. Here \(f\) is the center frequency of sinusoidal carrier wave, and \(\theta\) is its orientation.

Using a data-fitting term, and Gabor prior terms, the final cost function to be minimized for each PCA band image \(m = 1, 2, ..., K\) can be written as,

\[ \epsilon_m = ||y_m - D_m z_m||^2 + \lambda_1 \sum_{j=1}^{Q} ||G_j y_m - G_j(D_m z_m)||^2 + \lambda_2 \sum_{j=1}^{Q} ||G_j \hat{z}_m - G_j z_m||^2, \] (5.6)

where \(y_m\) is \(m^{th}\) band of LR, \(D_m\) represents degradation matrix for \(m^{th}\) band, estimated using the LR and initial SR image. Here \(\hat{z}_m\) and \(z_m\) are the SR images of \(m^{th}\) band of initial estimate and final estimate, respectively. \(G_j\) is \(j^{th}\) Gabor filter matrix representing the impulse response given in equation (5.5), and \(Q\) is the total number of filters in the Gabor filter bank. \(\lambda_1\) and \(\lambda_2\) represent the weightages given to the second and third term, respectively, chosen empirically. This way we obtain \(K\) number of primary SR PCA components. It is to be noted that while solving the HSI SR problem degradation matrix is usually considered as fixed entries for all spectral bands [13, 5]. However, in this work a matrix with different entries is considered for all spectral bands. This way an optimum linear observation model is considered for all primary bands. Our prior in second term of equation (5.6) imposes the condition that degraded SR image should possess features
similar to that of the LR test image when viewed at different frequencies. This means we look for a solution i.e., SR image whose downsampled version has the same Gabor features as that of LR input image when passed thorough the same Gabor filter bank. This is illustrated in Figure 5.2. Similarly, final term in equation (5.6) indicates that features of different frequency contents in final SR image should be identical to that of initial SR estimate. Use of the available initial SR estimate as an initial solution speed-up the convergence. Applying inverse PCA to all primary SR PCA images results in SR hyperspectral image in spatial domain.

![Figure 5.2: Gabor prior for SR image $Z_m$. Here the output $G_j D_m Z_m$ represents the image details at a particular frequency band which have to match with the details of $Y_m$ when it is passed through the same filter $G_j$. $\{G_j\}, j = 1, \ldots, Q$ represents a Gabor filter bank.](image)

### 5.5 Experiments and Result Analysis

In this section, we show the effectiveness of the proposed method by conducting experiments on two different data sets: (1) Natural hyperspectral images, and (2) Remotely sensed HSIs (AVIRIS). The data sets used in the experiments constitute images with 31 and 224 spectral bands, respectively. Detailed analysis of the results is performed on 224 band AVIRIS HSI. We performed experimentations for $r = 2$ as well as for $r = 4$. Due to the space constraint, we are demonstrating results only for the case of $r = 4$. For all our experiments, the step size for gradient descent algorithm was chosen as 0.01. The weightage to second and third terms were set as $\lambda_1 = 0.14$ and $\lambda_2 = 0.17$, respectively in the regularization equation (5.6). These were chosen by trial-and-error procedure for all experiments. Here, one can use a generalized cross-validation technique [156] to identify
the optimum values of $\lambda_1$ and $\lambda_2$, but it is computationally expensive and it is specific to a given image only. We show the visual as well as quantitative comparison for experiments on both data sets. Different quantitative measures used in our experiments are described in the following section.

5.5.1 Quantitative Evaluation Measures

For quantifying the results on 31-band natural HSI we used mean squared error (MSE) as a preliminary evaluation index which is discussed in chapter 3 section 3.3.1 (see equation 3.15), where $F_{i,j}$ and $\hat{F}_{i,j}$ represent the true HR (groundtruth) and the SR images, respectively. Detailed quantitative evaluation of spatial and spectral fidelity of super-resolved hyperspectral images is performed using different measures such as correlation coefficient (CC), erreur relative globale adimensionnelle de synthese (ERGAS), and $Q^{2n}$. CC and ERGAS are defined in chapter 4 section 4.4.1 (see equations 4.6, 4.7 and 4.9). What follows is the description of $Q^{2n}$.

$Q^{2n}$ [157] index is derived from the theory of hyper-complex numbers of $2^n$-ons (pro- nunciation: two-to-the-any-ons) [158]. It takes into account the correlation, mean of each spectral band, intra-band local variance, and the spectral angle. Both spectral and spatial distortion metrics are encapsulated in this index. It takes a real value in the interval 0 to 1, with 1 being the best value. It is defined between $k^{th}$ super-resolved ($F$) and groundtruth ($\hat{F}$) bands as:

$$Q^{2n}_k = \frac{\text{cov}(F, \hat{F})}{\sigma_F \sigma_{\hat{F}}} \cdot \frac{2||\hat{F}|| ||\hat{F}||}{||\hat{F}||^2 + ||\hat{F}||^2} \cdot \frac{2\sigma_F \sigma_{\hat{F}}}{\sigma_F^2 + \sigma_{\hat{F}}^2},$$  \hspace{1cm} (5.7)

where $k = 1, ..., B$ and $\text{cov}(F, \hat{F})$ is the covariance between bands $F$ and $\hat{F}$. $\sigma_F^2$ and $\sigma_{\hat{F}}^2$ are the variances of $F$ and $\hat{F}$, respectively. Here we averaged $Q^{2n}$ over the all HSI bands to get a global measure of spatial and spectral distortion of the super-resolved HSI i.e.,

$$Q^{2n} = \frac{1}{B} \sum_{k=1}^{B} Q^{2n}_k,$$  \hspace{1cm} (5.8)

where B is the total number of bands in hyperspectral image, which is 196 in our case.
5.5.2 Experiments on Hyperspectral Images

The first set of our experiment consists of 31-band reflectance images of natural scene, having spectral range of 400nm – 700nm all acquired under the direct sunlight in clear or almost clear sky [149]. Our second HSI data set is comprised of 224 bands of AVIRIS HSI cube \(^1\). After discarding few bands having low signal to noise ratio (SNR), 196 bands were used for super-resolving by a factor of 2 and 4, respectively. The band removal was based on visual inspection of the images. The above data sets have high spatial dimensions and hence specific regions are cropped from them and experiments are carried out on the cropped regions. Here, we do not have the true LR-HR pairs of HSIs. Hence the low spatial resolution (LR) images were created from these cropped images by using filtering and downsampling operations. The whole HSI cubes are used to generate raw dictionaries by random selection of patches. Use of same cube to create raw dictionaries ensures inclusion of large number of materials and objects of interest. Note that we need LR-HR pairs to construct dictionaries. If these images are acquired offline they can be utilized to form the pairs. One may also use the LR and HR images of the same scene captured by using different sensors but after applying the radiometric and geometric (registration) corrections. In order to evaluate the performance of our approach using quantitative measures, we need the groundtruth images. Since these images are not available, we consider original cropped HSIs of size 256 × 256 as ground truths and generated the LR HSIs of size 128 × 128 and 64 × 64 by applying downsampling operation by a factor of \(r = 2\) and \(r = 4\), respectively. The SR algorithm was then applied on these LR HSIs. In order to restrict the maximum spatial frequency in the image we use low pass filtering operation before downsampling. The low pass filtering operation was performed and tested using Gaussian filter with standard deviation of 0.5. For this purpose we used filter mask of size 5 × 5. While performing joint training of dictionaries weightages given to HR patches and LR patches were 0.65 and 0.35, respectively.

5.5.3 Experiments on 31-band Natural Hyperspectral Image

In this section, effectiveness of the proposed algorithm on the 31-band natural HSI is evaluated. Here we use cropped region of “Scene 5” of hyperspectral images of natural

5.5 Experiments and Result Analysis

Figure 5.3: Experimental results on PCA-1 of 31-band natural HSI for $r = 4$. (a) LR test image of size $64 \times 64$, (b) Ground truth of size $256 \times 256$, (c) Bicubic interpolation [148], (d) Iterative backprojection method [152] (e) SR image using Yang et al. method [79], and (f) SR image using the proposed approach.

scenes 2002² as test data. In this case we found that 99.4% of spectral variance is covered by first three principal components. Hence we retained these three PCA components and applied SR algorithm in our experiment. Figure 5.3 displays the results for the PCA-I component. Figures 5.3(a) and (b) display the LR test image and the ground truth image of size $64 \times 64$ and $256 \times 256$, respectively. The results obtained using different methods are shown in Figures 5.3(c-f). Visual inspection of images in Figures 5.3(c-e) indicate that the white borders of “D” shape mounted on the box appear blurred in the bicubic interpolated image, iterative backprojection (IBP), and the method proposed by Yang et al. [79]. One can see in Figure 5.3(f) that the SR image obtained using the proposed method compares well with the groundtruth. The white border of ”D“ shape is sharper

²http://personalpages.manchester.ac.uk/staff/david.foster/Hyperspectral_images_of_natural_scenes_02.html
5.5 Experiments and Result Analysis

Table 5.1: Quantitative evaluation measures for SR of 31-band Natural hyperspectral image using different techniques for \( r = 4 \)

<table>
<thead>
<tr>
<th>Quantitative Measures</th>
<th>Bicubic interpolation [148]</th>
<th>Iterative backprojection [152]</th>
<th>Yang et al. [79]</th>
<th>Proposed approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE-PCA-1</td>
<td>0.0072</td>
<td>0.0069</td>
<td>0.0065</td>
<td>0.0053</td>
</tr>
<tr>
<td>MSE-PCA-2</td>
<td>0.0388</td>
<td>0.0368</td>
<td>0.0563</td>
<td>0.0308</td>
</tr>
<tr>
<td>MSE-PCA-3</td>
<td>0.7487</td>
<td>0.6716</td>
<td>0.5940</td>
<td>0.5935</td>
</tr>
<tr>
<td>( CC_{avg} )</td>
<td>0.9681</td>
<td>0.9806</td>
<td>0.9827</td>
<td>0.9857</td>
</tr>
<tr>
<td>ERGAS</td>
<td>6.6613</td>
<td>5.2086</td>
<td>4.9888</td>
<td>4.6388</td>
</tr>
<tr>
<td>( Q^2 )</td>
<td>0.9643</td>
<td>0.9789</td>
<td>0.9796</td>
<td>0.9801</td>
</tr>
</tbody>
</table>

and closer to groundtruth. Similarly, the number "60" in the image appear more clear in Figure 5.3(f) compared to others shown in Figures 5.3(c-e). Quantification of this experiment is provided in Table 5.1. From the table we can see that the MSEs between the true and the estimated SR PCA components are significantly less for the proposed method. The use of CS based approach using learned dictionaries and regularization using Gabor prior improves the results in our approach as evident from quantitative evaluation measures such as \( CC_{avg} \), ERGAS, and \( Q^2 \). Note that the MSE is computed on PCA bands directly while all other measures are computed on 31 bands after performing the inverse PCA. These measures show that proposed approach better preserves spatial and spectral fidelity in the super-resolved hyperspectral images.

5.5.4 Experiments on AVIRIS Hyperspectral Image

We now discuss the SR results for 224-band AVIRIS hyperspectral image. This data set is comprised of 224-band real hyperspectral image of Moffett Field acquired by AVIRIS hyperspectral imaging system. In this experiment, cropped region of an urban area in Moffett Field is used as test data. This cropped region is specifically chosen to include various bandpass components in the image to evaluate the performance of the proposed method. The SR results on remotely sensed data acquired using AVIRIS hyperspectral imager is shown with reduced dimension. Figures 5.4, 5.5 and 5.6 show the SR results of first, second and third PCA bands, respectively. Quantitative results are listed for the first three PCA bands that include 97.56% of spectral variance of HSI. Here the measures listed in Table 5.2 are computed over 196 bands. Figures 5.4-5.6 (a) show the LR PCA test images of size 64 \( \times \) 64 and the original PCA bands of size 256 \( \times \) 256 are displayed in
5.5 Experiments and Result Analysis

Figure 5.4: Experimental results on PCA-I band of AVIRIS data for \( r = 4 \). (a) LR test image of size 64 \( \times \) 64, (b) Ground truth of size 256 \( \times \) 256, (c) Bicubic interpolation [148], (d) Iterative backprojection method [152], (e) SR image using Yang et al. method [79], and (f) SR image using the proposed approach.

Figures 5.4-5.6 (b).

From Figures 5.4-5.6(c) we can see that when the PCA images are upsampled using bicubic interpolation they become blurred and the high frequency spatial details are lost. Roads and buildings are no longer visible in bicubic interpolated image in the PCA-I and PCA-III results displayed in Figure 5.4(c) and Figure 5.6(c), respectively. One may notice that bicubic interpolation in Figure 5.3(c) appears better than the result in Figure 5.4(c). This is because there is significant high frequency content in AVIRIS data when compared to natural HSI displayed in Figure 5.3(c) and the interpolation fails to preserve the high frequency details. This indicates that the interpolation techniques are not suitable for solving the SR problem, hence they are not considered as SR techniques. Quantitative comparisons for bicubic interpolation given in Table 5.1 and Table 5.2 further proves this observation. SR result on PCA-I of IBP [152] shown in Figure 5.4(d)
is less blurred compared to bicubic interpolated image but the overall contrast of the image is not preserved by this method. Visibility of roads and buildings has improved over bicubic interpolation in all super-resolved PCA components as seen from Figures 5.4-5.6(d). Borders of objects appear blurred in SR PCA images obtained using IBP method (Figures 5.4-5.6(d)). Sparsity based SR results of [79] method shown in Figures 5.4-5.6(e) are visually better than bicubic interpolation and IBP method, but it fails to preserve high frequency details as evident from top half portions of the images. As seen from Figures 5.4-5.6(f), the use of compressed sensing and Gabor priors regularization results has reduced artifacts and also takes care of preservation of different frequency details. Sharpness of different objects such as roads and buildings has improved over all other methods, particularly noticeable in PCA-I and PCA-II SR images shown in Figures 5.4(f) and 5.5(f), respectively. The visual quality of SR PCA images in Figures 5.4-5.6(f) is closely matching with the groundtruth. We can see that the white patches visible in LR PCA-1 observation appear grayish in Yang et al. [79] method (See Figure 5.4(e)), but the result is improved in the proposed approach as seen from Figure 5.4(f). One can clearly discriminate the road lines joining the top right (1/4 way down) to bottom left corner of the image in Figure 5.4(f) indicating that edge details are well preserved in the proposed approach.

As far as the quantitative comparison is concerned, it is clear from Table 5.2 that the proposed method provides scores that are closer to the reference values shown in the same table when compared to bicubic interpolation [148], IBP [152], and Yang et al. approach [79]. Note that the quantitative measures $CC_{avg}$ and $Q2^n$ are averaged over the 196 HSI bands. Lower value of ERGAS in the proposed method indicates lesser global distortion in super-resolved HSI. Generally a value of ERGAS below 3 is believed to be an image with good quality [124]. We see that when compared to other approaches $CC_{avg}$ and $Q2^n$ are also better for the proposed method. Lower value of $Q2^n$ in Table 5.2 indicates minimum spatial as well as spectral distortions by the proposed approach. To further support the performance improvement using our approach, we show the plot of bands Vs CC in Figure 5.7 for various methods. From the average CC value listed in Table 5.2 and the plots in Figure 5.7, we can see that the proposed method better preserves the spatial details.

In order to compare the performance in terms of spectral fidelity, we show the spectral
5.5 Experiments and Result Analysis

reflectances of groundtruth and outputs of different SR algorithms at different regions. We have chosen three different regions to show the performance of various algorithms at different frequency bands. These regions include: (i) Uniform region (A) (very low frequency) (ii) Smooth edge region (B) (mid range frequency), and (iii) High frequency region (C) having sharp variation of texture. Spatial locations of these regions are shown in Figure 5.8(a). Spectral reflectances of different SR methods for the selected regions A, B and C in Figure 5.8(a) are shown in Figures 5.8(b), (c) and (d), respectively. Here the spectral reflectance for each region is computed by using a $3 \times 3$ patch in every region and computing the average reflectance. So, we have 3 vectors of $9 \times 1$ for each band corresponding to three regions and there are total of 196 bands. The plots showing the bands Vs spectral reflectance for various approaches including the original are shown in Figures 5.8(b), (c) and (d). Separate plots are shown for each region. The average computed over the bands is given in Table 3 for quantitative comparison. In Figure 5.8(a) region A represents smooth region having no significant reflectance variations. One can see from Figure 5.8(b) that the plots of all SR algorithms closely match that of the groundtruth as far as the region A is concerned. This is also evident in Table 5.3 where we can see that average spectral reflectances of bicubic interpolation, IBP, Yang et al., and proposed approach are closer to groundtruth. This indicates that low frequency regions are better preserved by most of the approaches. Region B has mid frequency content and in this case IBP as well as Yang et al. approaches perform better in addition to the proposed method. But, there is obvious deviation in bicubic interpolation as evident from Table 5.3 entries for region B. We can see that average spectral reflectance of bicubic interpolation is significantly deviating from the groundtruth when compared to IBP and Yang et al. methods. Region C has sharp variations in texture, and in this case the spectral reflectance plot of the proposed method is closer to groundtruth as evident from Figure 5.8(d). From Table 5.3, one can see that although the approach by Yang et al. [79] performs better than other two approaches, the proposed approach performs even better.

Before we conclude we would like to compare performance of this method with the previous method discussed in chapter 4. Since ERGAS represents the overall error in the super-resolved image irrespective of resolution factor, we use the same for comparison. From chapter 4 Table 4.1, one can see that the value of ERGAS is 7.065 in proposed
Figure 5.5: Experimental results on PCA-II band of AVIRIS data for $r = 4$. (a) LR test image of size $64 \times 64$, (b) Ground truth of size $256 \times 256$, (c) Bicubic interpolation [148], (d) Iterative backprojection method [152], (e) SR image using Yang et al. method [79], and (f) SR image using the proposed approach.

approach when we considered a resolution factor of 2, while it is 2.9725 (see Table 5.2) by the approach proposed in this chapter, even though the resolution difference between LR and SR is 4. This indicates reduced global distortion for the approach proposed in this chapter. The use of learned dictionaries in compressive sensing based approach on all significant components improves initial estimate in the present method when compared to the use of raw dictionaries in chapter 4. Bicubic interpolation of remaining significant components (except first one) cannot preserve high frequency details in the initial SR estimate discussed in the previous chapter. The performance is further improved due to the use of estimated entries of decimation matrix here, as well as use of Gabor prior. In this approach, the trained dictionaries have the tendency to adapt to local structures of the images, and the regularization based on Gabor prior preserves the spectral as well as spatial information better.
Figure 5.6: Experimental results on PCA-III band of AVIRIS data for $r = 4$. (a) LR test image of size $64 \times 64$, (b) Ground truth of size $256 \times 256$, (c) Bicubic interpolation [148], (d) Iterative backprojection method [152], (e) SR image using Yang et al. method [79], and (f) SR image using the proposed approach.

5.6 Conclusion

We have presented SR algorithm for HSIs based on the compressed sensing theory in which jointly learned dictionaries are used to obtain SR images in reduced dimension space. We construct the raw dictionaries of LR and HR from a training database and used K-SVD algorithm to obtain learned dictionaries for all primary PCA components. Using learned dictionaries in CS based approach we obtain initial estimates of SR for each primary PCA component in the PCA domain. High spatial resolution initial SR images and the corresponding low-resolution observed images in PCA domain were used to estimate the decimation. Subsequently, a regularization scheme is employed using Gabor priors considering varying degradation or PSF in the spectral space. Gabor prior was considered on downsampled as well as HR versions of initial estimates.
Table 5.2: Quantitative evaluation metrics of AVIRIS SR for $r = 4$

<table>
<thead>
<tr>
<th>Method</th>
<th>$CC_{avg}$</th>
<th>ERGAS</th>
<th>Q2$^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bicubic interpolation [148]</td>
<td>0.9163</td>
<td>5.2020</td>
<td>0.9074</td>
</tr>
<tr>
<td>IBP [152]</td>
<td>0.9509</td>
<td>3.9862</td>
<td>0.9451</td>
</tr>
<tr>
<td>Yang et al. [79]</td>
<td>0.9635</td>
<td>3.4362</td>
<td>0.9513</td>
</tr>
<tr>
<td>Proposed Approach</td>
<td>0.9807</td>
<td>2.9725</td>
<td>0.9681</td>
</tr>
<tr>
<td>Reference</td>
<td>1.0000</td>
<td>0.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Figure 5.7: Correlation coefficient Vs band number for SR on AVIRIS data for $r = 4$

Table 5.3: Spectral reflectance (%) of $3 \times 3$ pixels averaged over 196 bands at different region locations shown in Figure 5.8(a) of AVIRIS SR for $r = 4$

<table>
<thead>
<tr>
<th>Method</th>
<th>Region A</th>
<th>Region B</th>
<th>Region C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bicubic interpolation [148]</td>
<td>81.2244</td>
<td>67.7482</td>
<td>58.9277</td>
</tr>
<tr>
<td>IBP [152]</td>
<td>82.3980</td>
<td>76.5466</td>
<td>66.7565</td>
</tr>
<tr>
<td>Yang et al. [79]</td>
<td>82.9710</td>
<td>86.1222</td>
<td>68.2510</td>
</tr>
<tr>
<td>Proposed Approach</td>
<td>85.2926</td>
<td>81.9088</td>
<td>72.9670</td>
</tr>
<tr>
<td>Ground truth</td>
<td>85.9464</td>
<td>82.9792</td>
<td>74.2430</td>
</tr>
</tbody>
</table>

The advantage of the proposed technique is that there is no need of auxiliary registered HR image or multiple LR observations of the same scene with subpixel shifts while super-resolving. Use of Gabor prior in regularization preserves features at bandpass spatial frequencies. Use of estimated entries of degradation matrices for all significant PCA components represent the optimum PSF in regularization that aids in obtaining better solution. Super-resolution results obtained using proposed method show better preservation of spatial details over those obtained using raw dictionaries and averaged PSF. Quantitative comparison of score indices show that our method enhances spatial information without introducing significant spectral distortion.
The proposed super-resolution technique uses learned dictionaries of LR and HR images to produce the initial super-resolved images in reduced dimensional space. It may be noted here that we estimate degradation using the initial estimate of SR. It is to be noted that degradation matrix entries considered here are non-overlapping and used while obtaining final results of SR. In the next, chapter we consider the estimation of degradation in the form of PSF representing the low-pass filtering by estimating filter coefficients. The estimated wavelet filter coefficients are used to define the degradation matrix with overlapping entries while obtaining initial as well as final super-resolved HSI.