Chapter 1  Introduction and Statement of the Problem

Inventory can be viewed as a necessary evil in the industry. In an ideal world, where the demand upon a business is known exactly and well in advance, and where suppliers keep to their due dates, there would be little need to hold any form of inventory other than a limited amount of in-process stocks caused as a by-product of the manufacturing process. As the process of production and distribution of goods take time, processes are not so much perfect, consumer preferences and demands are not predictable, therefore, imbalance between supply and demand occurs so, it would be great if a business can arrange supply to equal demand, but in reality things are not that simple hence a proper inventory is required.

Maintaining optimum level of inventory is the main aim of inventory management. In practice however, both demands in advance and exact delivery date are not known. Even if the supply side of the balance equation was under control, it would still be difficult to control the demand side. As demand is always uncertain and also the consumer taste and requirement can change with respect to time. It is the lead time in producing and delivering a product which create the concern. Customers will certainly go to a competitor if their usual supplier is out of stock.

For an organization having the ability to find out inventory in a timely and accurate manner is critical for having uninterrupted business operations because inventory is often one of the largest current assets on a company's balance sheet. Operations research plays a strategic and tactical role in satisfying customer needs and making firms strong international competitors. If a business expects the cost of obtaining or producing their product to rise in the future, it may prefer to produce large quantities today and hold inventory for future demand. This tactic may provide a competitive edge over competition and secure greater market share. Keeping the right stock is a very strong casual factor when it comes to customer satisfaction and good financial results. But in practice, inventory control is far from simple matter. Effective control of inventory is considered a key factor to competitiveness for businesses and integrating supply with demand is a fundamental management challenge. Inventory is a primary part of many of today's businesses. Essentially, inventory is the storage of products that are
sold to consumers to help the business for making profit. Further, in some instances, inventory also includes what the company uses to keep the business up and running.

1.1 Introduction

Operations Research (OR) is a discipline which is focused on the application of information technology for informed decision-making, hence, the term OR can be associated with an art of winning the war without actually fighting it. The first mathematical technique in this field was developed by George B. Dentiz. Today, the impact of OR can be felt in many areas. The main aim of operations research is to provide researcher rational bases for making decisions by seeking to understand and structure of complex situations. Much of the actual work is conducted by using analytical and numerical techniques to develop and manipulate mathematical models of organizational systems that are composed of people, machines and procedures. The model is actually a simplified representation of the problem in which only the most important features are considered for reasons of simplicity.

Operations research provides us a wide range of techniques which are used in problem-solving and also provide methods applied in the pursuit of improved decision-making and efficiency. The methods of operations research are very often used in inventory control, management science, industrial engineering, mathematics and economics etc., operations research has an important part to play in proper and objective decision-making in industry, agriculture, business and other sphere of economy. OR provides top-level administrators with a quantitative basis for decisions that will increase the effectiveness of such organizations in carrying out their basic purpose. The actual OR process can in general be described via three steps.

(i) A set of potential solutions to a problem is identified and developed.
(ii) The alternatives derived in the first step are analyzed and reduced to a smaller set of solutions.
(iii) The alternatives derived in the second step are subjected to have simulated implementation and, if feasible exposed to an actual analysis in a real-world environment.

Thus, OR makes use of experience and proficiency of people from various disciplines for the developing new methods and procedures, which are more effective as these methods evolve from
specific tools and techniques of various disciplines and may be applied with or without some modification or refinements to the business problems. OR methods deal with taking appropriate decision with an objective of optimizing an objective function associated with the nature of problem. OR increases the creative ability of the decision making and replaces management by personality.

1.2 Inventory Control and Management

The role of capital is crucial in the context of industrial development. It is much more true in the case of developing countries like India. The capital raised by a firm is invested in fixed assets as well as in current assets for performing its business activities. The portion invested in current assets is called the working capital and the inventory constitutes the largest proportion of it. Thus, inventories call for efficient management. Good inventory management is good financial management as well as.

One must agree with the observation that “when you need money, look at your inventories before you look to your bankers”. Inventory control is a process by which one could obtain results in minimum cost. Two factors which influence the inventories of all types are the accuracy of the final forecast and the available storage space.

The concept of inventory management has been one of the many analytical aspects of management. It involves optimization of resources available for holding stock of various materials. Lack of inventory can lead to stock-outs, causing stoppage of production, but a very high inventory, on the other hand, can result in increased cost of production due to high cost of carrying inventory, thus optimization of inventory should ensure that stocks are neither too low nor too high.

For solving production problems, one can require a solid foundation in operations research fundamentals as production problems are operational research problems. Also the solution of production system’s problems frequently draws in more than one of the primary areas of operations research; therefore, for the successful production, researcher cannot be based on one-dimensional. It should be multi-dimensional. Furthermore, for solving the production system problems, an in-depth understanding of the real problem is required since involving assumptions that simplify the mathematical structure of the problem may lead to an elegant solution for the wrong problem. The efficient use of capital in an undertaking helps improve customer service and earn profit in the
process. It is also difficult to raise the margin of profit extensively due to competition in business. Thus the capital turnover and productivity of capital often become totally ineffective.

Inventories like finished products, work-in-progress, components, raw materials, stores, spares, etc., accounted for 80 percent or more of the working capital in some of the representative industries studied in the past. It would appear that any effort put in towards rationalization of inventories can bring about an appreciable saving. For example, a scientific system of control can reduce investment in inventories considerably, sometimes by as much as 50 per cent or even more. At the current time, the field of OR is extremely dynamic and ever evolving. To name a few of the contemporary (primary) research projects, current work in OR seeks to develop software for material flow analysis and design of flexible manufacturing facilities using pattern recognition and graph theory algorithms. Further, approaches for the design of re-configurable manufacturing systems and progressive automation of discrete manufacturing systems are under development. Additional OR projects focus on the industrial deployment of computer-based methods for assembly line balancing, business process, re engineering, capacity planning, pull scheduling, and setup reduction, primarily through the integration of the philosophies of the theory of constraints and lean manufacturing. Industrialization is an important means of modernization.

The increased pace of industrialization in India, in its wake, has given rise to a number of managerial problems. Among them, the problem of inventory management is significant. The need for efficient management of available resources in any business organization requires no emphasis as every industrial undertaking is expected to be running efficiently. The industrial undertakings, on the one hand, are witnessing the ever-increasing cost of inputs which constitutes nearly 60 percent of the total cost. On the other hand, there are constraints on increasing the price of finished products on account of fierce competition in the business. Left with no choice, the industrial undertakings are forced to manage their available resources more efficiently.

Several modern techniques have been developed and employed by managers as a solution to this problem. Among these, inventory management is the most effective. It enables a manager to increase productivity of capital by reducing material costs, preventing blocking up of large working capital for long periods and improving capital turnover.
1.3 Inventory Modelling and Terminology

An inventory control problem can be solved by using several methods starting from trial-and-error methods to mathematical and simulation models. Mathematical models help in deriving certain rules which may suggest how to minimize the total (or incremental) inventory cost in case of deterministic demand or how to minimize expected cost in case of probabilistic demand. In the present study, only deterministic inventory control models were used and also, the derivation of economic order quantity (EOQ) for a given inventory situation was done.

(a) Steps of Inventory Model Building

The steps to build up a suitable inventory model and then to derive decision rules are as follows:

(i) Collect the data regarding the pattern of demand, the replenishment policy, planning horizon, relevant inventory costs, etc.

(ii) Examine the inventory situation carefully, listing characteristics and assumptions concerning the situation.

(iii) Develop the total annual relevant cost equation in narrative form as follows: Total annual relevant cost = Cost of the item + Procurement costs + Carrying costs (Cycle stocks + Safety stocks) + Stock-out costs.

(iv) Transform the total annual cost equation from narrative into the shorthand logic of mathematics.

(v) Optimize the cost equation by finding the optimum for how much to order (order quantity), when to re-order (re-order point) and the total relevant cost.

The inventory situations can be classified as either deterministic (variables are known with certainty) or stochastic (variables are probabilistic).

In older times, an individual’s wealth was usually assessed by the size of his blocks, granaries and warehouses. But with the advent of industrialization, wealth has become to be identified with money. There has been a strong tendency towards possessing the means to purchase goods and services rather than goods themselves. Inventories are now often referred to as the graveyard of business as surplus stocks have been the principal cause of business failure.
Modern managers have made a complete change in their outlook on inventory holdings on account of:

(i) Ever-increasing size of the business establishments.
(ii) Wide variety and complexity of modern requirements.
(iii) Urgency of modern requirements.

Material management is an important function in the industrial undertakings and its efficient implementation can indeed be a significant factor in the struggle for improved earnings. As long as the business is on a small scale, the management can keep itself well informed of all phases of its activities. Thus, an undertaking can make best choices and decisions from various alternatives that are available. When the firm’s business expands, the number of specialists needed to control and manage various activities. These specialists make decisions that are money savers. At the same time, some decisions which may save costs in one specialist’s area, may have negative effects on the costs in another specialist’s area, virtually, none being aware of the reasons for the increased cost. This flaw can be overcome only through an effective organizational set-up. Thus, the management of physical goods or commodities is one of the most important aspects of a logistic system.

Inventory can be thought of as important to an organization as blood is in the human body. Different groups of people within the different organization put different views on inventory. Inventory management in a business serves much as the suspension system of automobile. Ups and downs in sales can be absorbed by inventory just as car springs absorb bumps in the road.

(b) Inventory Categories

There are five basic types of inventory: raw material, work-in-process, finished goods, distribution inventory and maintenance, repair and operating (MRO) supplies.

(i) Raw Material

This includes all the purchased parts and direct materials, that go into the end product. This type of inventory has value added to it as it flows together as sub assemblies, assemblies and finally into the
shippable product, Yu et al. [1] presented an inventory control model for supply chain with deteriorating raw materials and products.

(ii) Work-In-Process

This is inventory in the process of being assembled into final products. Raw materials are released from inventory and moved to work center. People (direct labor) and/or machines are used to add value by putting the parts together as sub-assemblies, assemblies, and then into final products. These parts may be restocked temporarily until withdrawn for use later in the production process. While they are in this state, they may be referred to as semi-finished assemblies.

(iii) Finished Goods

These are shippable inventories ready to be delivered at distribution centers, retailers, wholesalers or directly to customers.

(iv) Distribution Inventory

This is an inventory that held at points as close to the customer as possible. Distribution points, such as warehouses or stores, may be owned and operated by the manufacturer or may be independently owned and operated. However, managing inventories is necessary regardless of ownership. So, the term “distribution centers” is used for intermediate storage locations, pending delivery to the final customer.

(v) Maintenance, Repair and Operating (MRO) Supplies

As we know, it is natural that the accounting choices vary between distribution and manufacturing settings. The MRO items are held by most of companies. These inventories are often low cost, and include office and operating supplies and services. Distribution business tend to carry mostly finished goods for resale while manufacturing companies tend to have less finished goods and more raw materials and work in progress yet the amount of each category varies greatly depending on the specifics of your industry and business. Given these differences, it is obvious, that all organizations—whether manufacturers, wholesalers, retailers, banks, hospitals or even the Federal Reserve have some inventory concern.
(c) Costs of Inventory

The costs incurred in operating an inventory system play major role in determining what the operating doctrine should be. The following types of costs are considered relevant to inventory decision making namely;

(i) Inventory carrying cost or holding cost.
(ii) Ordering cost or set-up cost.
(iii) Shortage cost or stock-out cost.
(iv) System control cost or cost of operating the information processing system.
(v) Production cost or purchase price.
(vi) Selling price.

(i) Inventory Carrying Cost or Holding Cost

It is defined as the cost of holding material inside and outside the stores. It is associated with the level of inventories. Holding cost, which is the cost of keeping an item in inventory for a period of time consists of the opportunity costs of tied-up capital, material handling costs and storage costs. The holding cost is typically determined as a percentage of the rupees per unit of time. Components of inventory carrying cost are:

(i) Cost of money or capital tied up in inventories, e. g., interest on the locked up capital.
(ii) Cost of storage space and staff, e. g., rent and other maintenance costs including part of salary of stores personnel.
(iii) Inventory maintenance cost, e. g., security, stock checking and monitoring of inventory.
(iv) Cost of maintaining inventory records.
(v) Damage and deterioration cost e. g., cost of damage in storage and quality losses.
(vi) Pilferage cost, e. g., cost of missing items due to various causes.
(vii) Obsolescence cost, e. g., certain items would become obsolete or outdated and related costs.
(viii) Insurance cost, e. g., premium to be paid while in storage.

Maintenance of inventory means storage costs. These include expenditure incurred on inventory staff, expenditure on providing various facilities like heating, lighting, floor space, shelves and racks,
bins and containers, material handling equipment and other provisions for safe and proper storage of items. These costs generally depend upon the volume to value ratio of an item.

(ii) Ordering Cost or Set-Up Cost

This cost is associated with the placement of an order for the acquisition of inventories. It is determined on the basis of expenses incurred in the purchase and accounting department. In case, a firm produces its own inventory instead of purchasing from an outside source, production set up costs are analogous to ordering costs. Some of the components of cost under this category are:

(i) Man power cost e.g., money spent in sending enquiries, receiving quotations, comparing etc.
(ii) Finalizing orders and placing orders, expediting and follow up cost of an order.
(iii) Transportation cost and stationary cost.
(iv) Inspection cost and cost of settlement for payment.
(v) Requisition cost of handling of invoices, stationary, payments, etc.

(iii) Shortage Cost or Stock-Out Cost

These costs are associated with either a delay in meeting demand or the inability to meet it at all. Therefore, shortage costs are usually interpreted in two ways. In case of unfulfilled demand can be fulfilled at a later stage (backlog case), these costs are proportional to quantity which is short as well as the delay time. In case, the unfulfilled demand is lost (no backlog case), these costs become proportional to only the quantity that is short. These results in cancelled orders, lost sales, profit and even the business itself.

(iv) System Control Cost or Cost of Operating the Information Processing System

As stock levels change, someone must update records whether by hand or by computer. Where the inventory levels are not recorded daily, the operating cost is incurred in obtaining accurate physical counts of inventories.
(v) Production Cost or Purchase Price

The cost of the product is the amount paid to the supplier (purchase price) for the product received or the direct production cost, if manufactured. In the situation of fluctuating prices, planning for inventory depends on the average price. Thus, the cost/price factor is of great significance, in cases where price discounts can be obtained or when large production runs results into lower production cost.

(vi) Selling Price

In some cases, the demands may be influenced by the quantity stocked, and thus the inventory model will be based on a profit maximization criterion. This includes the revenue from selling the product. The unit selling price may be constant or variable, depending upon the quantity discount [2].

(d) Inventory Terminology

The following terms are also used in the classification of inventory control:

(i) Demand

According to Rao et al. [3], the demand of a product is the number of units taken from its inventory and can be categorized according to its size and pattern. Demand size is the quantity required to satisfy the demand for inventory. When the demand size is known, the system is referred to as deterministic and when it is not known, it is called non-deterministic system. Sometimes it depends on various parameters namely time, stock, price etc., The demand may be characterized as:

(i) Constant.
(ii) Time dependent.
(iii) Stock dependent.
(iv) Price dependent.
(v) Probabilistic with known or unknown distribution.
(ii) Re-order Point

Re-order point is the level of inventory of a given commodity at which a purchase requisition is initiated.

(iii) Re-order Level

According to Longo and Arivarignan [4], the re-order level is that level of the inventory at which the fresh order for that item must be placed. This is also defined as the point fixed between maximum and minimum stock levels at which time it is essential to initiate purchase requisition and manufacturing requisition for fresh supplies of the material.

(iv) Partial Backlogging

According to Zangwill [5], a supply chain system in which a supplier prepares for the selling season by building stock levels prior to the beginning of the season and shortages realized at the beginning of the season are represented as mixtures of back orders and lost sales. The many real-life situations and practical experiences reveal that some but not all customers will wait for backlogged items during a shortage period, such as for fashionable commodities or high-tech products with short product life cycle.

(v) Order Level

The level of stock of any item at which an order is initiated for more supplies of that item, is called order level.

(vi) Delivery Lag or Lead Time

According to Rob and Silver [6], the time between the requisition for an item and its receipt is known as delivery lag or lead time. In general, the lead time may be deterministic or probabilistic having the following four components:

(i) Administrative lead time can be fixed in nature.
(ii) Supplier’s lead time can be fixed in nature.
(iii) Transportation lead time cannot be fixed, and
(iv) Inspection lead time cannot be fixed.

(vii) Safety Stock

It is the extra units of inventory, which is carried as a protection against possible stock outs. It is held when an organization cannot accurately predict demand and/or lead time for the product. It is also known as buffer stock or buffer inventory. It balances carrying costs and stock out costs. Companies use supplies of safety stock to reduce the risk that they will sell out and not have stock available for customers who want to buy it. Also having a proper amount of safety stock in hand of companies to meet the sales demand which exceeds the demand they fore casted without altering their production plan.

(viii) Quantity Discount and Order Quantity

Quantity discounts are price reductions designed to induce large orders. According to Moon and Lee [7], a quantity discount is often offered by sellers to entice buyers to purchase in larger quantities. Hence, the buyer's goal in this case is to select the order quantity that will minimize total costs, where total cost is the sum of carrying cost, ordering cost, and purchase cost. Quantity discounts provide a practical foundation for coordinating inventory control models. An incentive is offered to a buyer that results in a decreased cost per unit of goods or materials, when purchased in greater numbers.

It is a common business practice for pricing schedule to display economies of scale with prices decreasing as lot size increases. Such pricing schedules offer discounts based on the quantity ordered in a single lot. This encourages the retailers to order in larger lots to take advantage of price discounts. This adds to the average inventory and flow time in a supply chain. Unlike the EOQ model, the purchase cost now becomes an important criterion in determining the optimal order size and the corresponding total annual inventory cost.

Sometimes, manufacturers use trade promotions to increase sales by offering a discounted price over a pre specified period of time over which the discount is effective. In some cases, the manufacturer may require some specific actions from the retailer to qualify for the discount, such as, displays, advertising, promotion, and so on. Trade promotions are quite common in the consumer packaged goods industry, with manufacturers promoting different things at different times of the year. The goal
of the trade promotions is to influence retailers to act in a way that helps the manufacturer achieve its goal.

(ix) Time Horizon

This refers to the planning period over which inventory is to be controlled. The planning period may be finite or infinite. In general, inventory planning is carried out on an annual basis. If the time period is long, the time value of cost should be taken into consideration using proper discount factors. On the other hand, it is also possible that the item can be stored only for a limited time period due to perishability or obsolescence. The model should, therefore, minimize the cost over the specific period of time. Jaggi et al. [8] presented the optimal inventory replenishment policy for deteriorating items under inflationary conditions using a discounted cash flow (DCF) approach over a finite time horizon.

(x) Deterioration

According to Ghare and Schrader [9], deterioration means that a product fails to regularly implement its function. Deterioration occurs for most products in the real world. The deterioration for different product is categorized into the following three parts:

(i) Direct spoilage, e.g., vegetable, fruit, and fresh food, etc.
(ii) Physical depletion, e.g., gasoline and alcohol, etc.
(iii) Deterioration such as radiation changing, negative spoiling, loss of efficacy in inventory, e.g., electronic components and medicine.

From another point of view, deterioration can also be classified by the time-value or the products life of inventory.

(i) Utility Constant: Its utility does not change significantly as time passes within its valid usage period, e.g., liquid medicine.
(ii) Utility Increasing: Its utility increases as time passes, e.g., some alcoholic drinks.
(iii) Utility Decreasing: Its utility decreases as time passes, e.g., vegetables, fruits, and fresh foods, etc.
(xi) Trade Credit

Trade credit is defined as an agreement where a customer can purchase goods on account of paying the supplier at a later date. Therefore for many businesses, trade credit is an essential tool for financial growth. It makes economic sense for the retailer to delay the settlement of the replenishment account up to the last moment of the permissible period allowed by the supplier. There are multiple functions of trade credit. First model based on trade credits was developed by Goyal [10]. A business must also consider the positive and negative costs of trade credit and its potential financial impact. A higher interest is charged if the payment is not settled by the end of the trade credit period.

Using trade credit, a purchase of supplies without making the immediate payment is concern, therefore, it is commonly used by many business organizations as a source of making short-term financing. The effect of supplier’s trade credit policy on inventory problem has received the attention of many researchers as it is granted to those customers who have reasonable amount of financial standing and goodwill.

(xii) Inflation

The literal meaning of the word inflation is to blow up or get bigger. Inflation in inventory modeling, first time was included by Buzacott [11]. Inflation occurs when the prices of goods and services increase over time. These might be raw material costs or production costs which have risen.

Inflation is an economic concept. It plays an important role in the inventory control models because of its impotency. Its effects cannot be ignored. Before 1970, in EOQ models, the effect of inflation was not considered because it was assumed that inflation would not influence the inventory policy to any significant degree.

It was found that when inventory control systems ware analyzed, inflation had their positive as well as negative effects. In case of negative effects of inflation it includes a decrease in the real value of money and other monetary items over time; uncertainty about future inflation may discourage investment and saving, or may lead to reductions in investment of productive capital and increase savings in non-producing assets, e.g., selling stocks and buying gold.
High inflation means that there is a lot of money power in the hands of the people and as a result there is a huge demand in the market, this demand is more than the supply as a result there is a shortage of goods. Thus prices are increased so as to curb the rising demand. As a result, the effect of inflation on demand cannot be ignored.

**(xiii) Deteriorating Items**

Deterioration of an item may be defined as decay, evaporation, obsolescence, loss of utility or marginal value of an item that results in the decreasing usefulness of an inventory from the original condition. It was introduced first time by Ghare and Schrader [9]. Commodities such as fruits, vegetables and foodstuffs suffer from depletion by direct spoilage while kept in store. Highly volatile liquids such as alcohol, gasoline, etc., undergo physical depletion over time through the process of evaporation. Electronic goods, photographic film, grain, chemicals, pharmaceuticals etc., deteriorate through a gradual loss of potential or utility with the passage of time.

Therefore, while determining the optimal inventory policy of that type of products, the general decay in the originality of the product, the effect of deterioration cannot be ignored. The inventory control models to explain more practical features of the real inventory systems and the deteriorating inventory control models have been continually modified. In recent years, mathematical ideas have been used in different areas in real life problems, particularly for controlling inventory. One of the important concerns of the management is to decide when and how much to order or to manufacture so that the total cost associated with the inventory system should be minimum. This is somewhat more important, when the inventory undergo decay or deterioration. Most of the researchers in inventory system were directed towards non-deteriorating products. However, there are certain substances, whose utility do not remain same with the passage of time. Such type of items cannot be stored for a long time, therefore, the deterioration of such items in the inventory control problems plays a vital role. Therefore, as we store items of the commodity for facing the future demand situation, there may be the deterioration of items in the inventory system, which may occur due to one or many factors i.e., storage conditions, weather conditions or due to humidity.
(xiv) Permissible Delay in Payments

There have been a lot of discussions in the existing literature for the possible extension of the basic economic order quantity model to a number of situations where its assumptions are not valid up to now. A business either manufactures the products or it purchases the products or it sells from other businesses. In either case, an increase in inventory usually involves a corresponding increase in accounts payable. Jamal et al. [12] introduced the concept of permissible delay in payments. Raw materials used in the production process are purchased on credit, and many other manufacturing costs are not paid immediately. Products from other businesses are bought on credit. Instead of making immediate cash payment when inventory is increased, a business delays payment, perhaps by a month or so.

In today’s business tractions, it is more and more common to see that the customers are allowed some grace period before they settle the account with the supplier. This gives an advantage to the customers due to fact that they do not have to pay the supplier immediately after receiving the product, but instead, they can delay their payment until the end of the allowed period. The customer pays no interest during the fixed period, they have to settle the account, but if the payment is delayed beyond that period, interest will be charged. The permissible delay in payments brings some advantages to the buyer, as he would try to earn some interest for the payment received during this period. When a supplier allows a fixed time for settling the account, he is actually giving a loan to the buyer without interest during this period. Therefore, it is economical to delay in the settlement of accounts to the last moment of the permissible delay in payments.

(xv) Partial Backlogging

An important issue in the inventory theory is related to how to deal with the unfulfilled demands which occur during shortages or stock outs. In most of the developed models, researchers assumed that the shortages are either completely backlogged or completely lost. The first case, known as backordered or backlogging case, represents a situation where the unfulfilled demand is completely back ordered. In the second case, also known as lost sale case, we assume that the unfulfilled demand is completely lost. Zangwill [5] developed an inventory model with backlogging.
Furthermore, when the shortages occur, some customers are willing to wait for backorder and others would turn to buy from other sellers. In many cases, customers are conditioned to a shipping delay and may be willing to wait for a short time in order to get their first choice. For instance, for fashionable commodities and high-tech products with short product life cycle, the willingness of a customer to wait for backlogging is decreasing as the length of the waiting time is increasing.

Therefore, when backlogging of the product is accepted or rejected, is based on the length of the waiting time for the next replenishment. In many real life situations, during a shortage period, the longer the waiting time is, the smaller is the backlogging rate would be. Therefore, for realistic business situations the backlogging rate should be variable and dependent on the waiting time for the next replenishment. Many researchers have modified inventory policies by considering the “time proportional partial backlogging rate”.

1.4 Introduction to Fuzzy Theory

In mathematics, fuzzy sets are sets whose elements have degrees of membership. Fuzzy sets were first proposed by Lofti A. Zadeh in his 1965 paper entitled none other than: Fuzzy Sets. This paper laid the foundation for all fuzzy logic that are followed by mathematically defining fuzzy sets and their properties. Fuzzy logic is an approach to computing based on "degrees of truth" rather than the usual "true or false". Therefore a fuzzy set can be defined as: Let X be a space of points, with a generic element of X denoted by x. Thus \( X = \{ x \} \). A fuzzy set \( A \) in \( X \) is characterized by a membership function \( f_A(x) \) which associates with each point in \( X \) a real number in the interval \([0,1]\), with the values of \( f_A(x) \) at \( x \) representing the "grade of membership" of \( x \) in \( A \). [13]

(i) Membership Function

In fuzzy logic, it represents the degree of truth as an extension of valuation. The membership function of a fuzzy set is a generalization of the indicator function in classical sets. Membership functions allow us to graphically represent a fuzzy set. For any set, a membership function on \( X \) is any function from \( X \) to the real unit interval \([0,1]\). Membership function on \( X \) represent fuzzy subset of \( X \). The membership function which represent a fuzzy set \( \tilde{A} \) is usually denoted by \( f_A \). For an element \( x \) of \( X \) the value \( f_A(x) \) is called the membership degree of \( X \) in the fuzzy set \( \tilde{A} \).
(ii) Defuzzification

The process of interpreting the membership degrees of the fuzzy sets into a specific decision or real value is known as defuzzification. Also, it is the process of producing a quantifiable result in fuzzy logic, given fuzzy sets and corresponding membership degrees. It is typically needed in fuzzy control systems. These will have a number of rules that transform a number of variables into a fuzzy result, that is, the result is described in terms of membership in fuzzy sets.

1.5 Statement of the Problem

The problem can be stated as follows “Development of Inventory Model for Perishable Products with Trade Credits”

This can be divided into following sub problems:

(i) To analyse the effect of partial backlogging and trade credits on two warehouse model for decaying items where the demand is stock dependent.

(ii) To analyse a production inventory model for time deteriorating items with linear trend in demand.

(iii) To analyse an inventory model for deteriorating items with linearly time dependent demand rate and lost sales and inflation.

(iv) To analyse an inventory model for time dependent deteriorating items with stock and production dependent demand.

(v) To analyse a production model for reduction in selling price when capacity of own warehouse is limited.

(vi) To analyse the production economic inventory control model for deteriorating items with price sensitive demands.

1.6 Organization of the Thesis

An attempt has been made in this thesis, to develop inventory models with trade credits and deterioration. The work has been organized as follows:
Chapter 1 is introductory in nature. It includes introduction to operations research, objectives and importance of inventory control, inventory control and management, types of inventory, various costs and terms related to inventory.

Chapter 2 presents an extensive updated review of the previous relevant literature based on the critical review, also the research gaps have been identified.

An inventory model having two warehouses for decaying items with stock dependent demand under trade credits for both warehouses, owned warehouse (OW) and rented warehouse (RW) has been proposed in chapter 3 and the proposed, technique is validated by numerical example.

Production inventory model for time dependent deteriorating items with linear trend in demand deals in chapter 4. Deterioration rate at any item is assumed to follow three parameter Weibull distribution function of time. Also, this deterioration rate is suitable for items with and without life-period. Shortages are allowed in this chapter with partial backlogging. Production rate is taken decision variable and unit production cost is a function of the production rate. Profit maximization technique is used to get the expression of the profit function.

In chapter 5, we have developed an inventory model for deteriorating items with linearly time dependent demand rate and lost sales under inflation. We have assumed that the supplier offers a trade-credit period to the retailer. The demand rate is a linear function of the time, shortages allow and time dependent backlogging rate is considered. The model is validated with numerical examples. Sensitivity analysis is also carried out.

An inventory model for time dependent deteriorating items with stock and price dependent demand has been developed in chapter 6. Shortages are allowed with partial backlogging. Backlogging rate is a waiting time for the next replenishment. In this chapter, we have considered two models: first is crisp model with trade credits and second is fuzzy model with trade credits. Numerical examples are given to illustrate the results, also, the sensitivity analysis with respect to system parameters is discussed.

In chapter 7, which deals with the development of a production inventory control model, is developed for a reduction in the selling price when the capacity of OW is limited and RW is used, if needed. The demand rate is fixed ($d_1$) up to time $\mu$; and after time $\mu$ the demand rate $d_2$ dependent on its
reduction rate r. The demand rate $d_2(r)$ has an exponential trend that can be estimated/fitted using a curve fitting method. All possible cases of the model with the facility of allowable delay in payment are formulated. The cost minimization technique is used in this study. Finally, numerical examples are given to illustrate the results.

In chapter 8, we have discussed that in classical inventory models basically, the demand rate are regularly assumed to be either constant or time-dependent, but independent of the stock levels. Since the demand rate is not only influenced by stock level, but also is associated with the selling price. Therefore, we also take into account the selling price and then establish an EPQ model, in which the demand rate is a function of the selling price. We use numerical examples to illustrate the model with sensitivity analysis.

In chapter 9, we discussed that with today’s high competitive market; to afford the cost of warehouses is a very difficult task for the management in most of the countries. So, it is economical to order the inventory according to available storage space. This chapter is developed with the concept of space restriction in which demand is exponential, and deterioration is time dependent. Production is taken as a function of demand.

Chapter 10 presents the outcomes of the thesis with the scope for future work.