CHAPTER VI
MHD MIXED CONVECTIVE OSCILLATORY FLOW OF VISCO-ELASTIC FLUID IN A POROUS CHANNEL

6.1 Introduction

The phenomenon of free convective flow in presence of heat source has been of great interest in a number of industrial applications such as fiber and granular insulation, geothermal systems etc. Such flows arise either due to unsteady motion of boundary or boundary temperature. The study of fluctuating flow finds applications in paper industry and many other technological fields. Unsteady oscillatory free convective flows play an important role in chemical engineering, turbo-machinery and in aerospace technology. In view of these, much attention has been given by several researchers towards fluctuating flows. Magnetohydrodynamics has attracted the attention of the scholars due to its diverse applications in astrophysics and geophysics as it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc. Convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage etc. The phenomenon of mass transfer is also very common in the theory of stellar structure and its observable effects are detectable at least on the solar surface. The study of the effects of magnetic field on free convection flow is important in liquid metals, electrolytes and ionized gases. The phenomenon of slip flow regime has attracted the researchers due to its wide ranging applications. The problem of the slip flow regime is very important in the era of science, technology and industrialization. Cooper et al. (1993), Makinde and Mhone (2005), Mehmood and Ali (2007) have made investigations in the broad area of heat transfer schemes and the use of oscillatory flow in a channel filled with porous medium. Choudhury and Jain (2007) have discussed the combined heat and mass transfer effects on MHD free convection flow past an oscillatory plate embedded in porous medium. Convection flows in porous medium have been investigated to observe the effects of thermal radiation and space.

- Accepted in the JP Journal of Heat and Mass Transfer, ISSN 0973-5763.
porosity by Makinde (1994). Srinivas and Muthuraj (2010) and Prakash et al. (2011) have studied the influence of chemical reaction analysis to give a mathematical model for the system to predict the reactor performance. The thermal radiation effect in fluid flow has been investigated by Grosan and Pop (2007) and Pal and Talukdar (2010), Sivaraj and Kumar (2011) have investigated the influence of buoyancy, heat source and chemical reaction on unsteady MHD slip flow in a porous channel.

This paper deals with an investigation of unsteady oscillatory slip flow of a visco-elastic fluid with variable temperature and concentration through a planar channel. The governing equations of the flow field are solved and the expressions for velocity, temperature, concentration, skin friction, rate of heat transfer and rate of mass transfer are obtained. The velocity profile and the skin friction are analyzed graphically to observe the effect of the visco-elastic parameter involved in the solution.

6.2 Mathematical Formulation

We consider the two dimensional unsteady MHD oscillatory slip flow of an electrically conducting second-order fluid through a planar channel, where $\tilde{x}$-axis is taken along the flow and $\tilde{y}$-axis is taken normal to the flow. The system is independent of $\tilde{x}$-axis as the flat wall is infinite in length. The equations governing the flow are:

Continuity Equation: \[ \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0 \] (6.2.1)

Momentum Equation: \[ \frac{\partial \tilde{u}}{\partial \tilde{t}} = -\frac{1}{\rho} \frac{\partial p}{\partial \tilde{x}} + \nu_1 \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} - \frac{v_1}{k} \tilde{u} + \nu_2 \frac{\partial^3 \tilde{u}}{\partial \tilde{y}^3 \partial \tilde{t}} - \frac{\sigma B_0^2}{\rho} \tilde{u} + g\beta (\tilde{T} - \tilde{T}_0) + g\beta (\tilde{C} - \tilde{C}_0) \] (6.2.2)

Equation of Temperature: \[ \frac{\partial \tilde{T}}{\partial \tilde{t}} = \frac{\kappa}{\rho c_p} \frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2} - \frac{1}{\rho c_p} \frac{\partial \tilde{q}}{\partial \tilde{y}} + \frac{Q}{\rho c_p} (\tilde{T} - \tilde{T}_0) \] (6.2.3)

Equation of Concentration: \[ \frac{\partial \tilde{C}}{\partial \tilde{t}} = D \frac{\partial^2 \tilde{C}}{\partial \tilde{y}^2} - K_R (\tilde{C} - \tilde{C}_0) \] (6.2.4)

with the boundary conditions

\[ \tilde{y} = 0: \tilde{u} = \frac{\tilde{h}}{\partial \tilde{y}}, \tilde{T} = \tilde{T}_0 + \delta_T \frac{\partial \tilde{T}}{\partial \tilde{y}}, \tilde{C} = \tilde{C}_0 + \delta_c \frac{\partial \tilde{C}}{\partial \tilde{y}} \]

\[ \tilde{y} = d: \tilde{u} = 0, \tilde{T} = \tilde{T}_1 + \delta_T \frac{\partial \tilde{T}}{\partial \tilde{y}}, \tilde{C} = \tilde{C}_1 + \delta_c \frac{\partial \tilde{C}}{\partial \tilde{y}} \] (6.2.5)
The radiative heat flux (Cogley et al. (1968)) is given by
\[
\frac{\partial q}{\partial y} = 4(T_0 - \bar{T})\Lambda^2, \quad \Lambda^2 = \int_0^\infty K_{\lambda\omega} \frac{\partial e_{\lambda\omega}}{\partial T} d\lambda
\]

We now introduce the following non-dimensional quantities
\[
x = \frac{x}{d}, \quad y = \frac{y}{d}, \quad p = \frac{dp}{\mu U_0}, \quad u = \frac{u}{U_0}, \quad \theta = \frac{T - \bar{T}}{T_1 - \bar{T}_0}, \quad \phi = \frac{\bar{C} - \bar{C}_0}{\bar{C}_1 - \bar{C}_0},
\]

\[
t = \frac{\hat{t}U_0}{d}, \quad h = \frac{\tilde{h}}{d}, \quad \delta_T = \frac{\delta_T}{d}, \quad \delta_C = \frac{\delta_C}{d}
\] (6.2.6)

In view of (6.2.6), equations (6.2.2)-(6.2.4) reduce to the following non-dimensional forms:
\[
R \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \left(\frac{1}{K} + M^2\right) u + \alpha \frac{\partial^3 u}{\partial y^2 \partial t} + G_r \theta + G_m \phi,
\]

\[
P_e \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + (F + S) \theta,
\]

\[
\frac{\partial \phi}{\partial t} = S \phi - K_r \phi
\] (6.2.7)

subject to boundary conditions
\[
y = 0: \quad u = h \frac{\partial u}{\partial y}, \quad \theta = \delta_T \frac{\partial \theta}{\partial y}, \quad \phi = \delta_C \frac{\partial \phi}{\partial y}
\]

\[
y = 1: \quad u = 0, \theta = 1 + \delta_T \frac{\partial \theta}{\partial y}, \quad \phi = 1 + \delta_C \frac{\partial \phi}{\partial y}
\] (6.2.8)

where \( R = \frac{U_0 d}{v_1} \) is the Reynolds number, \( M = \sqrt{\frac{\alpha \beta_0^2 d^2}{\mu_1}} \) is the Hartmann number,

\[
G_r = \frac{g\beta(\bar{T}_1 - \bar{T}_0) d^2}{v_1 U_0} \] is the Grashof number for heat transfer, \( G_m = \frac{g\beta(\bar{C}_1 - \bar{C}_0) d^2}{v_1 U_0} \) is the Grashof number for Mass transfer, \( \alpha = \frac{v_2 U_0}{v_1 d} \) is the visco-elastic parameter, \( K = \frac{\bar{K}}{d^2} \) is the permeability coefficient of porous medium, \( P_e = \frac{\rho C_p U_0 d}{\kappa} \) is the Peclet number, \( F = \frac{4d^2}{\kappa} \) is the Thermal radiation parameter, \( S = \frac{Q d^2}{\kappa} \) is the heat source parameter, \( S_c = \frac{D}{U_0 d} \) is the Schmidt number and \( K_r = \frac{K_{rd}}{U_0} \) is the chemical reaction parameter.

### 6.3 Method of Solution

For purely oscillatory flow, we take the solutions of equations (6.2.7) are of the form
\[
-\frac{\partial p}{\partial x} = \lambda e^{i\omega t}, \quad u(y, t) = u_0(y) e^{i\omega t},
\]

\[
\theta(y, t) = \theta_0(y) e^{i\omega t}, \quad \phi(y, t) = \phi_0(y) e^{i\omega t}
\] (6.3.1)
where \( \omega \) is the frequency of oscillation.

Substituting the equation (6.3.1) in equation (6.2.7), we get

\[
\begin{align*}
\frac{d^2 u'_0}{dt^2} - \frac{A_1^2}{1 + \alpha^2 \omega^2} u_0 &= - \frac{1}{1 + \alpha^2 \omega^2} (\lambda + G_r \theta_0 + G_m \phi_0) \quad (6.3.2) \\
\theta_0'' + A_2^2 \theta_0 &= 0 \quad (6.3.3) \\
\phi_0'' - A_3^2 \phi_0 &= 0 \quad (6.3.4)
\end{align*}
\]

where

\[ A_1 = \sqrt{\frac{1}{K} + M^2 + i \omega R}, \quad A_2 = \sqrt{F + S - i \omega P_e}, \quad A_3 = \sqrt{K_r + i \omega} \]

together with boundary conditions

\[
\begin{align*}
y = 0: & \quad u_0 = \frac{\partial u_0}{\partial y}, \quad \theta_0 = \frac{\partial \theta_0}{\partial y}, \quad \phi_0 = \frac{\partial \phi_0}{\partial y} \\
y = 1: & \quad u_0 = 0, \quad \theta_0 = 1 + \delta \frac{\partial \theta_0}{\partial y}, \quad \phi_0 = 1 + \delta \frac{\partial \phi_0}{\partial y} \quad (6.3.5)
\end{align*}
\]

Solving equations (6.3.2)-(6.3.4) under boundary conditions (6.3.5), we obtain the solutions for fluid velocity, temperature and concentration as

\[
\begin{align*}
u(y, t) &= [A_8 + A_9 \sinh A_3 y + A_{10} \cosh A_3 y + A_{11} \sin A_2 y + A_{12} \cos A_2 y] \\
&\quad + A_{13} \sinh \frac{A_1}{\sqrt{1 + \alpha^2 \omega^2}} y + A_{14} \cosh \frac{A_1}{\sqrt{1 + \alpha^2 \omega^2}} y] e^{i \omega t} \quad (6.3.6) \\
\theta(y, t) &= [A_6 \sin A_2 y + A_7 \cos A_2 y] e^{i \omega t} \quad (6.3.7) \\
\phi(y, t) &= [A_4 \sinh A_3 y + A_5 \cosh A_3 y] e^{i \omega t} \quad (6.3.8)
\end{align*}
\]

where

\[
\begin{align*}
A_4 &= \frac{1}{(1 - \delta^2 A_3^2) \sinh A_3}, \\
A_5 &= \delta A_3 A_4, \quad A_6 = \frac{1}{(1 + \delta^2 A_2^2) \sin A_2}, \quad A_7 = \delta A_2 A_6, \quad A_8 = \frac{\lambda}{A_1^2}, \\
A_9 &= \frac{G_m A_4}{A_1^2 - (1 + \alpha^2 \omega^2) A_3^2}, \quad A_{10} = \frac{G_m A_5}{A_1^2 - (1 + \alpha^2 \omega^2) A_2^2}, \quad A_{11} = \frac{G_r A_6}{A_1^2 + (1 + \alpha^2 \omega^2) A_2^2}, \\
A_{12} &= \frac{G_r A_7}{A_1^2 + (1 + \alpha^2 \omega^2) A_2^2}.
\end{align*}
\]
\[ A_{13} = \frac{hA_1}{\sqrt{1 + \alpha^2 \omega^2}} \cosh \left( \frac{A_1}{\sqrt{1 + \alpha^2 \omega^2}} \right) + \sinh \left( \frac{A_1}{\sqrt{1 + \alpha^2 \omega^2}} \right) \left\{ \frac{\lambda}{A_1^2} \right\} \]
\[ + \frac{G_r (A_7 - A_6 A_2 h)}{A_1^2 + (1 + \alpha^2 \omega^2)A_2^2} + \frac{G_m (A_5 - A_4 A_3 h)}{A_1^2 - (1 + \alpha^2 \omega^2)A_3^2} \cosh \left( \frac{A_1}{\sqrt{1 + \alpha^2 \omega^2}} \right) - \frac{\lambda}{A_1^2} \]
\[ - \frac{G_r}{A_1^2 + (1 + \alpha^2 \omega^2)A_2^2} (A_6 \sin A_2 + A_7 \cos A_2) \]
\[ + \frac{G_m}{A_1^2 - (1 + \alpha^2 \omega^2)A_3^2} (A_4 \sin h A_3 + A_5 \cosh A_3) \] 
\[ A_{14} = - \left[ \frac{\lambda}{A_1^2} + \frac{G_r (A_7 - A_6 A_2 h)}{A_1^2 + (1 + \alpha^2 \omega^2)A_2^2} + \frac{G_m (A_5 - A_4 A_3 h)}{A_1^2 - (1 + \alpha^2 \omega^2)A_3^2} - \frac{hA_3 A_1 h}{\sqrt{1 + \alpha^2 \omega^2}} \right] \]

6.4 Skin Friction, Rate of Heat and Mass Transfer

The non-dimensional skin friction at the plate is given by
\[ \tau_w = u'(0) + \alpha \frac{\partial}{\partial t} \{u'(0)\} = A_3 A_9 + A_2 A_{11} + \frac{A_1 A_{13}}{\sqrt{1 + \alpha^2 \omega^2}} (1 + a \iota \omega) e^{i\omega t} \quad (6.3.9) \]

The coefficient of the rate of heat transfer at the plate, which in the non-dimensional form in terms of Nusselt number \( N_u \) is given by
\[ N_u = -(\frac{\partial \theta}{\partial y})_{y=0} = A_6 A_2 e^{i\omega t} \quad (6.3.10) \]

and the coefficient of the rate of mass transfer at the plate, which in the non-dimensional form in terms of Sherwood number \( S_h \) is given by
\[ S_h = -(\frac{\partial \phi}{\partial y})_{y=0} = A_4 A_3 e^{i\omega t} \quad (6.3.11) \]

6.5 Results and Discussion

The purpose of the present study is to discuss the effects of visco-elasticity on two dimensional unsteady mixed convective oscillatory flow of an electrically conducting fluid through a planar channel with variable temperature and concentration. The visco-
elastic effect is exhibited through the non-dimensional parameter $\alpha$. All the corresponding results for Newtonian fluid is obtained by setting $\alpha=0$.

Figures 6.1 to 6.4 illustrate the variations of velocity profile against $y$ for various values of visco-elastic parameter $(\alpha = 0, -0.25, -0.4)$ along with different values of Hartmann number $(M)$, Grashof number for heat transfer $(G_r)$, Grashof number for mass transfer $(G_m)$, Permeability coefficient $(K)$, Reynolds number $(R)$, radiation parameter $(F)$, Schmidt number $(S_c)$, Peclet number $(P_e)$, chemical reaction parameter $(K_r)$, heat source parameter $(S)$ and some fixed values $\omega=0.1$, $\lambda=1$, $\delta_T=0.002$, $\delta_C=0.002$, $h=1$, $t=1$. The graphs show that the velocity profile increases in the neighbourhood of the first plate but decreases as the fluid moves towards the second plate in both Newtonian and non-Newtonian cases. It is also noted that the fluid flow experiences an enhanced trend due to the decreasing values of the visco-elastic parameter as compared to their values for Newtonian fluid.

Also, we notice that the velocity profile is subdued during the rising behaviour of Hartmann number (figures 6.1 and 6.2) and Grashof number for mass transfer (figures 6.1 and 6.4), but reverse behaviour is observed in case of Grashof number for heat transfer (Figures 6.1 and 6.3) in both Newtonian and non-Newtonian cases.

The variations of shearing stress at the plate $(y=0)$ against various values of flow parameters are depicted in figures 6.5 to 6.8. The figures notify that the shearing stress at the plate increases due to the increase of absolute values of visco-elastic parameter. The rising nature of Hartmann number (figure 6.5), frequency of oscillation (figure 6.6) and Grashof number for mass transfer (figure 6.8) decline the shearing stress at the plate but the reverse behaviour is seen in case of enlarging values of Grashof number for heat transfer (figure 6.7). Also, it is noticed from the figures that the shearing stress rises with the increasing trend of the absolute value of the visco-elastic parameter in combination with other flow parameters involved in the solution.

The temperature and concentration fields are not significantly affected by visco-elastic parameter.
6.6 Conclusion

The problem of unsteady MHD oscillatory slip flow of a visco-elastic fluid in a planar channel with variable temperature and concentration is analyzed. The key observations are:

- The velocity of the fluid is enhanced with a rise in the absolute value of the visco-elastic parameter.
- An increase in Hartmann number/Grashof number for mass transfer leads to decrease in the velocity profile, while increasing Grashof number for heat transfer produces the opposite effect in both Newtonian and non-Newtonian cases.
- The rising trend of Hartmann number/frequency of oscillation/Grashof number for mass transfer diminishes the shearing stress but a reverse effect is observed in case of the growth of Grashof number for heat transfer in both Newtonian and non-Newtonian cases.
- The enhanced trend of shearing stress is noticed in the flow field due to the increase of the absolute value of the visco-elastic parameter.
- The temperature and concentration fields are not significantly affected by the variation of visco-elastic parameter.
Fig. 6.1 Variation of u against y for M=2, G_r=2, G_m=1, K=2, R=2, F=1, S=2, P_e=4, K_r=4, S_c=0.02, \( \omega = 0.1 \), \( \lambda = 1 \), \( \delta_T = 0.002 \), \( \delta_c = 0.002 \), h=1, t=1

Fig. 6.2 Variation of u against y for M=4, G_r=2, G_m=1, K=2, R=2, F=1, S=2, P_e=4, K_r=4, S_c=0.02, \( \omega = 0.1 \), \( \lambda = 1 \), \( \delta_T = 0.002 \), \( \delta_c = 0.002 \), h=1, t=1
Fig. 6.3 Variation of $u$ against $y$ for $G_r=4$, $M=2$, $G_m=1$, $K=2$, $R=2$, $F=1$, $S=2$, $P_e=4$, $K_r=4$, $S_c=0.02$, $\omega = 0.1$, $\lambda=1$, $\delta_T=0.002$, $\delta_C=0.002$, $h=1$, $t=1$

Fig. 6.4 Variation of $u$ against $y$ for $G_m=3$, $M=2$, $G_r=2$, $K=2$, $R=2$, $F=1$, $S=2$, $P_e=4$, $K_r=4$, $S_c=0.02$, $\omega = 0.1$, $\lambda=1$, $\delta_T=0.002$, $\delta_C=0.002$, $h=1$, $t=1$
Fig. 6.5 Shearing stress $\tau_w$ against $M$ for $G_t=2$, $G_m=1$, $K=2$, $R=2$, $F=1$, $S=2$, $P_e=4$, $K_r=4$, $Sc=0.02$, $\omega=0.1$, $\lambda=1$, $\delta_T=0.002$, $\delta_C=0.002$, $h=1$, $t=1$.

Fig. 6.6 Shearing stress $\tau_w$ against $\tau_w$ for $M=2$, $G_t=2$, $G_m=1$, $K=2$, $R=2$, $F=1$, $S=2$, $P_e=4$, $K_r=4$, $Sc=0.02$, $\omega=0.1$, $\lambda=1$, $\delta_T=0.002$, $\delta_C=0.002$, $h=1$, $t=1$. 

Fig. 6.7 Shearing stress $\tau_w$ against $G_r$ for $M=2$, $G_m=1$, $K=2$, $R=2$, $F=1$, $S=2$, $P_\varepsilon=4$, $K_r=4$, $S_c=0.02$, $\omega=0.1$, $\lambda=1$, $\delta_T=0.002$, $\delta_C=0.002$, $h=1$, $t=1$

Fig. 6.8 Shearing stress $\tau_w$ against $G_m$ for $M=2$, $G_r=2$, $K=2$, $R=2$, $F=1$, $S=2$, $P_\varepsilon=4$, $K_r=4$, $S_c=0.02$, $\omega=0.1$, $\lambda=1$, $\delta_T=0.002$, $\delta_C=0.002$, $h=1$, $t=1$