Chapter 7  Effect of Trade-credit on Two-warehouse Production Policy for Different Demands

7.1 Introduction

In last few decades, production companies and business enterprises have executed broad information systems in order to improve their performances. In most of the scenarios, the yields come out to be much less than anticipated. Earlier, the researchers observed that in order to comprehend maximum functioning improvements, it is necessary to get well-timed information about customer’s demand. Functioning procedures like inventory management and accumulation arrangement must be valuable in improving the firms’ or enterprises’ inventory performance. Keeping this point in mind, shops or companies use to store inventories in their own warehouse (OW) or rented warehouse (RW), in case of need. In general, when suppliers offer price discounts or permissible delay period for bulk purchases, or when the item under consideration is a seasonal product such as the harvest output or for stock-dependent demand, the administrator may procure more goods than the capacity of its own warehouse. As a result, the surplus inventories are stored in a rented warehouse. The inventory costs including the holding cost and the deterioration cost in the RW are generally higher than those in the OW due to the extra cost of safeguarding and material holding.

In this chapter, two-warehouse production models are developed for different demand under the effect of permissible delay in payment. In reality, there exist such industries in which after a particular time point, the products starts to decay or deteriorate or become obsolete. Due to this phenomenon, an efficient administrator aims at clearing the stock by selling the large amount of the items at reduced prices. In this chapter, we have considered the fixed demand rate in the beginning until the certain time point occurs, while demand is assumed to follow the pattern of nonlinear increasing power function of the reduction rate. The total cost function is formulated that involves the owned warehouse and rented warehouse holding costs, set up cost, production cost, purchasing cost of raw material, interest earned and interest charged. Numerical illustrations are given to exemplify the model.
7.2 Assumptions and Notations

(i) The model is developed for a single item.

(ii) The replenishment rate is infinite but replenishment size is finite.

(iii) The lead time is zero.

(iv) No shortages are permitted.

(v) Own Warehouse (OW) and Rented Warehouse (RW) are considered.

(vi) The time horizon is infinite.

The followings are the notations used throughout the chapter.

(i) \( I_i(t) \) : On-hand inventory at different phases at any time \( t \).

(ii) \( C_s \) : Set up cost per cycle.

(ii) \( h_o \) : Inventory holding cost per unit per unit time at OW.

(iv) \( h_r \) Inventory holding cost per unit per unit time at RW.

(v) \( C_p \) : Production cost per unit.

(vi) \( C \) : Purchasing cost per unit.

(vii) \( i_e \) : Interest earned per unit time.

(viii) \( i_c \) : Interest charged per unit time.

(ix) \( M \) : Permissible delay in settling the accounts.

(x) \( r \) : Reduction rate in selling price.

(xi) \( P \) : Production rate.

(xii) \( d_1 \) : Demand rate for items before time point \( \mu \).

(xiii) \( d_2 \) : Demand rate for items after time point \( \mu \).

(xiv) \( \mu \) : Time after which retailers/suppliers offer discounts on the selling price.
(xv) W: Capacity of OW.

(xvi) $t_1$: Total time elapsed for storage of items at RW.

(xvii) $t_2$: Production time.

(xviii) $t_3$: Time up to which inventory level becomes zero at RW.

(xix) $t_4$: Cycle length.

(xx) $s$: Per unit selling price.

(xxi) $s_1 = (1-r)s$: Per unit reduced selling price.

### 7.3 Model Formulation

The production starts at $t = 0$ with a rate $P$ and due to the combined effect of production and demand, inventory level increases up to $W$ till time $t = t_1$ in OW. After that, the inventory continues to store at RW till the production stops at $t = t_2$. The inventory level in RW is depleted gradually due to demand and that stock is cleared up to $t = t_3$. Further, the inventory stored at OW starts depleting and reaches zero at $t = t_4$. The demand for items ($d_1$) is met during the time span $[0, \mu]$, $\mu$ is the expected time after which manufacturer starts giving discount on selling price. Here, $\mu$ is measured using an appropriate distribution such as normal distribution, uniform distribution, over time, according to the quality, the marketplace climate and the age of the commodity. The management seeks to clear stock to minimize loss. To enhance these sales, reductions are proposed on the selling price. It can be justified that the demand $d_2$ is a monotonic increasing function of the reduction rate $r$. Here, the demand rate $d_2$ is assumed to be $d_2(r) = ab^r$, $a > 0, b > 1, 0 < r < 1$,

where $d_2(r)$ is an exponential monotonic increasing function of $r$.

For convenience, $d_2$ is used rather than $d_2(r)$ throughout this chapter. In the proposed model, the following cases may arise:-

Case 1  \( 0 \leq \mu \leq t_1 \)

![Graphical representation of the system in case 1](image)

**Fig. 7.1** Graphical representation of the system in case 1

The system is governed by the following differential equations in case 1:

\[
\frac{dI_{11}(t)}{dt} = P - d_1, \quad 0 \leq t \leq \mu \tag{7.1}
\]

\[
\frac{dI_{12}(t)}{dt} = P - d_2, \quad \mu \leq t \leq t_1 \tag{7.2}
\]

\[
\frac{dI_2(t)}{dt} = P - d_2, \quad t_1 \leq t \leq t_2 \tag{7.3}
\]

\[
\frac{dI_3(t)}{dt} = -d_2, \quad t_2 \leq t \leq t_3 \tag{7.4}
\]

\[
\frac{dI_4(t)}{dt} = 0, \quad t_1 \leq t \leq t_3 \tag{7.5}
\]

\[
\frac{dI_5(t)}{dt} = -d_2, \quad t_3 \leq t \leq t_4 \tag{7.6}
\]

The above equations can be solved by using the boundary conditions \( I_{11}(0) = 0, I_{12}(t_1) = W, I_2(t_1) = 0, I_3(t_3) = 0, I_4(t_1) = W \) and \( I_5(t_4) = 0 \), respectively. The solution of the eq.(7.1) - (7.6) are given below:-
\[ I_{11}(t) = (P - d_1)t, \quad 0 \leq t \leq \mu \] (7.7)

\[ I_{12}(t) = (P - d_2)(t - t_1) + W, \quad \mu \leq t \leq t_1 \] (7.8)

\[ I_{2}(t) = (P - d_2)(t - t_1), \quad t_1 \leq t \leq t_2 \] (7.9)

\[ I_{3}(t) = -d_2(t - t_3), \quad t_2 \leq t \leq t_3 \] (7.10)

\[ I_{4}(t) = W, \quad t_1 \leq t \leq t_3 \] (7.11)

\[ I_{5}(t) = -d_2(t - t_4), \quad t_3 \leq t \leq t_4 \] (7.12)

Since \( I_{11}(\mu) = I_{12}(\mu) \) \( \Rightarrow \mu = \frac{W + d_2 t_1}{d_2 - d_1} \) (7.13)

\[ I_{2}(t_2) = I_{1}(t_2) \Rightarrow t_1 = \frac{Pt_2 - d_2 t_3}{P} \] (7.14)

\[ I_{5}(t_3) = W \Rightarrow t_3 = \frac{d_2 t_4 - W}{d_2} \] (7.15)

**Case 1.1.** \( M \leq t_4 \)

Total relevant cost of the system = Set-up cost + Production cost + Holding costs in (OW and RW) + Purchasing cost of raw material - Interest earned + Interest charged

\[ TC_{11} = C_s + C_p P t_2 + h_s \left\{ \frac{(P - d_1)}{2} \mu^2 - \frac{(P - d_2)}{2}(t_1 - \mu)^2 + W(t_1 + t_3) + \frac{d_2}{2}(t_4 - t_3)^2 \right\} \]

\[ + h_r \left\{ \frac{(P - d_2)}{2}(t_2 - t_1)^2 + \frac{d_2}{2}(t_3 - t_2)^2 \right\} + CPt_2 - i_r \left\{ \frac{s d_1 \mu^2}{2} \right\} \]

\[ + s(M - \mu)d_1 \mu + \frac{s d_2}{2}(M^2 - \mu^2) \right\} + i_r C d_2 \left\{ \frac{(t_4^2 - M^2)}{2} - t_3 M \right\} \] (7.16)
Case 1.2. $M \geq t_4$

Total relevant cost of the system = Set-up cost + Production cost + Holding costs in (OW and RW) + Purchasing cost of raw material-Interest earned

$$TC_{12} = C_s + C_pP_{t_2} + h_o \left\{ \frac{(P-d_1)}{2} \mu^2 - \frac{(P-d_2)}{2}(t_1 - \mu)^2 + W(t_1 + t_3) + \frac{d_2}{2}(t_4 - t_3)^2 \right\}$$

$$+ h_r \left\{ \frac{(P-d_2)}{2}(t_2 - t_1)^2 + \frac{d_2}{2}(t_3 - t_2)^2 \right\} + CPt_2 - i_e \left\{ - \frac{sd_1\mu^2}{2} + sd_1\mu M \right\} + \frac{sd_2\left(t_4^2 - \mu^2 \right)}{2} + s_2d_2\left(M - t_4\right)(t_4 - \mu) \right\} \quad (7.17)$$

Case 2. $t_1 \leq \mu \leq t_2$

![Graphical representation of the system in case 2](image)

The system is governed by the following differential equations in case 2:

$$\frac{dI_1(t)}{dt} = P - d_1 \quad , 0 \leq t \leq t_1 \quad (7.18)$$

$$\frac{dI_{21}(t)}{dt} = P - d_1 \quad , t_1 \leq t \leq \mu \quad (7.19)$$

$$\frac{dI_{22}(t)}{dt} = P - d_2 \quad , \mu \leq t \leq t_2 \quad (7.20)$$
\[
\frac{dI_1(t)}{dt} = -d_1, \quad t_1 \leq t \leq t_2 \tag{7.21}
\]

\[
\frac{dI_2(t)}{dt} = -d_2, \quad t_3 \leq t \leq t_4 \tag{7.22}
\]

\(I_4(t)\) is represented by eq. (7.5).

The above equations can be solved by using the boundary conditions \(I_1(0) = 0, I_{21}(t_1) = 0, I_{21}(\mu) = I_{22}(\mu), I_3(t_3) = 0\) and \(I_5(t_4) = 0\), respectively. The solution of the eq.(7.18) - (7.22) are given below:-

\[I_1(t) = (P-d_1) t, \quad 0 \leq t \leq t_1 \tag{7.23}\]

\[I_{21}(t) = (P-d_1)(t-t_1), \quad t_1 \leq t \leq \mu \tag{7.24}\]

\[I_{22}(t) = (P-d_2)t-d_1(\mu-t_1)+d_2\mu, \quad \mu \leq t \leq t_2 \tag{7.25}\]

\[I_3(t) = -d_2(t-t_3), \quad t_2 \leq t \leq t_3 \tag{7.26}\]

\[I_5(t) = -d_2(t-t_4), \quad t_3 \leq t \leq t_4 \tag{7.27}\]

Since \(I_3(t_2) = I_{22}(t_2) \Rightarrow \mu = \frac{d_1 t_4 + d_2 (t_2-t_3)}{d_1-P} \tag{7.28}\)

\[I_5(t_3) = W \Rightarrow t_3 = \frac{d_1 t_4 - W}{d_2} \tag{7.29}\]

\[I_1(t_1) = W \Rightarrow t_1 = \frac{W}{P-d_1} \tag{7.30}\]

**Case 2.1.** \(M \leq t_4\)

Total relevant cost of the system is given by:-


\[ TC_{21} = C_s + C_p P t_2 + \frac{(P-d_1)}{2} t_1^2 + W(t_3-t_1) - \frac{d_2}{2} (t_4-t_3)^2 \]

\[ + h_r \left\{ \frac{(P-d_1)}{2} (t_1-\mu)^2 + \frac{(P-d_2)}{2} (t_2-\mu)^2 + \{d_2 \mu - d_1 (\mu-t_1)\}(t_2-\mu) + \frac{d_2}{2} (t_3-t_2)^2 \right\} \]

\[ + C P t_2 - i_e \left\{ \frac{sd_1 \mu^2}{2} + s(M-\mu) d_1 \mu + \frac{s_d d_2 (M^2-\mu^2)}{2} \right\} + i_c C d_2 \left\{ \frac{(t_4^2-M^2)}{2} - t_4 M \right\} \]

\[ (7.31) \]

**Case 2.2.** \( M \geq t_4 \)

Total relevant cost of the system is given by:-

\[ TC_{22} = C_s + C_p P t_2 + \frac{(P-d_1)}{2} t_1^2 + W(t_3-t_1) - \frac{d_2}{2} (t_4-t_3)^2 \]

\[ + h_r \left\{ \frac{(P-d_1)}{2} (t_1-\mu)^2 + \frac{(P-d_2)}{2} (t_2-\mu)^2 + \{d_2 \mu - d_1 (\mu-t_1)\}(t_2-\mu) + \frac{d_2}{2} (t_3-t_2)^2 \right\} \]

\[ + C P t_2 - i_e \left\{ \frac{sd_1 \mu^2}{2} + sd_1 \mu M + \frac{s_d d_2 (t_4^2-\mu^2)}{2} + s_d d_2 (M-t_4)(t_4-\mu) \right\} \]

\[ (7.32) \]

**Case 3** \( t_2 \leq \mu \leq t_3 \)

The system is governed by the following differential equations in case 3:

\[ \frac{dI_1(t)}{dt} = P - d_1 \quad , \quad 0 \leq t \leq t_1 \]

\[ (7.33) \]

\[ \frac{dI_2(t)}{dt} = P - d_1 \quad , \quad t_1 \leq t \leq t_2 \]

\[ (7.34) \]
\[
\frac{dI_{31}(t)}{dt} = -d_1, \quad t_2 \leq t \leq \mu \quad (7.35)
\]

\[
\frac{dI_{32}(t)}{dt} = -d_2, \quad \mu \leq t \leq t_3 \quad (7.36)
\]

\[
\frac{dI_3(t)}{dt} = -d_2, \quad t_3 \leq t \leq t_4 \quad (7.37)
\]

\(I_4(t)\) is represented by eq. (7.5).

The above equations can be solved by using the boundary conditions \(I_1(0) = 0, I_2(t_1) = 0, I_{31}(t_2) = I_2(t_2), I_{32}(t_3) = 0\) and \(I_3(t_4) = 0\), respectively. The solution of the eq.(7.33) - (7.37) are given below:

\[
I_1(t) = (P - d_1)t, \quad 0 \leq t \leq t_1 \quad (7.38)
\]

\[
I_2(t) = (P - d_1)(t - t_1), \quad t_1 \leq t \leq t_2 \quad (7.39)
\]

\[
I_{31}(t) = -d_1t - (P - d_1)t_1, \quad t_2 \leq t \leq \mu \quad (7.40)
\]

\[
I_{32}(t) = -d_2(t - t_3), \quad \mu \leq t \leq t_3 \quad (7.41)
\]

\[
I_3(t) = -d_2(t - t_4), \quad t_3 \leq t \leq t_4 \quad (7.42)
\]

Also, \(I_1(t_1) = W \Rightarrow t_1 = \frac{W}{P - d_1} \quad (7.43)\)

\[
I_{31}(\mu) = I_{32}(\mu) \Rightarrow \mu = \frac{(P - d_1)t_1 - d_2t_3}{d_2 - d_1} \quad (7.44)
\]

\[
I_3(t_3) = W \Rightarrow t_3 = \frac{d_2t_4 - W}{d_2} \quad (7.45)
\]
Case 3.1. $M \leq t_4$

Total relevant cost of the system is given by:-

$$TC_{31} = C_s + C_p P_{t_2} + h_r \left\{ \left( \frac{P-d_1}{2} \right) t_1^2 + W (t_3 - t_1) + \frac{d_2}{2} (t_4 - t_3)^2 \right\}$$

$$+ h_r \left\{ \left( \frac{P-d_1}{2} \right) (t_2 - t_1)^2 - \frac{d_4}{2} (\mu - t_2)^2 - \left( P - d_1 \right) t_1 (\mu - t_2) + \frac{d_2}{2} (t_3 - \mu)^2 \right\}$$

$$+ C P_{t_2} - i_c \left\{ \frac{s_d \mu^2}{2} + s (M - \mu) d_i \mu + \frac{s_d^2 (M^2 - \mu^2)}{2} \right\} + i_c C d_2 \left\{ \frac{(t_4^2 - M^2)}{2} - t_4 M \right\}.$$

(7.46)

Case 3.2. $M \geq t_4$

Total relevant cost of the system is given by:-

$$TC_{32} = C_s + C_p P_{t_2} + h_r \left\{ \left( \frac{P-d_1}{2} \right) t_1^2 + W (t_3 - t_1) + \frac{d_2}{2} (t_4 - t_3)^2 \right\}$$

$$+ h_r \left\{ \left( \frac{P-d_1}{2} \right) (t_2 - t_1)^2 - \frac{d_4}{2} (\mu - t_2)^2 - \left( P - d_1 \right) t_1 (\mu - t_2) + \frac{d_2}{2} (t_3 - \mu)^2 \right\}$$

$$+ C P_{t_2} - i_c \left\{ \frac{s_d \mu^2}{2} + s_d \mu M + \frac{s_d^2 (M^2 - \mu^2)}{2} + s_d (M - t_4) (t_4 - \mu) \right\}.$$  

(7.47)

Case 4. $t_3 \leq \mu \leq t_4$

![Graphical representation of the system in case 4](image)

Fig. 7.4 Graphical representation of the system in case 4
The system is governed by the following differential equations in case 4:

\[
\frac{dI_1(t)}{dt} = P - d_1, \quad 0 \leq t \leq t_1 \tag{7.48}
\]

\[
\frac{dI_2(t)}{dt} = P - d_1, \quad t_1 \leq t \leq t_2 \tag{7.49}
\]

\[
\frac{dI_3(t)}{dt} = -d_1, \quad t_2 \leq t \leq t_3 \tag{7.50}
\]

\[
\frac{dI_{s1}(t)}{dt} = -d_1, \quad t_3 \leq t \leq \mu \tag{7.51}
\]

\[
\frac{dI_{s2}(t)}{dt} = -d_2, \quad \mu \leq t \leq t_4 \tag{7.52}
\]

The above equations can be solved by using the boundary conditions \( I_1(0) = 0, \ I_2(t_1) = 0, \ I_3(t_2) = 0, \ I_{s1}(t_3) = W \) and \( I_{s2}(t_4) = 0, \) respectively. The solution of the eq.\((7.48) - (7.52)\) are given below:-

\[
I_1(t) = (P - d_1)t, \quad 0 \leq t \leq t_1 \tag{7.53}
\]

\[
I_2(t) = (P - d_1)(t - t_1), \quad t_1 \leq t \leq t_2 \tag{7.54}
\]

\[
I_3(t) = -d_1(t - t_3), \quad t_2 \leq t \leq t_3 \tag{7.55}
\]

\[
I_{s1}(t) = -d_1(t - t_3) + W, \quad t_3 \leq t \leq \mu \tag{7.56}
\]

\[
I_{s2}(t) = -d_2(t - t_4), \quad \mu \leq t \leq t_4 \tag{7.57}
\]

Also, \( I_1(t_1) = W \Rightarrow t_1 = \frac{W}{P - d_1} \tag{7.58} \)

\[
I_2(t_2) = I_3(t_2) \Rightarrow t_3 = \frac{1}{d_1}\left\{Pt_2 - (P - d_1)t_1\right\} \tag{7.59}
\]
\[ I_{33}(\mu) = I_{22}(\mu) \Rightarrow \mu = -\frac{d_1 t_3 + d_2 t_4 - W}{d_2 - d_1} \]  

(7.60)

**Case 4.1.** \( M \leq t_4 \)

Total relevant cost of the system is given by:

\[ TC_{4i} = C_s + C_p P_t + h_o \left\{ \frac{(P-d_1)}{2} t_1^2 + W(t_3-t_1) - \frac{d_1}{2} (\mu-t_3)^2 + W(\mu-t_3) - \frac{d_2}{2}(t_4-\mu)^2 \right\} \]  

(7.61)

**Case 4.2.** \( M \geq t_4 \)

Total relevant cost of the system is given by:

\[
TC_{42} = C_s + C_p P_t + h_o \left\{ \frac{(P-d_1)}{2} t_1^2 + W(t_3-t_1) - \frac{d_1}{2} (\mu-t_3)^2 + W(\mu-t_3) - \frac{d_2}{2}(t_4-\mu)^2 \right\} \\
+ h_r \left\{ \frac{(P-d_1)}{2} (t_2-t_1)^2 - \frac{d_1}{2} (t_3-t_2)^2 \right\} + C P_t \left\{ -\frac{sd_4 \mu^2}{2} + sd_4 \mu M + \frac{s_2 d_2 (t_4^2 - \mu^2)}{2} + s_2 d_2 (M-t_4)(t_4-\mu) \right\}
\]  

(7.62)

The problem is to minimize \( TC_i (t_2, r) \).

\[ TC_i (t_2, r) = \min \left\{ \begin{array}{ll}
TC_{41}, & M \leq t_4 \\
TC_{42}, & M \geq t_4
\end{array} \right. \]

\( i = 1, 2, 3, 4 \)

**7.4 Numerical Example**

The data which is taken in this chapter is based on [41]. The values of the parameters are considered in appropriate units as follows: \( C_s = 120, d_1 = 10, a = 0.4, b = 1, h_o = 1, h_r = 1.3, i_c = 0.10, i_c = 0.11, C = 25, s = 220, W = 45, t_4 = 10. \) Optimal solutions exist when \( M \geq t_4 \), the results are mentioned below (when \( M = 13 \)) with the path of convergence in each case.

**Case 1:** \( 0 \leq \mu \leq t_4 \).

\[
r = 0.437, P = 15.243, t_2 = 0.16 \text{ and } TC_1 = 85342.54
\]
Case 2: \( t_1 \leq \mu \leq t_2 \).

\[ r = 0.375, \quad P = 15.527 \quad t_2 = 0.4 \quad \text{and} \quad TC_2 = 974.29 \]

Case 3: \( t_2 \leq \mu \leq t_3 \).

\[ r = 0.586, \quad P = 25.34, \quad t_2 = 0.43 \quad \text{and} \quad TC_3 = 1028.44 \]

Case 4: \( t_3 \leq \mu \leq t_4 \).

\[ r = 0.489, \quad P = 16.2, \quad t_2 = 0.68 \quad \text{and} \quad TC_4 = 863.28 \]

7.5 Conclusion

We found that producer’s cost minimization strategy while facing the special sale/reduction offer on the selling price of items and a trade-credit period is given by the supplier to the producer and the following conclusion are found.

(i) We have conclude that during the delay period producer can accumulate revenue and earn interest by selling the manufactured products and no interest is payable during this delay period. It’s a kind of price discount which is offered by the supplier to attract the producer and also this facility is beneficial for the producer as he/she does not have to pay the purchasing cost of raw material as they are received.

(ii) In this model, we have found that effect of obsolete or out of season products trade on demand and particularly on the retailer’s reaction to speed-up sales, so as to alleviate the effect of the larger order’s loss results into the price cut of the item.

(iii) Normally customers are encouraged to purchase more at more reduced prices, therefore, the demand rate during a special offer/reduction can be taken as increasing function of the reduction rate (r). This function is found to be suitable as per a prior survey of the market.

(iv) Demand function which is formulated so that all possible cases of the cost function are minimized by trading off; the inventory costs of the OW and the RW, the production cost, the setup cost and the different selling prices. The formulation of this demand function \( (d_2) \) with a limited capacity of OW under the facility of allowable delay in payment is novel and very much realistic in the field of inventory control.