Chapter 3

A Model Based Approach To Multiresolution Fusion Using Compressive Sensing Theory

Many remote sensing applications like urban-area monitoring, map updating, precision farming, land use classification, and hazard monitoring require images with both high spatial and high spectral resolution. Given technological limitations related to spatial and spectral resolution of imaging sensors, MS images having high spectral resolution are acquired with a larger IFOV i.e., lower spatial resolution than panchromatic images having lower spectral resolution. Many of the earth observation satellites like Advanced Land Imager (ALI), IKONOS, Indian Remote Sensing satellites (IRS), Landsat, Systeme Probatoire d’ Observation de la Terre (SPOT), and recently launched Worldview-2 provide data consisting of a panchromatic channel of high spatial resolution and several multispectral channels at a lower spatial resolution. Acquisition of panchromatic image enables the accurate geometric analysis of materials and the MS channels provide the spectral information, required for accurate discrimination between classes of different ground materials. A combination of spatial features of high-resolution PAN image and spectral features of MS can add valuable information in these applications. Image fusion refers to the algorithmic approach to generate images with high spectral as well as spatial resolution by combining high spatial resolution panchromatic (PAN) image with the high spectral resolution multispectral (MS) images. An image obtained using a fusion tech-
nique improves the interpretation of the image. An ideal fusion technique has to account for the enhancement of high spatial resolution as well as minimization of the spectral distortion.

Many researchers have investigated strategies for the fusion of multiresolution satellite images. Among the existing multiresolution fusion techniques, the methods most widely used are: intensity-hue-saturation (IHS)-based methods [111, 112], high-pass filter (HPF) [113], principal component analysis [114], Brovey transform [115] and the wavelet-based approach [116]. In these methods, first of all multispectral images are upscaled before the fusion to the spatial resolution of the high-resolution PAN image using bicubic interpolation. IHS based method is one of the most commonly used fusion techniques for sharpening of the multispectral images. IHS fusion converts a color MS image from the RGB space into the IHS color space which decomposes it to intensity and color components. In the IHS space, spectral information is reflected on the hue and the saturation while intensity change has little effect on the spectral information. The fusion is obtained by replacing the intensity (I) component in IHS image by the PAN image and converting the IHS back to RGB domain. In the high-pass filter (HPF) based fusion, a high-pass filter kernel is used to filter the high-resolution PAN data. Subsequently HP filtered image is added to each MS band by taking care of mean, standard deviation and spatial resolution ratio. It gives acceptable results, but the edges are too much emphasized. The PCA transform based fusion approach first obtains the principal components of the multispectral bands where the first principal component contains the most information (i.e. variance) of the MS image. After that, the first principle component is substituted by the panchromatic image. Applying inverse PCA results in new RGB (Red, Green, and Blue) bands of multispectral image having spatial resolution of PAN image. Being statistical method it is sensitive to the area to be sharpened hence the fusion results may vary depending on the selected MS image. Brovey transformation is due to the result of combination of arithmetic operations and normalizes the spectral bands before they are multiplied with the panchromatic image. The spectral properties, however, are usually not well preserved by this method. In a different approach based on multiresolution fusion, wavelet transform is widely used due to its compactness, orthogonality and the availability of directional information. In this method, a wavelet transform is applied to the PAN image, resulting in a four components namely coarser, horizontal, vertical, and
diagonal. Coarser component contains low-resolution information while remaining three components have the spatial details. The LR coarser component is replaced by the MS band and the process is repeated for all MS bands. Applying inverse wavelet transform results in the fused MS image. More recent works on the multiresolution fusion can be found in [117, 118]. Recently, the authors in [14] have proposed a learning based approach for multiresolution fusion using contourlet based learning approach. Zhu et al. [119] have proposed fusion using compressive sensing theory. They construct dictionary of LR-HR pairs using MS image and PAN image and obtain sparse representation of image patches by estimating sparse coefficients and finally reconstruct HR multispectral image.

In this chapter, we propose a new two step approach for multiresolution fusion in remotely sensed multispectral images. The edge details of the fused multispectral images are learned using wavelet transform and compressive sensing. Given the registered panchromatic (PAN) image and a multispectral (MS) image, we first decompose the PAN image using discrete wavelet transform. Compressive sensing (CS) technique is then applied to derive an initial estimate of fusion by first computing the the sparseness of observed MS patches and then using them for obtaining detail coefficients. Taking the inverse discrete wavelet transform gives us the initial estimate of the fused image. In the second step, we model the image acquisition process using a linear system and solve the fusion problem by formulating it as a maximum a posteriori (MAP) framework that enforces smoothness constraint on the fused image. The cost function consisting of a data fitting term and the prior term is minimized using simple gradient optimization technique. The quantitative and perceptual comparison is carried out to evaluate the proposed technique w.r.t the recently proposed image fusion techniques.

3.1 Use of CS Theory for Initial Estimate

Compressive sensing presents a new approach in signal processing for sparse signal recovery [12] which directly measures a compressed representation of the signal. The key assumption is that the most signals that arise in nature are sparse whose digital representation requires few nonzero coefficients. CS exploits this sparsity, by allowing a digital signal to be reconstructed using only few linear measurements when compared to the size of the original signal. As long as the measurement matrix satisfies a Restricted Isometry
3.1 Use of CS Theory for Initial Estimate

Property (RIP), the exact signal recovery is possible from these measurements [120].

3.1.1 Compressive Sensing

Suppose $x$ is an unknown digital image vector in $\mathbb{R}^N$ that we want to acquire. Normally this should require $N$ pixels. But if we know apriori that $x$ is compressible in certain transform domain (e.g. wavelet, Fourier), then as per CS theory we can acquire $x$ by measuring only $M$ linear projections rather than all $N$ pixels. If these projections are properly chosen (projection matrix or the measurement matrix satisfy the RIP), the size of $M$ can be smaller than $N$, the size of image. If the image is $K$ sparse, random projection works if $M = O(K \log(N/K))$ [121]. Mathematically, under the sparsity assumption, a signal $x \in \mathbb{R}^N$, can be recovered by solving the $l_1$-minimization using standard optimization tool such as linear programming [122] i.e., the problem can be posed as

$$\min_{x \in \mathbb{R}^N} ||x||_1 \text{subject to } y = Dx,$$

where $||x||_1 = \sum_{i=1}^{N} |x_i| \tag{3.1}$

In proposed approach, observation $y \in \mathbb{R}^M$ is represented as a linear combination of few number of elements of dictionary $D$ using sparse vector $x$ i.e., $x$ has few number of nonzero elements. The main component in the $l_1$-minimization is the dictionary $D$. The choice of dictionary is dependent on the application. In proposed approach we construct the dictionaries from the empirical data and use them in finding initial estimate of fusion.

3.1.2 Finding Initial Estimate

We construct various dictionaries from PAN image and use them in CS based multiresolution approach to obtain initial estimate of fused image. Our method fuses high frequency details from panchromatic image into the multispectral bands assuming that they are registered. As a powerful multiresolution analysis tool, we use wavelet transform to obtain high frequency details in the fused image without altering the spectral contents of the multispectral images. Considering a spatial resolution difference of 2 between PAN and MS images, the learning process can be explained as follows.

As shown in Figure 3.1, single level wavelet decomposition of panchromatic image
3.1 Use of CS Theory for Initial Estimate

![Image of MS, I, II, IV, III quadrants](image)

Figure 3.1: (a) MS Image of size $M \times M$, (b) $P_{wt}$: One level wavelet decomposition of PAN image. Here I, II, III and IV quadrants represent the coarse, vertical, diagonal and horizontal details each of size $M \times M$.

results in coarse coefficients in the top-left quadrant I, having the same dimension as that of MS image. Remaining three quadrants (II-IV) show vertical, diagonal and horizontal edge details, respectively. These edge details are used to obtain finer details of fused MS image. We know that PAN and MS images are obtained from the same geographic region with the difference that PAN image is acquired with high spatial resolution covering wide spectral features. This results in high spatial correlation between the MS image and the coarser coefficients of the PAN image. Hence patches of MS image can be well-represented as a sparse linear-combination of coarse elements of the PAN image having high spectral width. Since the finer details fused MS and PAN are similar, we make use of the same sparseness in order to obtain the finer details of fused MS image. Thus our method fuses fine details from PAN image to MS bands by deriving the sparseness of low-resolution observation using the coarser part of PAN image and the same sparsity is used to obtain the detail coefficients of the initial fused estimate. This reduces spectral distortion of the fused image. The proposed algorithm to obtain initial fused image is described below.

Suppose we have the MS image of size $M \times M$, and the corresponding PAN image of size $qM \times qM$, where $q$ is the resolution factor. Here, we describe the algorithm by considering a value of $q = 2$.

1. Obtain one level wavelet decomposition of PAN image $P_{wt}$ as shown in Figure 3.1(b).

2. For each $2 \times 2$ patch of MS image choose corresponding patches from all four quadrants of $P_{wt}$. Arrange them in lexicographical order to form the vectors $v_{MS}$, $v_{PA}$, $v_{PV}$, $v_{PH}$ and $v_{PD}$. Here, $v_{MS}$ corresponds to vector of size $4 \times 1$ formed by using MS image, while $v_{PA}$, $v_{PV}$, $v_{PH}$ and $v_{PD}$ are the corresponding vectors.
3.1 Use of CS Theory for Initial Estimate

Figure 3.2: Detailed understanding of CS based approach. Construction of dictionaries \( D_{PA}, D_{PV}, D_{PH}, \) and \( D_{PD} \) from coarser, vertical, horizontal, and diagonal quadrants of \( P_{wt} \), respectively, each of size \( 4 \times M^2/4 \). Obtaining sparse vector \( x \) using coarser dictionary, and use it to obtain diagonal details of fused image, assuming mean subtracted \( D_{MS} \) (hence \( y \)) and \( D_{PA} \) (i.e. \( D_{MSm} \) and \( D_{PAm} \)).

3. Create dictionaries \( D_{MS}, D_{PA}, D_{PV}, D_{PH} \) and \( D_{PD} \) each of size \( 4 \times M^2/4 \) using the corresponding vectors formed in step 2. Construction of dictionaries \( D_{PA} \) and \( D_{PD} \) are shown in Figure 3.2. Dictionaries \( D_{PV} \) and \( D_{PH} \) are constructed in a similar way.

4. Find the mean subtracted dictionaries of MS image and the coarse quadrant of the PAN image i.e.,

\[
\text{for } j = 1 \text{ to } M^2/4
\]
\[
D_{MSm}(\cdot,j) = D_{MS}(\cdot,j) - m_{MS}, \]
\[
D_{PAm}(\cdot,j) = D_{PA}(\cdot,j) - m_{PA},
\]

where \( D_{MSm} \) and \( D_{PAm} \) represent the mean subtracted dictionaries. Here, \( m_{MS} \) and \( m_{PA} \) represent the mean of the MS and PAN image patches.
5. Consider a mean subtracted vector $y$ of MS test image taken from $D_{MSm}$.

6. Now solve for sparse vector $x$ using compressive sensing based $l_1$-minimization problem found in [12] i.e.,

$$
\min_{x \in \mathbb{R}^N} ||x||_1 \text{ such that } y = D_{PAm}x
$$

Here, $x$ represents the sparseness vector of size $N \times 1$, where $N = M^2/4$. It gives sparse representation of test patch $y$ in terms of dictionary ($D_{PAm}$) elements adaptively. Remember $y$ and $D_{PAm}$ resembles observation vector and sensing matrix, respectively of the compressive sensing theory.

7. Obtain the finer detailed patches of the MS image i.e., the patches of horizontal, vertical and diagonal by using the sparsity vector $x$, found in step 6.

$$
P_H = D_{PH}x; \quad P_V = D_{PV}x; \quad P_D = D_{PD}x$$

Convert these into a block of size $2 \times 2$. This indicates that the finer details of initial fused MS image are obtained as a linear combination their respective dictionary elements (also referred to as patches) where the sparseness obtained using LR MS image is used. Steps 2-7 are explained graphically in Figure 3.2.

8. Create the fused decomposed patch

$$
Fp = [P_M, P_V; P_H, P_D],
$$

where $P_M$ is the patch of MS image of size $2 \times 2$; $P_V$, $P_D$ and $P_H$ are the patches corresponding to vertical, diagonal and horizontal edges each of size $2 \times 2$.

9. Repeat steps 6-8 for all elements of $D_{MSm}$ and append the MS patches and the obtained finer detailed patches to $Fp$ created in step 8 to their respective locations.

10. Take the inverse wavelet of derived initial estimate $Fp$ of size $2M \times 2M$ to obtain the initial fused image $Z$.

These steps give us the initial fused image $Z$ having size of $2M \times 2M$. 
3.2 Regularization

Since we are constructing the fused MS image by using patch based approach, the spatial homogenity is not taken into account. Hence regularize it further to obtain the final solution. We restrict the solution space for the fused image by using maximum a posteriori - Markov random field (MAP-MRF) approach. The MAP-MRF approach requires data fitting term. This can be done by considering an observation model that represents the MS image formation.

3.2.1 Observation Model

Let $Y$ be the observed MS image of the size $M \times M$ pixels and $Z$ be the fused high-resolution (HR) image, then the forward model for the image formation can be written as,

$$y = Dz + n,$$  \hspace{1cm} (3.2)

where $y$ and $z$ represent the lexicographically ordered vectors of size $M^2 \times 1$ and $q^2M^2 \times 1$, respectively. Here, $n$ is the independent and identically distributed (i.i.d.) noise vector with zero mean and variance $\sigma_n^2$ and has the same size as $y$. It is given by

$$P(n) = \frac{1}{(2\pi\sigma_n^2)^{M^2/2}} e^{-\frac{1}{2\sigma_n^2}n^2},$$  \hspace{1cm} (3.3)

where $D$ is the downsampling matrix which takes care of aliasing caused due to downsampling. For an integer downsampling factor of $q$, the matrix $D$ consists of $q^2$ non-zero elements along each row at appropriate locations [123].

$$D = \frac{1}{q^2} \begin{pmatrix} 1 1 \ldots 1 & 0 \\ 1 1 \ldots 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 1 1 \ldots 1 \end{pmatrix},$$  \hspace{1cm} (3.4)

Now our problem is to estimate fused $z$ given $y$, which is an ill-posed inverse problem and can be solved by using regularization methods.
3.2 Regularization

3.2.2 MRF Prior Model

The Markov Random Field (MRF) has emerged as a popular stochastic model for images due to its ability to capture local dependencies. MRF provides a convenient and consistent way of modeling context dependent entities. This is achieved through characterization of mutual influence among such entities. The practical use of MRF models is largely ascribed to the equivalence between the MRF and the Gibbs Random Fields (GRF). In proposed approach we characterize the final fused image using the MRF model. It finds out pixel intensity at current site using neighbourhood pixel intensities of that site and not anything else. This is justified because the changes in intensities in a scene is gradual and hence there is a local dependency.

Let $Z$ be a random field over regular $N \times N$ lattice of sites $L = \{(i,j)|0 \leq i, j \leq N-1\}$. The equivalence between MRF and GRF is established in Hammersley-Clifford theorem, hence we have

$$P(Z = z) = \frac{1}{Z_z} e^{-U(z)}, \quad (3.5)$$

where $z$ is a realization of $Z$, $Z_z$ is the partition function given by $Z_z = \Sigma_z e^{-U(z)}$ and $U(z)$ is the energy function given by $U(z) = \Sigma_{c \in C} V^z_c(z)$. Here, $V^z_c(z)$ denotes the potential function of clique $c$ and $C$ is the set of all cliques. The lexicographically ordered fused image $z$ satisfying Gibbs density function is now written as,

$$P(z) = \frac{1}{Z_z} exp\{-\Sigma_{c \in C} V^z_c(z)\}. \quad (3.6)$$

3.2.3 MAP-MRF Formulation

We use MAP-MRF framework for regularization approach in order to obtain the final fused image. The use of MAP estimation for fusion requires a suitable prior for the same. A method of specifying MRF prior on fused image involves considering cliques $c$ on a neighborhood. By using first order neighborhood, the energy function corresponding to the MRF prior for fused image can be written as,

$$\sum_{c \in C} V_c(z) = \gamma \sum_{k=1}^{N_1} \sum_{l=1}^{N_2} [(z_{k,l} - z_{k,l-1})^2 + (z_{k,l} - z_{k-1,l})^2], \quad (3.7)$$
where $\gamma$ represents the penalty for departure from smoothness in $z$. $C$ is the set of all cliques and $V_c(z)$ is the clique potential. Here, the MRF parameter $\gamma$ is estimated using the initial fused image and this avoids the tuning of the parameter.

The MRF model on the fused image serves as the prior for the MAP estimation. The MAP estimate of the high-resolution fused image comes about by an application of Bayes theorem,

$$P(z|y) = \frac{P(y|z)P(z)}{P(y)}.$$  \hspace{1cm} (3.8)

The left hand side is known as the posterior distribution over $z$ and $y$ represents observed data. Here, $P(y)$ may be considered as a normalization constant. We apply this to our problem. Given the LR observation $y$, the MAP estimate $\hat{z}$, using Bayesian rule, is given by,

$$\hat{z} = \arg\max_z P(z|y) = \arg\max_z P(y|z)P(z).$$  \hspace{1cm} (3.9)

Taking the log of the posterior probability we can write,

$$\hat{z} = \arg\max_z \left[ \log P(y|z) + \log P(z) \right].$$  \hspace{1cm} (3.10)

Since $n$ is independent. The above MAP formulation allows us to incorporate prior knowledge about $z$ for improving robustness during reconstruction. Using equations 3.2 and 3.3, we obtain

$$P(y|z) = P(n|y-Dz) = \frac{1}{(2\pi\sigma_n^2)^{\frac{M^2}{2}}} e^{-\frac{||y-Dz||^2}{2\sigma_n^2}}.$$  \hspace{1cm} (3.11)

Thus for MAP-MRF approach, the final cost function to be minimized can be expressed as,

$$\hat{z} = \arg\min_z \left[ \frac{||y-Dz||^2}{2\sigma_n^2} + \sum_{c \in C} V_c(z) \right].$$  \hspace{1cm} (3.12)

Convexity of this cost function allows us to use the simple gradient descent optimization technique, which quickly leads to the minima. Since the optimization process is iterative the choice of initial solution fed to the optimization process determines the speed of convergence. Use of the available fused approximation as an initial solution speed-up the convergence. It may be mentioned here that we obtain initial fused approximation separately for each of the MS observations and the optimization is carried out independently.
3.3 Experimental Results

In this section, we present the results of the proposed method for fusion. The experiments are conducted on real images captured using Landsat-7 Enhanced Thematic Mapper plus (ETM+) satellite. The original PAN image and the MS images are of size $512 \times 512$ and $256 \times 256$ pixels, respectively with ground resolution of each pixel as $30m \times 30m$ and $15m \times 15m$, respectively. These are considered as ground truth. In order to make the quantitative comparison, we downscaled both images by a factor of 2 and 4 and conducted the experiments using the downsampled versions. It is to be noted that the resolution difference between the MS image and PAN image captured by Landsat-7 satellite is of 2. But to evaluate the performance of the proposed algorithm for the higher resolution factor we conducted experiments by downsampling by a factor of 4 also. In both cases, the size of the fused MS image is $256 \times 256$. We compare the performance of the proposed method with other methods on the basis of qualitative as well as quantitative measures. Different quantitative measures used in our experiments are described in the following section.

3.3.1 Quantitative Evaluation Measures

For quantifying the results of fusion we used correlation coefficient (CC), structural similarity (SSIM), and mean squared error (MSE) as an evaluation index. These metrics have been widely used in the multiresolution fusion techniques in order to measure the spatial and spectral fidelity of the fused MS images. What follows is a brief review of these measures.

1. Correlation Coefficient (CC) [124]: The correlation coefficient is the most popular measure for checking spatial fidelity between the fused and the groundtruth (original) MS image. It shows the similarity between the fused and the groundtruth MS image band. $CC$ between groundtruth and fused image bands $F, \hat{F} \in R^{M \times N}$ is
defined as

\[
CC = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} (F_{i,j} - \bar{F})(\hat{F}_{i,j} - \bar{\hat{F}})\sqrt{\sum_{i=1}^{M} \sum_{j=1}^{N} (F_{i,j} - \bar{F})^2 \sum_{i=1}^{M} \sum_{j=1}^{N} (\hat{F}_{i,j} - \bar{\hat{F}})^2}}{\sqrt{\sum_{i=1}^{M} \sum_{j=1}^{N} (F_{i,j} - \bar{F})^2 \sum_{i=1}^{M} \sum_{j=1}^{N} (\hat{F}_{i,j} - \bar{\hat{F}})^2}},
\]

(3.13)

where \(F_{i,j}, \hat{F}_{i,j}\) are pixel values at location \((i, j)\) of groundtruth and fused images \(F\) and \(\hat{F}\), respectively and \(\bar{F}, \bar{\hat{F}}\) are the mean values. The \(CC\) has a value lying between zero and one, with zero representing the lowest correlation.

2. Structural similarity (SSIM) [125]: The SSIM combines a comparison of luminance, contrast, and structure. It is applied locally to \(8 \times 8\) square window. This window is moved pixel-by-pixel over the entire image and the SSIM is calculated within the window having range of 0 to 1. Values close to 1 show the highest correspondence of fused image with the groundtruth image. SSIM between groundtruth \((X)\) and fused \((Y)\) image windows is defined as below

\[
SSIM(X, Y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)},
\]

(3.14)

where \(\mu_x\) and \(\mu_y\) represent the mean intensities of \(X\) and \(Y\), respectively, while \(\sigma_x\) and \(\sigma_y\) represent the standard deviation of \(X\) and \(Y\), respectively. Here, \(\sigma_{xy}\) is the correlation between \(X\) and \(Y\). The constant \(C_1\) is included to avoid instability when \(\mu_x^2 + \mu_y^2\) is very close to zero. Similarly, the constant \(C_2\) is included to avoid instability when \(\sigma_x^2 + \sigma_y^2\) is very close to zero. SSIM is averaged over all windows to obtain average over entire fused image.

3. Mean squared error (MSE): It is a common measure to estimate the squared error between two entities, measuring the ratio of the power within the error to the signal power. It is given by [126]

\[
MSE = \frac{\sum_{i,j} [F_{i,j} - \hat{F}_{i,j}]^2}{\sum_{i,j} [F_{i,j}]^2},
\]

(3.15)

where \(F_{i,j}\) and \(\hat{F}_{i,j}\) represent the true HR (groundtruth) and the fused images, respectively.
3.3 Experimental Results

Figure 3.3: Fusion results of Band 2 for $q = 2$. (a) MS image of size 128 $\times$ 128, (b) Approach in [127], (c) Approach in [14], and (d) Proposed approach.

Figure 3.4: Fusion results of Band 2 for $q = 4$. (a) MS image of size 64 $\times$ 64, (b) Approach in [127], (c) Approach in [14], and (d) Proposed approach.

Figure 3.3 show the results of fusion for Band-2 using different approaches. Table 3.1 shows the quantitative comparison using correlation coefficient, structural similarity (SSIM) [125] and mean squared error (MSE) [127] for Band-2. We can see that proposed approach gives better quantitative values as compared to other approaches. We also observe perceptual improvement in the fused images obtained using the proposed method. Method in [127] uses interpolation of MS image and [14] uses patch matching criterion between MS image and coarse level of the PAN image to obtain initial estimate of the fused image, while proposed approach uses sparsity coefficients to represent high frequency details, which leads to better initial estimate compare to other methods. This results in better regularized final fused image because results of regularization depends on initial estimate also.
3.4 Conclusion

We have presented a new technique to recover the high spatial and high spectral resolution fused MS image using compressive sensing based learning and MAP-MRF approach. Since the initial estimate is obtained using DWT and compressive sensing theory, the suggested method gives finer details present in different directions with reduced spectral distortion. We considered MRF model to enforce smoothness constraint while regularizing. True MRF parameter can be known only if the fused image is known. Since this is not available, we make use of the close approximation of fusion (initial estimate) to obtain the same. A simple approximation method called maximum pseudo likelihood is used for estimating this parameter [128]. The quantitative results demonstrate that the proposed technique yields better solution as compared to those obtained using the recent approaches.

The proposed approach to increase spatial resolution of multispectral images uses auxiliary panchromatic image in registered form. The high frequency details in fusion result are obtained from high spatial resolution panchromatic image. The proposed approach recovers these high frequency details by exploiting the similarity in sparse representation of coarser resolution of PAN and observed MS image. Note that the accuracy of results is also dependent on registration. Many times auxiliary high-resolution image is not available onboard. Based on this discussion, one may conclude that spatial resolution enhancement without using auxiliary information is difficult. Contrary to this, another

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<tr>
<td></td>
<td>Correlation coefficient</td>
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Table 3.1: Performance comparison of band 2 for $q = 2$ and $q = 4$ in terms of correlation coefficient, SSIM and MSE.
alternative is to use a database of HR and LR training images unassociated with test image to form the dictionaries and recover the finer details from them to enhance the spatial resolution. In next chapter, we discuss the use of LR-HR dictionaries to increase spatial resolution of hyperspectral data where the auxiliary data is seldom available.