Chapter 4

Super-Resolution of Hyperspectral Images using Compressive Sensing Framework

Over the past decade hyperspectral (HS) image analysis has turned into one of the most powerful and growing technologies in the field of remote sensing. Hyperspectral sensors collect information in the form of reflectance spectra in very narrow contiguous bands simultaneously in the visible to mid infrared portion of the spectrum i.e., 0.4-2.5µm. They represent an evolution in technology from multispectral sensors, which typically collect spectral information in only a few discrete, noncontiguous bands [1]. In general HS images have hundreds of spectral bands [129]. Being spectrally overdetermined, they provide ample spectral information to identify and differentiate spectrally unique materials [2]. The high spectral resolution of hyperspectral sensors preserves important properties of the spectrum and makes possible better discrimination of different materials on the ground [1]. Hence they have been proven to be a powerful source for the monitoring of the Earth surface and the atmosphere on global as well as local scales [130]. Nowadays, HS images are widely used in wide range of military and defence applications which include target detection and tracking of objects [131], agriculture planning [132], forest inventory [133], and urban monitoring [134] to mention a few. These applications require high spectral and high spatial resolution data for accurate determination of object properties.

In practice, few satellites have high spatial resolution ( < 5m × 5m ) sensors available
onboard. For example, Quickbird satellite collects panchromatic (PAN) image of spatial resolution 0.7\(m\) and four multispectral bands of 2.8\(m\) resolution. Recently launched WorldView-1 and WorldView-2 satellites carry an imaging instrument specifically designed to meet the requirements of very high spatial resolution and more number of spectral bands. WorldView-1 provides a single PAN image of half a meter resolution, while WorldView-2 provides a high spatial resolution (0.46\(m\)) PAN image and eight spectral bands having resolution of 1.84\(m\). Although they have high spatial resolution data, they do not provide better discrimination of many surface materials and environmentally relevant information due to their limited spectral resolution. An improved spaceborne hyperspectral sensors (e.g. Hyperion and Chiris) provides very large number of narrow spectral bands, but their spatial resolution is very less (17 – 34\(m\)). For airborne hyperspectral sensors (e.g. AVIRIS, HYDICE, HyMap) the spatial resolution is dictated largely by the height of the aircraft [18]. As the height increases the extent of coverage also increases but this decreases the spatial resolution. At low spatial resolutions, the averaging effect on the pixels degrades the performance of spectral detection algorithm, which is used to identify the materials present in the scene [24]. Though the HS images cover large area at fine spectral resolution, their spatial resolutions are often limited for the use in various applications. Hence improving their resolution has a high payoff. This chapter presents a novel approach for super-resolution (SR) of HS images using compressive sensing (CS). Besides ill-posedness of SR problem, the main challenge in HS super-resolution is to preserve spectral contents among all bands while increasing their spatial resolutions.

In this work, given the hyperspectral (HS) images we first obtain an initial estimate of the super-resolution on a reduced dimension HS data. The dimensionality reduction is obtained by using principal component analysis (PCA). Our approach uses CS based method to super-resolve the most informative PCA transformed image representing highest spectral variance (i.e. the first principal component). We make use of low and high spatial resolution dictionaries of patches generated by random sampling of raw patches of PCA transformed images that are generated using the training images of LR and HR having similar statistical properties. Using the sparsity constraint, low-resolution test patch is represented as a sparse linear combination of relevant dictionary elements. Finally, assuming that same sparseness holds for LR and corresponding HR patches an
initial estimate of super-resolved PCA is obtained. Since SR is an ill-posed problem, we obtain the final solution using a regularization framework considering the sparse coefficients obtained by the CS approach and the autoregressive (AR) parameters obtained from the initial estimate. The SR for remaining PCA images is obtained by performing bicubic interpolation and regularization, considering the same AR parameters which were obtained from the initial SR estimate of first PCA component. Application of inverse PCA results in SR of HSI bands in original spatial domain. Experiments are conducted on two different kinds of HS images. Visual inspections and quantitative comparison confirm the effectiveness of the proposed method.

4.1 Previous Work

Super-resolution enhancement refers to an algorithmic approach for increasing the spatial details [135]. Many researchers have attempted to increase the spatial resolution of the HS images by fusing the PAN image and the hyperspectral data [15, 16, 23, 40]. Winter et al. [15] replaced first PCA component of LR hyperspectral image (HSI) with HR PAN image for resolution enhancement, but performance of this method decreases when correlation between the spectral response of the PAN and HSI decreases. First PCA component represents the intensity component, hence the resulting HR HSI has only the intensity variations at finer resolution. The main limitation of all this method is that the spectral characteristics are not preserved. In a different approach Bar et al. in [41] combined spectral and spatial analysis for detection and classification, respectively. In the detection stage they used high spectral resolution HSI to locate the target and in the classification stage high spatial resolution PAN image is fused with low spatial resolution HSI to reduce the false alarms. The limitation of all these algorithms is that they require the images to be registered. Besides this, many times high-resolution PAN imager is not available onboard, hence auxiliary high-resolution image of the same geographic area is not captured. In such circumstances our proposed method provides the solution of resolution enhancement. Recently, compressive sensing (CS) theory has drawn major attention in various image processing applications [80, 136, 81, 82, 137, 138]. Compressive sensing theory has many potential applications in signal and image processing applications. It is primarily concerned with the recovery of a vector \( \mathbf{x} \) that is sparse in some transform
domain. In this work, we present a novel approach for super-resolution of HS images from the perspective of CS which is not dependent on the registration of the different HS images. Our method needs only HR and LR registered HS images of any compatible scenery to create training dictionaries which is one time offline procedure.

4.2 Theoretical Background

In this work, we are using compressive sensing (CS) and principal component analysis (PCA). Since compressive sensing has been discussed in chapter 3 section 3.1.1 we only give the necessary details of CS and briefly discuss PCA here.

A fundamental ingredient to deploy CS theory in applications is the dictionary $D$ in equation 3.1 mentioned in section 3.1. There are three different ways to construct the dictionaries [122]: (1) Preconstructed dictionaries, like wavelets [139], contourlets [140] etc. They are generally used for “cartoon-like” images, assumed to be piecewise smooth having smooth boundaries [141, 142]. (2) Tunable dictionaries, in which a basis or frame is generated under the control of particular parameter (discrete or continuous): wavelet packets [143] (parameter is time-frequency subdivision) or bandelettes [144] (parameter is spatial position). (3) A training database of signal instances similar to those anticipated in the application, and build an empirically learned dictionary [80]. Here the entries in dictionary are chosen from the empirical data rather than from some theoretical model. In proposed approach a training database of signal instances similar to those anticipated in the application is used to build an empirically learned dictionary. Thus the entries in dictionaries are chosen from the empirical data rather than from some theoretical model. Validity of such dictionaries is examined by [80]. Such dictionary has the potential to outperform commonly used predetermined dictionaries [145]. Such a dictionary can then be used in the application as a fixed and redundant dictionary. In our application we explore the third option.

4.2.1 Principal Component Analysis

Hyperspectral images are composed of large number of spectral bands (e.g., AVIRIS acquires 224 bands). Hence, applying super-resolution technique to each band separately is prohibitive because of time complexity. In addition individual band SR does not make
use of the information present across the bands [33]. Information is present across these bands in the form of spectral signatures and the identification of ground materials of interest is based on their unique spectral signatures [1]. Besides the ill-posed nature of SR problem, the HS image SR task becomes difficult since preservation of spectral correlation combined with the SR is more challenging. The spectral content of HS images are inherently low dimensional, hence this must be exploited. Principal component analysis (PCA) plays a central role in the analysis of multivariate data [8].

Suppose we have a dataset of \( B \) hyperspectral bands with size of each band as \( M \times M \) pixels. Assume that \( \mathbf{F} \) represents the pixel vector of size \( B \times 1 \) along spectral dimension of HS image. A set of principal components of dimension \( M^2 \times K \) are computed from the first \( K \) eigen vectors \( \mathbf{E} = [\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_K]^T_{B \times K} \), which in turn is computed from the covariance matrix \( \Sigma \) of size \( B \times B \) from the given data set

\[
\Sigma = \frac{1}{M^2 - 1} \sum_{i=1}^{M^2} (\mathbf{F}_i - \mathbf{m}_F)(\mathbf{F}_i - \mathbf{m}_F)^T, \tag{4.1}
\]

where \( \mathbf{m}_F = \frac{1}{M^2} \sum_{i=1}^{M^2} \mathbf{F}_i \).

Here \( \mathbf{m}_F \) is the vector representing average band intensity. Here the top \( K \ll B \) eigen vectors corresponding to maximum eigenvalues contribute to maximum information of HS observations and the remaining \( B - K \) eigen vectors cover very less information. By projecting HS observations on these eigen vectors we obtain \( K \) number of primary PCA components and \( B - K \) number of secondary PCA components each of dimension \( M \times M \).

The most obvious feature of the principal components is that the maximum spectral variability of hyperspectral bands is contained in the first few principal components. Each component has variability in decreasing order of magnitude. Percentage of variability included in \( i^{th} \) principal component is given by,

\[
Variability = \frac{\lambda_i}{\sum_{i=1}^{B} \lambda_i}, \tag{4.2}
\]

where \( \lambda_i \) is eigen value of the \( i^{th} \) eigen vector. Since \( K \) is less than \( B \), one cannot reconstruct hyperspectral image exactly. However, the hyperspectral bands are highly correlated, and hence only a small value of \( K \) suffices to reconstruct HS image to obtain
the required details. Our algorithm works on $K$ number of primary PCA components to obtain super-resolution of HSI cube.

### 4.3 Block Diagram Description of the Proposed Method

Block diagram of the proposed approach is shown in Figure 4.1. In our approach, we first represent the HS observations from different wavelengths, as a weighted linear combination of small number of basis image planes (BIPs) using PCA transform described in section 4.2.1. The super-resolution is applied on the reduced set of PCA transformed images. To reduce computational burden of $l_1$-minimization, we first apply the CS based approach only to first PCA component to obtain initial SR estimate of PCA-I. Regularization based on AR parameters and sparsity priors is performed on this image to obtain the final SR PCA-I image. Remaining images corresponding to $K$ principal components are interpolated and regularized using AR parameters obtained from the same initial SR image. In our work we have used $K = 3$. Here our assumption is that the AR parameters learned from the initial SR PCA-I are valid for all $K$ PCA images, as we are learning the spatial dependence and not their explicit values. Applying inverse PCA on this set of SR PCA images yields super-resolved HS image.
4.3 Block Diagram Description of the Proposed Method

4.3.1 Close Approximation to Super-resolution using CS

Here, a dictionary based approach is used on the first LR principal component to learn the HR details. Training database of the same class is used to create a set of the HR and LR patches, so that BIPs of PCA better represent the materials of interest. Thus the choice of BIPs depends on the application. Note that dictionaries are not created directly from all bands of HS image, instead first PCA transformed image is used.

Suppose LR HS image of size $M \times M$ need to be super-resolved to a size of $qM \times qM$, where $q$ is the super-resolution factor. Due to similar statistical properties, HS test image patches can be represented as a sparse linear combination of LR dictionary elements using equation (3.1). The same sparsity holds good for the corresponding HR unknown image [146]. Hence we can recover the HR image using the HR dictionary. The proposed algorithm to obtain initial estimation of SR PCA-I image is described below.

1. Generate mean subtracted LR and HR training HS images of $B$ bands with $L_m = [L_1, L_2, \ldots, L_B]$ and $H_m = [H_1, H_2, \ldots, H_B]$, respectively. Here all bands of LR and HR HS images are arranged lexicographically to convert them in vectors of size $M^2 \times 1$ and $q^2M^2 \times 1$, respectively.

2. Determine basis eigen vectors corresponding to LR and HR training HS images. Retain basis eigen vectors $e_h$ and $e_l$ corresponding to maximum spectral variability of data (i.e., highest variance), each of size $1 \times B$.

3. Create HR and LR transformed images $B_H$ and $B_L$ by projecting $H_m$ and $L_m$ on their corresponding basis eigen vectors generated in step (2). $B_H = e_h \times H_m^T$; $B_L = e_l \times L_m^T$; Convert $B_H$ and $B_L$ into matrices of size $qM \times qM$ and $M \times M$ respectively.

4. For each $b \times b$ patch of $B_L$ choose the corresponding patch from $B_H$. Arrange them in lexicographic order to form the vectors $V_l$ and $V_h$ of size $b^2 \times 1$ and $q^2b^2 \times 1$, respectively.

5. Repeat step (4) to create dictionaries $D_L$ and $D_H$ of size $b^2 \times M^2/b^2$ and $q^2b^2 \times M^2/b^2$, respectively.
6. Project the mean subtracted LR hyperspectral test image on basis eigen vector $e_l$ and obtain the transformed LR test image $Y$ of size $M \times M$.

7. Consider a patch of size $b \times b$ from the transformed LR test image $Y$, convert it in lexicographic order to obtain a vector $y_p$.

8. Solve CS based $l_1$-minimization optimization problem

$$\min \|x_p\|_1 \text{ such that } y_p = D_L x_p + e.$$  

Here $x_p$ gives sparse representation of test patch $y_p$ in terms of LR dictionary ($D_L$) elements adaptively.

9. Obtain SR patch using $\hat{z}_p = D_H x_p$. This is SR patch in transform domain.

10. Repeat steps (7) to (9) for all patches of $Y$ to obtain transform domain SR PCA-I image $\hat{Z}$. Note that the spatial dependency still exists within the pixels of this image. However, different PCA components of HS image are uncorrelated. Hence there is no spatial dependency of pixels among different PCA components.

The above steps gives us the initial estimate of the SR PCA-I image $Z$ having a size of $qM \times qM$. In our experiments, we considered patch size of $b = 2$ for $q = 2$. We used a dictionary of 100000 raw patches. Here raw patches belonging to the same class were used in our experiment.

### 4.3.2 Final Solution using Regularization

Since we are estimating the initial SR image using patch based approach, the spatial correlation is not considered whenever there are discontinuities at the patch boundaries. Hence we need to regularize it further to obtain a better solution by restricting the solution space by using AR and sparsity priors. For regularization purpose one needs to have data fitting term and regularization term. Hence we first model the image formation in order to get the data fitting term.

Let $Y$ be the PCA transformed LR HS image of size $M \times M$ and and $Z$ be the corresponding HR HS image of size $qM \times qM$, then the model of image formation is represented as:

$$y = Dz + n,$$  \hspace{1cm} (4.3)
where \( y \) and \( z \) represent the lexicographically ordered vectors of size \( M^2 \times 1 \) and \( q^2 M^2 \times 1 \), respectively with \( z \) representing the SR vector. \( D \) is the downsampling matrix taking care of aliasing caused as a result of downsampling. For an integer downsampling factor of \( q \), matrix \( D \) consists of \( q^2 \) non-zero elements along each row at appropriate locations. Here \( n \) is the independent and identically distributed (i.i.d.) noise vector with zero mean and variance \( \sigma_n^2 \) and has same size as that of \( y \). In this model the LR intensity is the average of the HR intensities over a neighborhood of \( q^2 \) pixels corrupted with additive noise.

### 4.3.3 Estimation of Autoregressive Parameters

We consider learned super-resolved image as the initial estimate and regularize it further to obtain the final solution. We characterize the statistical dependence of pixel values on its neighbors, by using an AR model, where the pixel value at a location is expressed as a linear combination of its neighborhood pixel values and an additive noise [13]. We use initially estimated SR PCA-I image using CS approach as an AR model. Note that, PCA components are mutually uncorrelated across bands but there exist spatial dependency among the pixels in each of the PCA components. We estimate the AR parameters from the PCA-I component and use them for other bands. We use a homogeneous AR model and derive a set of parameters for entire SR image. Suppose \( z(s) \) is the gray level value of the image pixel at location \( s = (i, j) \) in an \( N \times N \) image, where \( i = 1, 2, ..., N \) and \( j = 1, 2, ..., N \). The AR model for \( z(i, j) \) can be expressed as [147]

\[
z(i, j) = \sum_{r \in N_s} B_r z(s + r) + \sqrt{\rho} n(s) \tag{4.4}
\]

where \( N_s \) is the neighborhood of pixel \( s \), \( r \) being a neighborhood index with \( r \in N_s \), and \( \rho \) are unknown parameters, \( n(.) \) is an independent and identically distributed noise sequence with zero mean and unit variance; \( \rho \) is the variance of the white noise that generates the spatial data for the given AR parameters. Here we use fifth order neighborhood as a compromise between local and global texture representation, that requires to estimate a total of eight AR model parameters \( B_r \) using the iteration scheme given in [147]. We are considering same neighborhood size around each pixel. The extracted AR parameters are also used to regularize other PCA images.
4.3.4 Regularization with Sparsity Coefficients and AR Parameters

Super-resolution is an ill-posed inverse problem. Prior information can enhance the quality of the solution considerably. Hence we obtain the final solution using regularization framework. Sparse coefficients obtained during the CS framework and AR parameters obtained from the initial super-resolved image are considered as prior informations. The prior knowledge about sparsity coefficients is used in determining weightage of dictionary atoms to represent HR patches. It provides the constraint of sparsity in final solution. The AR parameters plays the role of maintaining the contextual constraint used to regularize the solution. Using a data-fitting term, sparsity and AR prior terms, the final cost function is written as

\[ \epsilon = \| y - Dz \|^2 + \beta \| z - DHx \|^2 + \sum_i \sum_j \left( (z(i,j) - \sum_{l,k \in N_s} B_r z(i+l,j+k) \right)^2 \] (4.5)

Here \( \beta \) represents the regularization parameter. The above cost function is convex. Hence it can be minimized by using a simple optimization technique such as gradient descent. In order to provide a good initial guess and to speedup the convergence, the result obtained by CS based technique is used as the initial estimate for \( z \).

4.4 Experiments and Results Analysis

In order to evaluate the performance of the proposed SR technique, experiments are conducted on two different hyperspectral image data sets. The first data set is comprised of 31 band reflectance image of natural scene, corresponding to wavelengths between 0.4\( \mu m \) and 0.7\( \mu m \) in steps of 10 \( nm \) all acquired under the direct sunlight in clear or almost clear sky [149]. Here we used “Scene 5” of Hyperspectral images of natural scenes 2002\(^1\). The second data set is comprised of 224-band real hyperspectral image of Moffett Field acquired by AVIRIS hyperspectral imaging system\(^2\). First dataset is

\(^1\)http://personalpages.manchester.ac.uk/staff/david.foster/Hyperspectral_images_of_natural_scenes_02.html

Figure 4.2: SR results on PCA-I of natural HS image for $q = 2$. (a) LR image of size $100 \times 100$, (b) Groundtruth of size $200 \times 200$, (c) Bicubic interpolation [148], (d) CS based initial estimate, and (e) Proposed approach.

used for checking effectiveness of different steps of algorithm by visual comparison and second dataset is used to test the performance using visual comparison as well as different quantitative measures.

### 4.4.1 Quantitative Evaluation Measures

Detailed quantitative evaluation of spatial and spectral fidelity of super-resolved AVIRIS hyperspectral image is performed using different measures such as correlation coefficient (CC), spectral angle mapper (SAM), and erreur relative globale adimensionnelle de synthèse (ERGAS). Note that these metrics are used by the multiresolution fusion researchers in order to measure the spatial and spectral fidelity of the fused MS images. What follows is a brief review of these measures.

1. **Correlation Coefficient (CC) [124]**: The correlation coefficient is the most popular
measure for checking spatial fidelity between the SR and the original HSI. It shows the similarity between the super-resolved and the groundtruth HSIs for each band. $CC_k$ between two $k^{th}$ image bands $F, \hat{F} \in R^{M\times N}$ is defined as

$$CC_k = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} (F_{i,j} - \bar{F})(\hat{F}_{i,j} - \bar{\hat{F}})}{\sqrt{\sum_{i=1}^{M} \sum_{j=1}^{N} (F_{i,j} - \bar{F})^2 \sum_{i=1}^{M} \sum_{j=1}^{N} (\hat{F}_{i,j} - \bar{\hat{F}})^2}}, \quad (4.6)$$

where $F_{i,j}, \hat{F}_{i,j}$ are pixel values at location $(i, j)$ of groundtruth and super-resolved images $F$ and $\hat{F}$, respectively and $\bar{F}, \bar{\hat{F}}$ are the mean values. The $CC$ has a value lying between zero and one, with zero representing the lowest correlation. Since we have a number of spectral images, we average the correlation coefficient values computed over all the bands to obtain the $CC_{avg}$ i.e.,

$$CC_{avg} = \frac{1}{B} \sum_{k=1}^{B} CC_k, \quad (4.7)$$

where $B$ is the total number of bands i.e. 141 in this experiment.

2. Spectral Angle Mapper ($SAM$): Since this measure is insensitive to variable gain resulting from the topographic illumination effects [150], we have chosen it to measure spectral fidelity of super-resolved images. It is defined as the angle between two vectors. Low value of $SAM$ indicates less spectral distortions. Spectral angle between ground truth and super-resolved image is defined as:

$$SAM(v, \hat{v}) = \arccos \left( \frac{v \cdot \hat{v}}{||v||_2 \cdot ||\hat{v}||_2} \right), \quad (4.8)$$

where $v$ and $\hat{v}$ are the pixel vectors of groundtruth and super-resolved image, respectively.

For example for SR HSI with spatial size of $200 \times 200$ and spectral size of 141 bands we obtain $200 \times 200 = 40000$ values for SAM indicating spectral fidelity between groundtruth pixel and SR pixel along spectral dimension. These values are averaged over the entire image band to get a global measure of spectral distortion of the super-resolved image. Ideally this value has to be zero.
3. Relative Dimensionless Global Error in Synthesis (ERGAS): This is an indicator of the overall error in super-resolved HSI. The ERGAS value is defined as [151]

\[
ERGAS = 100 \frac{h}{l} \sqrt{\frac{1}{B} \sum_{k=1}^{B} \left( \frac{RMSE(k)}{\mu(k)} \right)^2},
\]

(4.9)

where \( h/l \) is the ratio of number of pixels in HR and LR images i.e., \( r \) in our case. \( RMSE(k) \) and \( \mu(k) \) are the root mean squared error and mean of the \( k \)th band, respectively. This value has to be small for better performance.

Both datasets have high spatial dimensions, hence specific regions are cropped from them and experiments are carried out on these regions. For the purpose of quantifying the results we consider original HS images as groundtruth and generated synthetic LR HS images by applying downsampling by a factor of \( q = 2 \), in both the spatial directions of HS data. We then applied SR algorithm to the synthetically generated LR HS images and compared the results against the original HS image. We limit AVIRIS HSI data in the wavelength range of \( 0.4 \mu m \) and \( 1.79 \mu m \) to conduct the simulation to reduce the time complexity, as this will not make any significant difference in the results of our proposed algorithm. After removing few bands having low signal to noise ratio (SNR), 141 bands from the original HS image of AVIRIS were super-resolved by a factor of 2. The band removal was based on visual inspection of the images. Effectiveness of the proposed algorithm on the natural scene is presented in Figure 4.2. Note that, here results are presented only on the PCA-I image instead of HS bands. Figures 4.2(a) and (b) display the LR test image and the ground truth image, respectively. The result obtained using bicubic interpolation is shown in Figure 4.2(c). To demonstrate the effectiveness of proposed approach, here also we show the initial SR estimate obtained using CS based approach in Figure 4.2(d) and final super-resolved PCA-I image in Figure 4.2(e). Figure 4.2(c) indicates that the borders of text and crossed lines appear blurred in the bicubic interpolated image. Visual inspection of Figure 4.2 (d) shows that initial estimate of SR PCA-I using CS based approach gives a quality, comparable to the groundtruth. Since the neighborhood relations are not considered in patches while obtaining the initial estimate, we observe shading of letters written on a ball and also observe little blockiness. Applying regularization to initially estimated SR PCA-I helps to reduce this effect considerably as
Figure 4.3: SR results on AVIRIS HS Band 100 for $q = 2$. (a) LR image of size 100 × 100, (b) Groundtruth of size 200 × 200, (c) Bicubic interpolation [148], (d) Iterative backprojection method [152], and (e) Proposed approach. Shown in Figure 4.2(e), which is closer to the groundtruth. This shows effectiveness of regularization in proposed algorithm.

In Figure 4.3 we display the SR results on band 100 of AVIRIS data. The LR test image and the groundtruth image are displayed in Figures 4.3(a) and (b) respectively. High resolution image obtained using bicubic interpolation [148] is shown in Figure 4.3(c). The results obtained using iterative backprojection [152] and proposed approach are shown in Figures 4.3(d) and (e), respectively. From Figure 4.3(c) we can see that high-resolution image obtained using bicubic interpolation is blurred and the high frequency spatial details are not preserved. We can see in Figure 4.3(d) that pure white patches in LR image is converted to grayish patches in SR image obtained using iterative backprojection method. One can see that the SR image obtained using the proposed method displayed in Figure 4.3(e) compares well with the groundtruth. The proposed method provides better visual quality compared to other approaches.
4.4 Experiments and Results Analysis

In order to compare the results on quantitative basis we use score indices such as correlation coefficient (CC) [124], relative dimensionless global error in synthesis (ERGAS) [151], and spectral angle mapper (SAM) [150]. These are generally used in the multiresolution fusion techniques in order to measure the spatial and spectral fidelity of the fused multispectral images. Since SAM measure is insensitive to variable gain that results from the topographic illumination effects [150] we choose it for measuring the spectral fidelity. Table 4.1 shows quantitative comparison among bicubic interpolation, IBP and proposed approach. Results are listed for different amount of spectral variability (I) retained after transformation. CC is averaged over all bands of HS image to obtain a global measurement of spatial distortion and SAM is averaged over all pixels to yield a global measurement of spectral distortion. As seen from the Table 4.1, our method provides scores that are more closer to reference values compared to bicubic interpolation and IBP. Lower value of ERGAS in the proposed method indicates less global distortion in super-resolved HS image. CC for all the HS bands plotted in Figure 4.4 shows that proposed method gives better spatial fidelity compared to bicubic interpolation and iterative backprojection. The surface plots in Figures 4.5(a), (b), and (c) represent spectral fidelity of each specific pixel in super-resolved HS images obtained using bicubic interpolation, iterative backprojection, and proposed approach, respectively for \( q = 2 \). Bicubic interpolation and iterative backprojection method give a maximum of 144.89 degree and 120.10 degree of SAM, while proposed method gives a maximum of 116.22 degree SAM as seen from surface plots of Figures 4.5(a), (b), and (c) respectively. Lower values of maximum as well as average SAM indicate that proposed method provides better spectral fidelity.

<table>
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<tr>
<th>Method</th>
<th>q</th>
<th>I</th>
<th>CC</th>
<th>SAM</th>
<th>ERGAS</th>
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<tr>
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<td>0.902</td>
<td>4.863</td>
<td>7.065</td>
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</table>

Table 4.1: Quantitative comparison of SR results on AVIRIS data for \( q = 2 \)
4.4 Experiments and Results Analysis

Figure 4.4: Plot showing detailed performance of correlation coefficients for $q = 2$ of AVIRIS HS bands 1-141

Figure 4.5: Surface plots showing SAM of all pixels for $q = 2$ on AVIRIS data. (a) Bicubic interpolation [148], (b) Iterative backprojection [152], and (c) Proposed approach.

The proposed method achieves higher spatial correlations indicated by the CC and ERGAS, which are more closer to reference value. In IBP the choice of back-projection filter is arbitrary and incorporation of prior information is difficult, which results in higher spatial as well as spectral distortion in SR image compared to proposed approach. In the proposed approach the use of regularization helps us to achieve better spectral fidelity in terms of lesser value of SAM. As we increase the variability retained in PCA components, spatial and spectral distortions are reduced considerably. Here we can extend the CS based approach to all significant PCA components in order to improve the performance.
4.5 Conclusion

We have presented a novel approach to recover the high spatial resolution and high spectral resolution HS image using CS based learning and global AR prior model. The advantages of the proposed technique are: 1) no need of supplementary spatial information in registered form, 2) has high spatial fidelity and low spectral distortions, and 3) once the low-resolution and high-resolution dictionaries are created from training dataset, the HS images captured by a low-resolution sensor can be super-resolved. Quantitative comparison of score indices show that our method enhances spatial information without introducing significant spectral distortion.

It is necessary to point out that proposed approach is using CS based learning on first PCA component only. Use of raw dictionary for CS based approach needs large number of patches, which in turn increases computation time while obtaining initial SR estimate. Another drawback is the use of an observation model which assumes that the LR pixel intensity is the average of the corresponding HR pixels intensities i.e., we assumed an averaging as degradation for all bands of hyperspectral image. In practice, many factors like diffraction, shape, location, physical construction and electronic response of the detectors contribute to the degradation (i.e., PSF) of any hyperspectral imager. Hence the degradation considered in proposed approach is not optimum for all spectral bands. It is typical for the degradation to degrade as distance from the center of the FOV is increased. Hence a better way is to estimate degradation that can optimally represent image formation for all spatial locations and spectral bands. But the spatially varying PSF requires estimation of PSF at each observed pixel which is quite involved. The work presented in the next chapter involves extension of CS based approach to multiple PCA components instead of single one. Instead of raw dictionaries jointly trained dictionaries that have few number of atoms are used. This reduces timings in initial estimates of SR. Besides this, it estimates PSF for each spectral component separately to represent optimum image observation model for each spectral band to obtain final SR results.