CHAPTER III
HEAT AND MASS TRANSFER IN A VISCO-ELASTIC MHD FLOW PAST A VERTICAL PLATE UNDER OSCILLATORY SUCTION VELOCITY

3.1 Introduction

The flow past an infinite porous plate and the phenomenon of heat and mass transfer have been of great interest due to its applications in industries. Flows which arise in fluids due to the unsteady motion of a boundary, boundary temperature, density differences caused by the diffusion of thermal energy etc. have many applications in geophysics, chemical engineering, turbo-machinery and aerospace technology. Some important contributions where the transfer of heat and mass take place simultaneously as a result of buoyancy reduced motions have been given by Singh and Singh (1983), Raptis and Soundelgekar (1984), Lai (1991), Jha and Prasad (1992), Abdussattar (1994), Singh (1994, 1996), Soundelgekar et al. (1995), Singh and Kumar (1995), Singh et al. (1996), Soundelgekar and Lahurikar (1996) etc. The investigations of the problems of free convective flow of a viscous fluid through a porous medium with heat and mass transfer have been studied by many researchers. The effects of permeability of free convective flow past a vertical porous wall have been investigated by Shreekanth et al. (1996). Singh et al. (1999) have discussed hydromagnetic free convective and mass transfer flow of such fluids considering permeability variation with direction. Acharya et al. (2000) have extended the study in steady flow with constant suction in the presence of magnetic field. Singh et al. (2003) have studied the effects of permeability variation and oscillatory suction velocity in presence of time dependent viscosity along with the uniform magnetic field.


Published in the International Journal of Computational Science and Mathematics, ISSN 0974-3189, Volume 2, Number 3 (2010), pp. 137-146.
The objective of the present paper is to study the unsteady hydromagnetic flow of an electrically conducting second-order fluid past an infinite vertical porous plate in a porous medium of time dependent permeability under oscillatory suction velocity normal to the plate. It is considered that the uniform magnetic field acts normal to the flow and the permeability of the porous medium fluctuates with time. The perturbation technique has been used to solve the problem. The profiles of velocity and skin friction have been presented graphically for different values of parameters involved in the solution to observe the effects of the visco-elastic parameter.

3.2 Mathematical Analysis

We introduce a co-ordinate system where \( \bar{x} \)-axis is taken along the infinite vertical plate in the direction of flow and \( \bar{y} \)-axis normal to it. The permeability of the porous medium is considered to be of the form \( K(t) = K_0(1 + \varepsilon e^{\text{i}t}) \) and the suction velocity is assumed to be \( v(t) = -v_0(1 + \varepsilon e^{\text{i}t}) \), where \( \varepsilon \ll 1 \) being the amplitude of the permeability variation, is a positive constant, \( v_0 > 0 \) is a constant and negative sign indicates that the suction is towards the plate, and \( K_0 \) the mean permeability of the medium. All the fluid properties are assumed to be constant except that the influence of the density variation with temperature. Let \( \bar{u} \) be the component of velocity in the \( \bar{x} \)-direction.

The governing equations for momentum, energy and concentration are

\[
\frac{1}{4} \frac{\partial \bar{u}}{\partial t} - v \frac{\partial \bar{u}}{\partial y} = g\beta \left( \bar{T} - \bar{T}_\infty \right) + g \beta (\bar{C} - \bar{C}_\infty) + v_1 \frac{\partial^2 \bar{u}}{\partial y^2} + v_2 \left[ \frac{\partial^2 \bar{u}}{\partial t^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \right] - \frac{v_1}{K} \bar{u} - \frac{\sigma B_0^2}{\rho} \bar{u} \tag{3.2.1}
\]

\[
\frac{\partial \bar{T}}{\partial t} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \kappa \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \tag{3.2.2}
\]

\[
\frac{\partial \bar{C}}{\partial t} + \bar{v} \frac{\partial \bar{C}}{\partial \bar{y}} = D \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \tag{3.2.3}
\]

The boundary conditions relevant to the problem are
\[ \bar{y} = 0 : \bar{u} = 0, \bar{T} = \bar{T}_w, \bar{C} = \bar{C}_w \]
\[ \bar{y} \to \infty : \bar{u} \to 0, \bar{T} = \bar{T}_\infty, \bar{C} = \bar{C}_\infty \] (3.2.4)

where \( \bar{T}_w, \bar{T}_\infty \) and \( \bar{C}_w, \bar{C}_\infty \) are respectively the temperature and the molar concentration of the fluid at the plate and far away from the plate.

Let us introduce the following non-dimensional quantities
\[
y = \frac{\bar{y} v_n}{v_1}, t = \frac{v_0^2 \bar{T}}{4 \nu_1}, u = \frac{\bar{u}}{v_0}, n = \frac{4 \nu_1 \bar{w}}{v_0^2}, T = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}, C = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty} \] (3.2.5)

Introducing the above non-dimensional variables in the governing equations for velocity, temperature and molar concentration neglecting the induced magnetic field of strength \( B_0 \) and taking the usual Boussinesq’s approximations, we obtain the following non-dimensional equations of the fluid motion:

\[
\frac{1}{4} \frac{\partial^2 u}{\partial t^2} - (1 + \alpha \epsilon^{int}) \frac{\partial^2 u}{\partial y^2} = G_r T + G_m C + \frac{\partial^2 u}{\partial y^2} + \frac{1}{4} \frac{\partial^2}{\partial y^2} \left( \frac{\partial u}{\partial y} \right) - (1 + \alpha \epsilon^{int}) \frac{\partial^3 u}{\partial y^3} \]
\[ - \frac{u}{K_0 (1 + \alpha \epsilon^{int})} - M^2 u \] (3.2.6)

\[
\frac{1}{4} \frac{\partial T}{\partial t} - (1 + \alpha \epsilon^{int}) \frac{\partial T}{\partial y} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2} \] (3.2.7)

\[
\frac{1}{4} \frac{\partial C}{\partial t} - (1 + \alpha \epsilon^{int}) \frac{\partial C}{\partial y} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} \] (3.2.8)

subject to boundary conditions

\[ y = 0 \quad u = 0, T = 1 + \alpha \epsilon^{int}, C = 1 + \alpha \epsilon^{int} \]
\[ y \to \infty \quad u \to 0, T \to 0, C \to 0. \] (3.2.9)

where \( S_c = \frac{v_1}{D} \) is the Schmidt number, \( P_r = \frac{v_1}{\kappa} \) is the Prandtl number,

\[
G_r = \frac{v_1 g \beta (T_w - T_\infty)}{\nu_0^3} \] is the Grashof number for heat transfer, \( G_m = \frac{v_1 g \beta (C_w - C_\infty)}{\nu_0^3} \)

is the Grashof number for mass transfer, \( M = \frac{ \sqrt{ \sigma v_1 B_0 } }{ \rho v_0 } \) is the Magnetic parameter,
\( \alpha = \frac{v_2 v_0}{v_1} \) is the visco-elastic parameter and \( \beta, \bar{\beta} \) are the coefficients of volume expansion for heat and mass transfer respectively, \( T \) is the fluid temperature, \( C \) is the molar concentration, \( \kappa \) is the thermal diffusivity, \( D \) is the chemical molar diffusivity, \( C_p \) is the specific heat at constant pressure, \( n \) is the frequency of oscillation, \( t \) is the time, \( \rho \) is the density of the fluid, \( v_1, v_2 \) are the kinematic coefficients of viscosity and elasticity, \( \sigma \) is the electric conductivity and \( g \) is the acceleration due to gravity.

### 3.3 Method of Solution

To solve the equations (3.2.6) to (3.2.8) subject to boundary conditions (3.2.9), we assume the solutions for \( \varepsilon_c \approx \varepsilon \ll 1 \) as follows:

\[
\begin{align*}
    u(y,t) &= u_0(y) + \varepsilon_c u_1(y)e^{\varepsilon t}, \\
    T(y,t) &= T_0(y) + \varepsilon_c T_1(y)e^{\varepsilon t}, \\
    C(y,t) &= C_0(y) + \varepsilon_c C_1(y)e^{\varepsilon t}
\end{align*}
\]

(3.3.1)

Substituting (3.3.1) into equations (3.2.6) to (3.2.8) and equating the harmonic and non-harmonic terms, we get

\[
\begin{align*}
    a u''_0 - u''_0 - u'_0 + a_1 u_0 &= G_c T_0 + G_m C_0 
    \tag{3.3.2}

    a u''_1 - (1 + \alpha) u''_1 - u'_1 + a_1 u_1 &= G_c T_1 + G_m C_1 - \alpha u''_0 + \alpha u'_0 + \frac{u_0}{K_0}
    \tag{3.3.3}

    T''_0 + P_c T'_0 &= 0
    \tag{3.3.4}

    T''_1 + P_c T'_1 - \frac{in}{4} P_c T_1 &= -P_c T'_0
    \tag{3.3.5}

    C''_0 + S_c C'_0 &= 0
    \tag{3.3.6}

    C''_1 + S_c C'_1 - \frac{in}{4} S_c C_1 &= -S_c C'_0
    \tag{3.3.7}
\end{align*}
\]

where the primes denote differentiation with respect to \( y \).

The corresponding boundary conditions are

\[
\begin{align*}
    y = 0 & \colon u_0 = 0 = u_1, T_0 = 1 = T_1, C_0 = 1 = C_1 \\
    y \to \infty & \colon u_0, u_1, T_0, T_1, C_0, C_1 \to 0
\end{align*}
\]

(3.3.8)

Substituting the solutions of equations (3.3.2) to (3.3.7) under the boundary conditions
(3.3.8), we obtain

\[ T_0(y) = e^{-P_y}, \]
\[ T_1(y) = (1 - i \frac{4P_y}{n}) e^{-m_y} + i \frac{4P_y}{n} e^{-P_y}, \]
\[ C_0(y) = e^{-S_y}, \]
\[ C_1(y) = (1 - i \frac{4S_y}{n}) e^{-m_y} + i \frac{4S_y}{n} e^{-S_y}, \]
\[ u_0(y) = (a_1 + a_4) e^{-m_y} - a_3 e^{-P_y} - a_4 e^{-S_y} + \alpha \{(A_1 - A_2) e^{-m_y} + A_3 e^{-P_y} + A_4 e^{-S_y}\}, \]
\[ u_1(y) = \{B_1 e^{-m_y} - G_y (B_2 e^{-m_y} + B_3 e^{-P_y}) - G_m (B_4 e^{-m_y} + B_5 e^{-S_y}) \]
\[ + B_6 e^{-m_y} - B_7 P_y e^{-P_y} - B_8 S_y e^{-S_y}\} + \alpha \{(C_1 + C_2) e^{-m_y} + C_3 e^{-P_y} + C_4 e^{-S_y} \]
\[ + C_5 e^{-m_y} + C_6 e^{-P_y} + C_7 e^{-S_y}\} \]

Hence the solutions for temperature, concentration and the velocity can be expressed as

\[ T(y, t) = e^{-P_y} e^{(1 - i \frac{4P_y}{n}) e^{-m_y} + i \frac{4P_y}{n} e^{-P_y}} e^{i \epsilon t} \] (3.3.9)
\[ C(y, t) = e^{-S_y} e^{(1 - i \frac{4S_y}{n}) e^{-m_y} + i \frac{4S_y}{n} e^{-S_y}} e^{i \epsilon t} \] (3.3.10)

and

\[ u(y, t) = [(a_1 + a_4) e^{-m_y} - a_3 e^{-P_y} - a_4 e^{-S_y} + \alpha \{(A_1 - A_2) e^{-m_y} + A_3 e^{-P_y} + A_4 e^{-S_y}\}] \]
\[ + \epsilon \{\cos nt + i \sin nt\} [B_1 e^{-m_y} - G_y (B_2 e^{-m_y} + B_3 e^{-P_y}) - G_m (B_4 e^{-m_y} + B_5 e^{-S_y}) \]
\[ + B_6 e^{-m_y} - B_7 P_y e^{-P_y} - B_8 S_y e^{-S_y}\} + \alpha \{(C_1 + C_2) e^{-m_y} + C_3 e^{-P_y} + C_4 e^{-S_y} \]
\[ + C_5 e^{-m_y} + C_6 e^{-P_y} + C_7 e^{-S_y}\}] \]

(3.3.11)

where

\[ m_1 = \frac{1}{2} [P_r + \sqrt{P_r^2 + \frac{m_2}{n}}], m_2 = \frac{1}{2} [S_c + \sqrt{S_c^2 + \frac{m_3}{n}}], \]
\[ a_1 = M^2 + \frac{1}{K_0}, a_2 = a_1 + \frac{m_3}{4}, a_3 = \frac{G_m}{P_r^2 - P_r - a_1}, a_4 = \frac{G_m}{S_c^2 - S_c - a_1}, \]
\[ 2m_3 = 1 + \sqrt{1 + 4a_1}, 2m_4 = [1 + \sqrt{1 + 4a_2}], A_i = \frac{m_3^2 (a_1 + a_4)}{m_3 - m_3 - a_1} - \frac{P_r^2 a_3}{P_r^2 - P_r - a_1} - \frac{S_c^3 a_4}{S_c^2 - S_c - a_1}, \]
\[ A_2 = \frac{m_3^2 (a_1 + a_4)}{m_3^2 - m_3 - a_1}, A_3 = \frac{P_r^2 a_3}{P_r^2 - P_r - a_1}, A_4 = \frac{S_c^3 a_4}{S_c^2 - S_c - a_1}, \]
\[ B_2 = \frac{1-i \cdot 4P}{m_1^n - m_1 - a_1}, B_3 = \frac{1-i \cdot 4P}{P_r^2 - P_r - a_2}, B_4 = \frac{1-i \cdot 4S_c}{m_2^n - m_2 - a_2}, B_5 = \frac{1-i \cdot 4S_c}{S_c^n - S_c - a_2}, \]
\[ B_6 = \frac{m_1(a_3 + a_4)}{m_1^n - m_1 - a_1}, B_7 = \frac{a_3}{P_r^2 - P_r - a_2}, B_8 = \frac{a_4}{S_c^n - S_c - a_2}, \]
\[ C_1 = -\left[ \frac{G_r m_1^2 (m_1 + \frac{in}{4}) B_2}{m_1^n - m_1 - a_1} + \frac{G_r m_2^2 (m_2 + \frac{in}{4}) B_4}{m_2^n - m_2 - a_2} + \frac{(1 - m_3^n - \frac{in}{4} m_3) B_6}{m_3^n - m_3 - a_2} \right], \]
\[ C_2 = \frac{G_r m_1^2 (m_1 + \frac{in}{4}) B_2}{m_1^n - m_1 - a_1}, C_3 = \frac{G_r m_2^2 (m_2 + \frac{in}{4}) B_4}{m_2^n - m_2 - a_2}, C_4 = \frac{(1 - m_3^n - \frac{in}{4} m_3) B_6}{m_3^n - m_3 - a_2}, \]
\[ C_5 = \frac{m_4^2 (-m_4 - \frac{in}{4}) B_1}{m_4^n - m_4 - a_1}, C_6 = \frac{G_r P_r^2 (P_r - \frac{in}{4}) B_3}{P_r^2 - P_r - a_2}, \]
\[ C_7 = \frac{G_r s_c^2 S_c (-B_8 + S_c^3 + \frac{in}{4} S_c^2) B_8}{S_c^n - S_c - a_2}. \]

Separating real and imaginary parts and taking only the real part, we obtain the velocity, temperature and concentration fields in terms of fluctuating parts in the form

\[ u(y,t) = u_0(y) + \varepsilon (M_r \cos nt - M_i \sin nt), \]
\[ T(y,t) = T_0(y) + \varepsilon (T_r \cos nt - T_i \sin nt), \]
\[ C(y,t) = C_0(y) + \varepsilon (C_r \cos nt - C_i \sin nt). \]

where the expressions for \( M_r, M_i, T_r, T_i, C_r \) and \( C_i \) are given as follows:

\[ M_r = G_r \left( \frac{1}{m_1^n - m_1 - a_1} + \frac{1}{P_r^2 - P_r - a_1} \right) e^{-m_4 y} \]
\[ + G_m \left( \frac{1}{m_2^n - m_2 - a_2} + \frac{1}{S_c^n - S_c - a_2} \right) e^{-m_4 y} + \frac{m_3(a_3 + a_4)}{m_3^n - m_3 - a_2} (e^{-m_3 y} - e^{-m_4 y}) + \frac{P_r a_3}{P_r^2 - P_r - a_1} e^{-m_4 y} + \frac{S_c a_4}{S_c^n - S_c - a_2} e^{-m_4 y} \]
\[ - \frac{1}{m_1^n - m_1 - a_1} e^{-m_4 y} + \frac{1}{P_r^2 - P_r - a_1} e^{-P_r y} \]
\[-G_m \left( \frac{1}{m_2^2 - m_2 - a_2} e^{-m_2 y} + \frac{1}{S_c^2 - S_c - a_2} e^{-s_c y} \right) - B_t e^{-p_t y} - B_8 e^{-s_c y} \]

\[+ \alpha \left\{ -G_r \left( \frac{m_3^3}{(m_1^2 - m_1 - a_1)(m_2^2 - m_2 - a_2)} \right) + G_m \left( \frac{m_3^3}{(m_2^2 - m_2 - a_2)^2} \right) \right. \]

\[+ \frac{(1 - m_3^3) m_3 (a_3 + a_4)}{(m_2^2 - m_2 - a_2)^2} \]

\[- \frac{m_4^3}{m_4^2 - m_4 - a_2} \left( \frac{G_r}{m_1^2 - m_1 - a_1} + \frac{G_r}{P_r^2 - P_r - a_1} + \frac{G_m}{m_2^2 - m_2 - a_2} \right) \]

\[+ \frac{G_m}{S_c^2 - S_c - a_2} - \frac{m_3 (a_3 + a_4)}{m_3^2 - m_3 - a_2} + \frac{P_r a_3}{P_r^2 - P_r - a_2} + \frac{S_c a_4}{S_c^2 - S_c - a_2} \]

\[+ \frac{G_r P_r^3}{(P_r^2 - P_r - a_2)^2} - \frac{P_r a_3^4}{(P_r^2 - P_r - a_2)^3} + \frac{G_m S_c^3}{(S_c^2 - S_c - a_2)^2} - \frac{S_c a_4^2}{(S_c^2 - S_c - a_2)^3} + \frac{a_4 S_c^4}{(S_c^2 - S_c - a_2)^3} \]

\[\left. \left\{ e^{-m_4 y} + \frac{G_m m_2^3}{(m_2^2 - m_2 - a_2)} e^{-m_1 y} \right. \right\} \]

\[+ \frac{(1 - m_3^3) m_3 (a_3 + a_4)}{(m_2^2 - m_2 - a_2)^2} e^{-m_2 y} \]

\[- \frac{m_4^3}{m_4^2 - m_4 - a_2} \left( \frac{G_r}{m_1^2 - m_1 - a_1} + \frac{G_r}{P_r^2 - P_r - a_1} + \frac{G_m}{m_2^2 - m_2 - a_2} \right) \]

\[+ \frac{G_m}{S_c^2 - S_c - a_2} - \frac{m_3 (a_3 + a_4)}{m_3^2 - m_3 - a_2} + \frac{P_r a_3}{P_r^2 - P_r - a_2} + \frac{S_c a_4}{S_c^2 - S_c - a_2} \]

\[+ \frac{G_r P_r^3}{(P_r^2 - P_r - a_2)^2} - \frac{P_r a_3^4}{(P_r^2 - P_r - a_2)^3} + \frac{G_m S_c^3}{(S_c^2 - S_c - a_2)^2} - \frac{S_c a_4^2}{(S_c^2 - S_c - a_2)^3} + \frac{a_4 S_c^4}{(S_c^2 - S_c - a_2)^3} \]

\[\left. \left\{ e^{-m_3 y} \right. \right\} \]

\[\left. \left. + \left( \frac{G_r P_r^3}{(P_r^2 - P_r - a_2)^2} + \frac{a_3 P_r^4}{(P_r^2 - P_r - a_2)^3} - \frac{P_r a_3^2}{(P_r^2 - P_r - a_2)^3} \right) e^{-p_r y} \right. \]

\[\left. + \left( \frac{G_m S_c^3}{(S_c^2 - S_c - a_2)^2} - \frac{S_c a_4^2}{(S_c^2 - S_c - a_2)^3} + \frac{a_4 S_c^4}{(S_c^2 - S_c - a_2)^3} \right) e^{-s_c y} \right\} , \]

\[M_i = -\frac{4G_r P_r}{n} \left( \frac{1}{m_1^2 - m_1 - a_1} + \frac{1}{P_r^2 - P_r - a_2} \right) e^{-m_4 y} \]

\[-\frac{4G_m S_c}{n} \left( \frac{1}{m_2^2 - m_2 - a_2} + \frac{1}{S_c^2 - S_c - a_2} \right) e^{-m_4 y} \]
\[ + \frac{4G_rP_r}{n} \left( \frac{1}{m_1^2 - m_1 - a_1} e^{-m_1y} + \frac{1}{P_r^2 - P_r P_r - a_2} e^{-p_r y} \right) \]
\[ + \frac{4G_mS_c}{n} \left( \frac{1}{m_2^2 - m_2 - a_2} e^{-m_2y} + \frac{1}{S_c^2 - S_c - a_2} e^{-s_c y} \right) \]
\[ + a \left\{ \frac{G_rP_r}{m_1^2 - m_1 - a_1} \frac{m_1^2}{(m_2^2 - m_2 - a_2)^2(m_1^2 - m_1 - a_1)} + \frac{G_mS_c m_2}{(m_2^2 - m_2 - a_2)^3} \right\} \]
\[ + \frac{n(1 - m_3^2)m_3^2(a_3 + a_4)}{4(m_2^2 - m_3 - a_2)^2} \]
\[ - \frac{m_4^2}{m_4 - m_4 - a_2} \left( \frac{G_rP_r}{m_1^2 - m_1 - a_1} + \frac{G_rP_r}{P_r^2 - P_r - a_2} + \frac{G_mS_c}{m_2^2 - m_2 - a_2} + \frac{G_mS_c}{S_c^2 - S_c - a_2} \right) \]
\[ - \frac{4P_r^4}{n} G_r + \frac{n}{4} G_r P_r^2 - \frac{n}{4} P_r^3 B_7 \frac{4S_c^4}{n} G_m + \frac{n}{4} G_m S_c^2 - \frac{n}{4} S_c^3 B_8 \left( e^{-m_4 y} \right) \]
\[ - \frac{4}{n} P_r G_r m_3^1 + \frac{n}{4} G_r m_1^2 \frac{(m_2^2 - m_1 - a_1)(m_2^2 - m_1 - a_2)}{e^{-m_1 y}} - \frac{4}{m_1^2 - m_1 - a_1)(m_2^2 - m_1 - a_2)} e^{-m_1 y} \]
\[ - \frac{n}{4} m_3 B_6 \frac{e^{-m_2 y}}{(m_2^2 - m_3 - a_2)} \]
\[ + \frac{G_rP_r}{m_1^2 - m_1 - a_1} \frac{1}{P_r^2 - P_r - a_2} + \frac{G_mS_c}{m_2^2 - m_2 - a_2} \frac{1}{S_c^2 - S_c - a_2} \frac{1}{e^{-m_3 y}} \]
\[ - \frac{4P_r^4}{n} G_r + \frac{n}{4} G_r P_r^2 - \frac{n}{4} P_r^3 B_7 \frac{4S_c^4}{n} G_m + \frac{n}{4} G_m S_c^2 - \frac{n}{4} S_c^3 B_8 \left\{ e^{-p_r y} \right\} - \frac{(S_c^2 - S_c - a_2)^2}{e^{-s_c y}} \]

\[ T_r = e^{-m_1 y}, T_i = \frac{4P_r}{n} (e^{-p_r y} - e^{-m_1 y}), \]
\[ C_r = e^{-m_2 y}, C_i = \frac{4S_c}{n} (e^{-s_c y} - e^{-m_2 y}), \]

Hence expressions for transient velocity, temperature and concentration field for
\[ nt = \pi \] are
\[ u(y, \frac{\pi}{n}) = u_0(y) - \varepsilon M_r, T(y, \frac{\pi}{n}) = T_0(y) - \varepsilon T_r, C(y, \frac{\pi}{n}) = C_0(y) - \varepsilon C_r, \]

\[ u(y, \frac{\pi}{n}) = u_0(y) - \varepsilon M_r, T(y, \frac{\pi}{n}) = T_0(y) - \varepsilon T_r, C(y, \frac{\pi}{n}) = C_0(y) - \varepsilon C_r, \]
3.4 Skin Friction, Rate of Heat and Mass Transfer

The non-dimensional skin friction $\tau_w$ at the plate ($y=0$) is given by

$$\tau_w = [u_0'(0) + \alpha u_0^\ast(0)] + \epsilon e^{im} [u_0'(0) + \alpha u_0^\ast(0)] \quad (3.4.1)$$

The dimensionless heat transfer coefficient at the plate is given by

$$N_u = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = -\left[T_0'(0) + \epsilon e^{in} T_0'(0)\right] = P_r + \epsilon |R| \cos(nt + \beta) \quad (3.4.2)$$

where $R$ and $\beta$ are the amplitude and the rate of heat transfer.

Similarly, the mass transfer co-efficient $S_h$ at the plate is given by

$$S_h = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = -\left[C_0'(0) + \epsilon e^{im} C_0'(0)\right] = S_c + \epsilon |Q| \cos(nt + \gamma) \quad (3.4.3)$$

where $Q$ and $\gamma$ are the amplitude and the rate of mass transfer.

3.5 Results and Discussion

The aim of the problem is to bring out the effects of the visco-elastic parameter on the flow characteristics. The visco-elastic effect is exhibited through the non-dimensional parameter $\alpha$. The corresponding results for Newtonian fluid can be deduced from the above results by setting $\alpha = 0$ and it is worth mentioning here that the results show conformity with that of Singh et al.

Figures 3.1 to 3.8 reveal the transient velocity $u$ against $y$ and the skin friction $\tau_w$ against the magnetic parameter $M$ with different values of the Grashof number for heat transfer $G_r$, the Grashof number for mass transfer $G_m$ and the visco-elastic parameter $\alpha$ with consideration of Prandtl number $P_r = 5$, permeability parameter $K_0 = 10$, frequency parameter $n=5$, Schmidt number $S_c = 5$, perturbation parameter $\epsilon = 0.005$ and $nt = \pi$. Two cases in general interest for Grashof number $G_r > 0$ corresponding to cooling of the plate and Grashof number $G_r < 0$ corresponding to heating of the plate are considered.

The figures reveal that due to cooling of the plate ($G_r > 0$), the transient velocity $u$ first increase and then decrease (Figs. 3.1, 3.2, 3.3) in both Newtonian and non-
Newtonian cases but the opposite pattern is observed (Figs. 3.4, 3.5, 3.6) due to heating of the plate ($G_r < 0$). Again, Fig.3.1 depicts that $u$ increases with increasing values of the visco-elastic parameter $|\alpha|$ as compared to their corresponding values for Newtonian fluid, but decreases when the values of the magnetic parameter $M$ increase with $|\alpha|$ (Figs. 3.2, 3.3) due to cooling of the plate but the reverse behaviour is observed due to heating of the plate (Figs. 3.4, 3.5, 3.6).

Figs. 3.7 and 3.8 demonstrate the variations of the skin friction $\tau_w$ against $M$. From the figures, it is seen that the values of $\tau_w$ decrease for $G_r > 0$ and increase for $G_r < 0$ in both Newtonian and non-Newtonian cases. Again, the profiles reveal that $\tau_w$ increases with the increasing values of $|\alpha|$ in comparison with their corresponding values for Newtonian fluid and the magnetic parameter $M$ due to cooling of the plate (Fig. 3.7) but decreases due to heating of the plate (Fig. 3.8) with the combination of other flow parameters.

It is noted that the dimensionless heat transfer co-efficient $N_u$ and the mass transfer co-efficient $S_q$ are not affected by the visco-elastic parameter.

### 3.6 Conclusion

In this study, the unsteady hydromagnetic flow of an electrically conducting visco-elastic fluid past an infinite vertical porous plate in a porous medium of time dependent permeability under oscillatory suction velocity normal to the plate has been investigated. The results of investigation may be summarized in the following conclusions:

- The velocity field is considerably affected by the variation of the visco-elastic parameter in the cases of cooling of the plate ($G_r > 0$) and heating of the plate ($G_r < 0$) with the change of magnetic parameter in both Newtonian and non-Newtonian cases.
• In the heated plate, the increase of the absolute values of the visco-elastic parameter leads to increase the skin friction but a reverse trend is noticed in case of cooled plate when the variation in the magnetic parameter is included.
• The temperature and concentration fields are not significantly affected by the variation of the visco-elastic parameter.
Fig. 3.1 Transient velocity $u$ against $y$ for $M=0.5$, $K_0=10$, $n=5.0$, $G_r=10$, $G_m=10$, $\varepsilon=0.005$, $P_r=5$, $S_c=5$, $nt=\pi$.

Fig. 3.2 Transient velocity $u$ against $y$ for $M=1.0$, $K_0=10$, $n=5.0$, $G_r=10$, $G_m=10$, $\varepsilon=0.005$, $P_r=5$, $S_c=5$, $nt=\pi$. 
Fig. 3.3 Transient velocity $u$ against $y$ for $M=1.5$, $K_0=10$, $n=5.0$, $G_r=10$, $G_m=10$, $\varepsilon=0.005$, $P_r=5$, $S_c=5$, $nt=\pi$

Fig. 3.4 Transient velocity $u$ against $y$ for $M=0.5$, $K_0=10$, $n=5.0$, $G_r=-10$, $G_m=-10$, $\varepsilon=0.005$, $P_r=5$, $S_c=5$, $nt=\pi$
Fig. 3.5 Transient velocity $u$ against $y$ for $M=1.0$, $K_0=10$, $n=5.0$, $G_r=-10$, $G_m=-10$, $\varepsilon=0.005$, $Pr=5$, $Sc=5$, $nt=\pi$

Fig. 3.6 Transient velocity $u$ against $y$ for $M=1.5$, $K_0=10$, $n=5.0$, $G_r=-10$, $G_m=-10$, $\varepsilon=0.005$, $Pr=5$, $Sc=5$, $nt=\pi$
Fig. 3.7 Skin Friction $\tau_w$ against $M$ for $K_0 = 10$, $n=5.0$, $Gr=10$, $Gm=10$, $\varepsilon=0.005$, $Pr=5$, $Sc=5$, $nt=\pi$

Fig. 3.8 Skin Friction $\tau_w$ against $M$ for $K_0 = 10$, $n=5.0$, $Gr=-10$, $Gm=-10$, $\varepsilon=0.005$, $Pr=5$, $Sc=5$, $nt=\pi$