Chapter 6  
Inventory System For Weibull Distribution Decaying Items, Multi - Variate Demand With Partial Backlogging and Trade Credits in Fuzzy Environment

6.1 Introduction

Many realistic experiences disclose that some but not all customers will wait for backlogged items during a shortage period, such as for fashionable supplies and the products with short life cycle. The longer the waiting time is, the lesser the backlogging rate would be. According to such phenomenon, backlogging rate should not be disregarded.

In this chapter, we have attempted to develop an inventory model for decaying items with a very realistic and practical demand rate, which depends upon both the stock level available and the selling price of the item. The factor of stock level takes care of the fact that the demand is influenced by the level of stock displayed and changes with respect to it. The selling price factor accounts for the fact that an increase in the selling price of the commodity discourages a repeat demand. The effect of permissible delay in payments in this study has been developed. Partially backlogged shortages with time dependent backlogging rate are permitted in this study. Both crisp and fuzzy model are discussed in this study. A numerical example is provided to illustrate the problem. Sensitivity analysis of the optimal solution with respect to the changes in the value of system parameters is also discussed.

6.2 Assumptions and Notations

The basic assumptions of the chapter are as follows:

(i)  The demand rate is a function of stock and selling price considered as \( f(t) = (a+bQ(t)− p) \), where \( a > 0, \ 0 < b < 1, \ a > b \) and \( p \) is selling price.

(ii)  Holding cost \( h(t) \) per item per time-unit is time dependent and is assumed to be \( h(t) = h + \delta t \) where \( \delta > 0, \ h > 0 \).

(iii)  Shortages are allowed and partially backlogged and backlogging rate is assumed to be \( 1/(1+\eta t) \) which is a decreasing function of time.

(iv)  The deterioration rate is time dependent.

(v)  \( T \) is the length of the cycle.
(vi) Replenishment is instantaneous and lead time is zero.
(vii) The order quantity in one cycle is \( Q \).
(viii) The selling price per unit item is \( p \).
(ix) \( A \) is the cost of placing an order.
(x) \( C_1 \) is the purchasing cost per unit per unit.
(xi) \( c_2 \) the backorder cost per unit per unit time.
(xii) \( c_3 \) the opportunity cost (i.e., goodwill cost) per unit.
(xiii) \( P(T, p) \) the total profit per unit time.
(xiv) The deterioration of units follows the two parameter Weibull distribution (say) \( \theta(t) = \alpha \beta t^{\beta - 1} \) where \( 0 < \alpha < 1 \) is the scale parameter and \( \beta > 1 \) is the shape parameter.
(xv) During time \( t_1 \), inventory is depleted due to deterioration and demand of the item. At time \( t_1 \), the inventory becomes zero and shortages start going on.
(xvi) \( I_e \) = Interest earned per unit time.
(xvii) \( I_r \) = Interest payable per unit time.
(xviii) \( a, A, c_1, c_2, c_3, h \) and \( \delta \) as fuzzy numbers.

6.3 Mathematical Formulation and Solution

We have discussed two models: first is crisp and second is fuzzy model.

Model I: Crisp model

Let \( Q(t) \) be the inventory level at time \( t \) \( (0 \leq t \leq T) \). During the time interval \([0, t_1]\), inventory level decreases due to the combined effect of demand and deterioration and at \( t_1 \), inventory level depletes up to zero. Shortages are allowed with partial backlogging during \([t_1, T]\), from Figure 6.1.
The differential equation to describe immediate state over \([0, t_1]\) is given by:
\[
Q(t) + \alpha \beta t^{\beta-1} Q(t) = -(a + bQ(t) - p), \quad 0 \leq t \leq t_1
\] (6.1)

Again, during time interval \([t_1, T]\) shortages start occurring and at \(T\) there are maximum shortages, due to partial backordering some sales are lost. The differential equation to describe instant state over \([t_1, T]\) is given by:
\[
Q'(t) = -\frac{(a - p)}{[1 + \eta(T - t)]}, \quad t_1 \leq t \leq T
\] (6.2)

with condition \(Q(t_1) = 0\)

Solving equation (6.1) and equation (6.2) and neglecting higher powers of \(\alpha\), we get
\[
Q(t) = (a - p) \left[ t_1 - t + \frac{b}{2} \left( t_1^2 - t^2 \right) + \frac{\alpha}{(\beta + 1)} \left( t_1^{\beta+1} - t^{\beta+1} \right) \right] e^{-bt-m^\beta}, \quad 0 \leq t \leq t_1
\] (6.3)

and
\[
Q(t) = \frac{(a - p)}{\eta} \left[ \log \{1 + \eta(T - t)\} - \log \{1 + \eta(T - t_1)\} \right], \quad t_1 \leq t \leq T
\] (6.4)

At time 0 inventory level is \(Q(0)\) and is given by:
\[
Q(0) = (a - p) \left( t_1 + \frac{bt_1}{2} + \frac{\alpha t_1^{\beta+1}}{\beta + 1} \right)
\]

At time \(T\) maximum shortages (\(Q_1\)) occurs and is given by:
\[
Q_1 = \frac{(a - p)}{\eta} \left[ \log \{1 + \eta(T - t_1)\} \right]
\]

The order quantity is \(Q\) and is given by:
\[ Q = (a - p) \left( t_i + \frac{bt_i}{2} + \frac{\alpha t_i^{\beta+1}}{\beta+1} + \frac{1}{\eta} \log(1 + \eta(T - t_i)) \right) \]

The purchasing cost is:

\[ PC = c_1(a - p) \left( t_i + \frac{bt_i}{2} + \frac{\alpha t_i^{\beta+1}}{\beta+1} + \frac{1}{\eta} \log(1 + \eta(T - t_i)) \right) \] (6.5)

Ordering cost is:

\[ OC = A \] (6.6)

Holding cost is:

\[ IHC = \int_0^T (h + \delta t)Q(t)dt \]

\[ = \int_0^T (h + \delta t)(a - p) \left[ t_i - t + \frac{b}{2}(t_i^2 - t^2) + \frac{\alpha}{(\beta+1)}(t_i^{\beta+1} - t^{\beta+1}) \right] e^{-bt+ar^\delta} dt \]

\[ = (a - p) \left[ \frac{ht_i^2}{2} + \frac{\delta t_i^3}{6} + \frac{bht_i^4}{6} + \frac{b^2\delta t_i^4}{8} - \frac{b^2ht_i^5}{15} + \frac{\alpha h t_i^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha h t_i^{\beta+2}}{(\beta+2)(\beta+3)} \right] \]

\[ + \frac{3\alpha\delta t_i^{\beta+3}}{2(\beta+3)} - \frac{\alpha bh t_i^{\beta+3}}{2(\beta+1)} - \frac{\alpha\delta t_i^{\beta+4}}{\beta+2} + \frac{\alpha\delta bh t_i^{\beta+4}}{6(\beta+4)} - \frac{\alpha\delta t_i^{\beta+4}}{2(\beta+2)} - \frac{\alpha^2\delta t_i^{2\beta+3}}{2(\beta+1)^2} \]

\[ - \frac{\alpha^2\delta t_i^{2\beta+3}}{(\beta+2)(2\beta+3)} \] (6.7)

Shortage cost due to backordered is:

\[ BC = c_2 \int_{t_i}^T [-Q(t)]dt \]

\[ = \frac{c_2(a - p)}{\eta} \left[ \eta(T - t_i) - \log \{1 + \eta(T - t_i)\} \right] \] (6.8)

Lost sales cost due to lost sales is:

\[ LS = c_3(a - p) \int_{t_i}^T \left[ 1 - \frac{1}{1 + \eta(T - t)} \right] dt \]

\[ = \frac{c_3(a - p)}{\eta} \left[ \eta(T - t_i) - \log \{1 + \eta(T - t_i)\} \right] \] (6.9)

Sales revenue is given by:

\[ SR = p \int_0^T (a + bQ(t) - p)dt + p \int_{t_i}^T (a - p)dt = p(a - p)T + b(a - p)p \left( \frac{t_i^2}{2} + \frac{bt_i^3}{3} + \frac{\alpha t_i^{\beta+2}}{\beta+2} \right) \] (6.10)
Now regarding the permissible delay period $M$ for settling the accounts, there arise two cases

$M \leq t_1$ or $M > t_1$.

**Case I: $M \leq t_1$ (i.e. when the permissible delay period is less than the inventory period)**

In this case, since the credit period ($M$) is smaller than the length of period with positive inventory stock of the items, therefore the buyer can use the sale revenue to earn the interest with the rate $I_e$ per unit time. The interest earned ($I_E$) is given by:

$$I_E = C_1 I_c \int_0^{t_1} (a - p) f(t) dt = C_1 I_c \left\{ \frac{(a - p) t_1^2}{2} + b(a - p) \frac{(1 - b) t_1^3}{3} + \frac{5b}{24} + \frac{\alpha t_1^{\beta+3}}{\beta + 1} \frac{t_1^2}{2} - \frac{t_1^{\beta+3}}{(\beta + 2)(\beta + 3)} \right\}$$

$$+ \frac{2\alpha t_1^{\beta+3}}{\beta + 2} - \frac{2\alpha(\beta + 2)t_1^{\beta+3}}{(\beta + 1)(\beta + 3)} \right\}$$

(6.11)

And the interest payable ($I_p$) is given by:

$$I_p = C_1 I_c \int_M^{t_1} I(t) dt = C_1 I_c (a - p) \left\{ t_1^2 M + t_1 M^2 - \frac{M^3}{3} - \frac{5t_1^3}{3} + \frac{b}{2} \left\{ \frac{5t_1^4}{12} - t_1^3 M + \frac{M^3 t_1}{3} + \frac{M^2 t_1^2}{3} - \frac{M^4}{4} \right\} \right\}$$

$$+ \frac{\alpha}{\beta + 1} \frac{t_1^{\beta+3}}{(\beta + 2)(\beta + 3)} - M t_1^{\beta+2} + t_1 M^{\beta+2} - \frac{t_1^{2\beta+3}}{2} + \frac{t_1^{\beta+1} M^{\beta+2}}{2} - \frac{M^{\beta+3}}{(\beta + 3)}$$

$$+ b M t_1^2 - b t_1^3 M^2 - \frac{b t_1^5}{3} + \frac{b M^4}{3} - \frac{\alpha t_1^{\beta+3}}{\beta + 1} + \frac{\alpha t_1 M^{\beta+1}}{\beta + 1}$$

(6.12)

From (6.6), (6.7), (6.8), (6.9), (6.10), (6.11) and (6.12), total profit per unit time is given by:

$$P(T, p) = \frac{1}{T} \left( SR - OC - PC - IHC - BC - LS - I_p + I_E \right)$$

$$= - (a - p) \left\{ \frac{ht_1^2}{2} + \frac{\delta t_1^3}{6} + \frac{bht_1^3}{24} - \frac{b^2ht_1^4}{8} + \frac{b^3t_1^5}{15} + \frac{\alpha t_1^{\beta+2}}{\beta + 2} - \frac{\alpha t_1^{\beta+2}}{(\beta + 1)(\beta + 2)} \right\}$$

$$+ \frac{3\alpha \delta t_1^{\beta+3}}{2(\beta + 3)} - \frac{\alpha bht_1^{\beta+3}}{(\beta + 1)} + \frac{\alpha \delta t_1^{\beta+4}}{6(\beta + 4)} - \frac{\alpha bht_1^{\beta+4}}{2(\beta + 2)}$$

$$- \frac{\alpha^2 t_1^{2\beta+2}}{(\beta + 1)^2} - \frac{\alpha^2 \delta t_1^{2\beta+3}}{2(\beta + 2)(\beta + 3)}$$

$$- C_1 I_c (a - p) \left\{ t_1^2 M + t_1 M^2 - \frac{M^3}{3} - \frac{5t_1^3}{3} + \frac{b}{2} \left\{ \frac{5t_1^4}{12} - t_1^3 M + \frac{M^3 t_1}{3} + \frac{M^2 t_1^2}{3} - \frac{M^4}{4} \right\} \right\}$$

$$+ \frac{\alpha}{\beta + 1} \frac{t_1^{\beta+3}}{(\beta + 2)(\beta + 3)} - M t_1^{\beta+2} + t_1 M^{\beta+2} - \frac{t_1^{2\beta+3}}{2} + \frac{t_1^{\beta+1} M^{\beta+2}}{2} - \frac{M^{\beta+3}}{(\beta + 3)}$$

5
\[+bMt_i^2 - bt_iM^2 - \frac{bt_i^3}{3} + \frac{bM^3}{3} - \frac{\alpha t_i^{\beta+3}}{\beta+1} + \frac{\alpha t_i M^{\beta+1}}{\beta+1}\]

\[+C/I\left[\frac{(a - p)t_i}{2} + b(a - p)\left(\frac{1-b}{3}t_i^3\right) + \frac{5b}{24} + \frac{\alpha}{\beta+1}\left(\frac{t_i^{\beta+3}}{2} - \frac{t_i^{\beta+3}}{(\beta+2)(\beta+3)}\right)\right.\]

\[+\frac{2\alpha t_i^{\beta+3}}{\beta+2} - \frac{2\alpha(\beta+2)t_i^{\beta+3}}{(\beta+1)(\beta+3)}\]}\]

(6.13)

Our objective is to maximize the profit function \(P(T, p)\). The necessary conditions for maximizing the profit are:

\[\frac{\partial P(T, p)}{\partial T} = 0 \] (6.14)

\[\frac{\partial P(T, p)}{\partial p} = 0 \] (6.15)

Using the software Mathematica-8.1, from equation (6.14) and (6.15) we can determine the optimum values of \(T^*\) and \(p^*\) simultaneously and the optimal value \(P^*(T, p)\) of the average net profit is determined by (6.13), provided they satisfy the sufficiency conditions for maximizing \(P^*(T, p)\) are

\[\frac{\partial^2 P(T, p)}{\partial T^2} > 0, \quad \frac{\partial^2 P(T, p)}{\partial p^2} > 0 \quad \text{and} \quad \frac{\partial^2 P(T, p)}{\partial T^2} \cdot \frac{\partial^2 P(T, p)}{\partial p^2} - \left(\frac{\partial^2 P(T, p)}{\partial T \partial p}\right)^2 > 0\]

Case II: \(M > t_1\) (i.e. when the permissible delay period is greater than the inventory period)

In this case, the customer sells \(D t_1\) units in total by the end of the replenishment cycle time \(t_1\) and has \(CDt_1\) to pay the supplier in full by the end of the credit period ‘M’. Thus there is no interest payable. The interest earned per year is given by:

\[I_e = \frac{C/I}{T}\left[\int_0^{t_1} (t_i - t) D(t)\, dt + \int_0^{t_1} (M - t_i) D(t)\, dt\right]\]
\[ P(T, p) = \frac{1}{T} \left( SR - OC - PC - IHC - BC - LS + I_e \right) \]

\[ -(a - p) \left( \frac{ht^2}{2} + \frac{b t^3}{6} + \frac{b h t^3}{24} + \frac{b^2 h t^4}{48} - \frac{a h t^2}{\beta + 2} \right) \]

\[ + \frac{a^2 h t^2}{(\beta + 1)(\beta + 2)} \]

\[ + C_{1} I_{1} \left( \frac{(a - p) t^2}{2} + b(a - p) \left( \frac{(1-b)t^3}{3} + \frac{5b}{24} + \frac{t_{1}^{\beta+3}}{2} - \frac{t_{1}^{\beta+3}}{(\beta+2)(\beta+3)} \right) \right) \]

\[ + \frac{2a t_{1}^{\beta+3}}{\beta + 2} \left( \frac{(a - p) t_{1} + b(a - p)}{(\beta + 1)(\beta + 3)} \right) \]

\[ + \frac{a t_{1}^{\beta+2}}{6} \left( \frac{t_{3}^3}{(\beta + 1)(\beta + 2)} \right) \] (6.17)

Our objective is to maximize the profit function \( P(T, p) \). The necessary conditions for maximizing the profit are:

\[ \frac{\partial P(T, p)}{\partial T} = 0 \] (6.18)

\[ \frac{\partial P(T, p)}{\partial p} = 0 \] (6.19)

Using the software Mathematica-8.1, from equation (6.18) and (6.19) we can determine the optimum values of \( T^* \) and \( p^* \) simultaneously and the optimal value \( P^*(T, p) \) of the average net profit is determined by (6.17) provided they satisfy the sufficiency conditions for maximizing \( P^*(T, p) \) are
\[
\frac{\partial^2 P(T, p)}{\partial T^2} > 0, \frac{\partial^2 P(T, p)}{\partial p^2} > 0 \text{ and } \frac{\partial^2 P(T, p)}{\partial T^2} \cdot \frac{\partial^2 P(T, p)}{\partial p^2} - \left( \frac{\partial^2 P(T, p)}{\partial T \partial p} \right)^2 > 0
\]

6.4 Numerical Example

To illustrate the model numerically the following parameter values are considered. Let \( A = 230, a = 200, b = 0.05, c_1 = 22, c_2 = 4, c_3 = 18, \alpha = 0.45, \beta = 3, \gamma = 0.05, \eta = 0.3, \delta = 0.3, I_r = 0.4, I_e = 0.14, t_1^* = 1. \) Based on these input data, the findings are as follows: \( T^* = 2.6821 \) and Profit \( P^*(T, p) = 657.36 \)

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Fig. 6.2 Variation in profit w.r.t. a for case \( M \leq t_1 \)

Fig. 6.3 Variation in profit w.r.t. b for case \( M \leq t_1 \)
Fig. 6.4 Variation in profit w.r.t. $\eta$ for case $M \leq t_1$

Fig. 6.5 Variation in profit w.r.t. $I_e$ for case $M \leq t_1$
Fig. 6.6 Variation in profit w.r.t. M for case $M \leq t_1$

Table 6.2 Sensitivity analysis of optimal solution for case $M > t_1$

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Fig. 6.7 Variation in profit w.r.t. 'a' for case $M > t_1$

Fig. 6.8 Variation in profit w.r.t. 'b' for case $M > t_1$
Fig. 6.9 Variation in profit w.r.t. $\eta$ for case $M > t_1$

Fig. 6.10 Variation in profit w.r.t. $I_e$ for case $M > t_1$
6.5 Result Analysis (Crisp model)

Some important inferences drawn from the table of both cases are as follows:

(i) From Table 6.1, Table 6.2, Figure 6.2, 6.3, 6.7 and Figure 6.8, it is observed that the total profit is increases with the increment of the demand parameters ‘a’ and ‘b’.

(ii) From Table 6.1, Table 6.2, Figure 6.4 and Figure 6.9, it is observed that the backlogging parameter gives the positive effect on total profit.

(iii) From Table 6.1, Table 6.2, Figure 6.5 and Figure 6.10, it is observed that the parameter of interest earned is directly proportional to total profit.

(iv) From Table 6.1, Table 6.2, Figure 6.6 and Figure 6.11, it is observed that the permissible delay period is slightly increases to the total profit.

Model II: Fuzzy Mathematical Model

We have also developed this chapter with the help of fuzzy theory. In this study we consider a, A, c₁, c₂, c₃, h and δ as fuzzy numbers i.e, as $\hat{a}, \hat{A}, \hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{h} and \hat{\delta}$. Then $P' (T, P)$ is regarded as the estimate of total profit per unit time in the fuzzy sense.

$$\hat{a} = (a - \Delta_1, a, a + \Delta_2), \text{ where } 0 < \Delta_1 < a \text{ and } \Delta_1, \Delta_2 > 0$$
\( \hat{A} = (A - \Delta_3, A, A + \Delta_4) \), where \( 0 < \Delta_3 < A \) and \( \Delta_3, \Delta_4 > 0 \)

\( \hat{c}_1 = (c_1 - \Delta_5, c_1, c_1 + \Delta_6) \), where \( 0 < \Delta_5 < c_1 \) and \( \Delta_5, \Delta_6 > 0 \)

\( \hat{c}_2 = (c_2 - \Delta_7, c_2, c_2 + \Delta_8) \), where \( 0 < \Delta_7 < c_2 \) and \( \Delta_7, \Delta_8 > 0 \)

\( \hat{c}_3 = (c_3 - \Delta_9, c_3, c_3 + \Delta_{10}) \), where \( 0 < \Delta_9 < c_3 \) and \( \Delta_9, \Delta_{10} > 0 \)

\( \hat{h} = (h - \Delta_{11}, h, h + \Delta_{12}) \), where \( 0 < \Delta_{11} < h \) and \( \Delta_{11}, \Delta_{12} > 0 \)

\( \hat{\delta} = (\delta - \Delta_{13}, \delta, \delta + \Delta_{14}) \), where \( 0 < \Delta_{13} < \delta \) and \( \Delta_{13}, \Delta_{14} > 0 \)

And the signed distance of \( \hat{a} \) to \( \hat{0} \) is given by the relation:

\[ d(\hat{a}, \hat{0}) = a + \frac{1}{4}(\Delta_2 - \Delta_1), \text{ where } d(\hat{a}, \hat{0}) > 0 \text{ and } [a - \Delta_1, a + \Delta_2] \]

Similarly, the signed distance of other parameters to \( \hat{0} \) is given by the relations:-

\[ d(\hat{A}, \hat{0}) = A + \frac{1}{4}(\Delta_4 - \Delta_3), \text{ where } d(\hat{A}, \hat{0}) > 0 \text{ and } [A - \Delta_3, A + \Delta_4] \]

\[ d(\hat{c}_1, \hat{0}) = c_1 + \frac{1}{4}(\Delta_6 - \Delta_5), \text{ where } d(\hat{c}_1, \hat{0}) > 0 \text{ and } [c_1 - \Delta_5, c_1 + \Delta_6] \]

\[ d(\hat{c}_2, \hat{0}) = c_2 + \frac{1}{4}(\Delta_8 - \Delta_7), \text{ where } d(\hat{c}_2, \hat{0}) > 0 \text{ and } [c_2 - \Delta_7, c_2 + \Delta_8] \]

\[ d(\hat{c}_3, \hat{0}) = c_3 + \frac{1}{4}(\Delta_{10} - \Delta_9), \text{ where } d(\hat{c}_3, \hat{0}) > 0 \text{ and } [c_3 - \Delta_9, c_3 + \Delta_{10}] \]

\[ d(\hat{h}, \hat{0}) = h + \frac{1}{4}(\Delta_{12} - \Delta_{11}), \text{ where } d(\hat{h}, \hat{0}) > 0 \text{ and } [h - \Delta_{11}, h + \Delta_{12}] \]

\[ d(\hat{\delta}, \hat{0}) = \delta + \frac{1}{4}(\Delta_{14} - \Delta_{13}), \text{ where } d(\hat{\delta}, \hat{0}) > 0 \text{ and } [\delta - \Delta_{13}, \delta + \Delta_{14}] \]

**Case I: \( M \leq t_1 \)**

Now, by the fuzzy triangular rule, fuzzy total profit per unit is \( FP(\hat{a}, \hat{A}, \hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{h}, \hat{\delta}) = (F_1, F_2, F_3) \)

And \( F_1, F_2, F_3 \) are obtained as:-
\[
F_1 = \frac{1}{T} \left[ \frac{p(a - \Delta_1 - p)T + b(a - \Delta_1 - p)p \left( \frac{\gamma^2 T^2}{2} + \frac{b\gamma^3 T^3}{3} + \frac{\alpha \gamma^{\beta + 2} T^{\beta + 2}}{\beta + 2} \right)}{(A + \Delta_4)} - (A + \Delta_4) \right]
\]

\[
-\left( c_1 + \Delta_6 \right)(a - \Delta_1 - p)(\gamma T + \frac{b\gamma^7 T^2}{2} + \frac{\alpha \gamma^{\beta + 1} T^{\beta + 1}}{\beta + 1} + \frac{1}{\eta} \log \left( 1 + \eta(T - \gamma T) \right) - \frac{(c_2 + \Delta_6)(a - \Delta_1 - p)}{\eta^2} \frac{\eta(T - \gamma T) - \log (1 + \eta(T - \gamma T))}{\eta}
\]

\[
\{ \eta(T - \gamma T) - \log (1 + \eta(T - \gamma T)) \} - \frac{c_1(a - \Delta_1 - p)}{\eta} \{ \eta(T - \gamma T) - \log (1 + \eta(T - \gamma T)) \}
\]

\[
-\left( a + \Delta_2 - p \right) \left( \frac{(h - \Delta_1)\gamma^2 T^2}{2} + \frac{(\delta - \Delta_1)\gamma^3 T^3}{6} + \frac{b(h - \Delta_1)\gamma^5 T^5}{24} - \frac{b^2(h - \Delta_1)\gamma^5 T^5}{8} \right)
\]

\[
- \frac{b^2(\delta - \Delta_1)\gamma^5 T^5}{15} + \frac{\alpha(h - \Delta_1)\gamma^{\beta + 2} T^{\beta + 2}}{\beta + 2} - \frac{\alpha(h - \Delta_1)\gamma^{\beta + 2} T^{\beta + 2}}{(\beta + 1)(\beta + 2)} + \frac{3\alpha(\delta - \Delta_1)\gamma^{\beta + 3} T^{\beta + 3}}{2(\beta + 3)}
\]

\[
- \frac{ab(h - \Delta_1)\gamma^{2 \beta + 3} T^{2 \beta + 3}}{2(\beta + 1)} - \frac{\alpha^2(\delta - \Delta_1)\gamma^{2 \beta + 3} T^{2 \beta + 3}}{(\beta + 2)(2 \beta + 3)}
\]

\[
- \frac{\alpha^2(\delta - \Delta_1)\gamma^{2 \beta + 3} T^{2 \beta + 3}}{(\beta + 2)(2 \beta + 3)} - \frac{\alpha^2(\delta - \Delta_1)\gamma^{2 \beta + 3} T^{2 \beta + 3}}{(\beta + 2)(2 \beta + 3)}
\]

\[
-C_i, (a - p)[t_i M + t_i^2 M^2 - \frac{M^3}{3} + \frac{5t_i^3}{3} + b^2 \left\{ \frac{5t_i^3}{12} - t_i^3 M + \frac{M^3 t_i}{3} + \frac{M^2 t_i^2}{3} - \frac{M^4}{4} \right\}]
\]

\[
+ \frac{\alpha}{\beta + 1} \left( t_i^{\beta + 3} - \frac{t_i^{\beta + 3}}{(\beta + 2)(\beta + 3)} - M t_i^{\beta + 2} + t_i M^{\beta + 2} + \frac{t_i^{2 \beta + 3}}{2} + t_i^{\beta + 1} M^{\beta + 2} + \frac{M^{\beta + 3}}{2} - \frac{M^{\beta + 3}}{(\beta + 3)} \right)
\]

\[
+ bM t_i^{\beta + 2} - \frac{b t_i^3}{3} + \frac{bM^2}{3} - \frac{\alpha t_i^{\beta + 3}}{\beta + 1} - \frac{\alpha t_i M^{\beta + 1}}{\beta + 1}
\]

\[
+ C_i, \left\{ \frac{(a - p) t_i^2}{2} + b(a - p) \left\{ \frac{(1 - b) t_i^3}{3} + \frac{5b}{24} + \frac{\alpha}{\beta + 1} \left( \frac{t_i^{\beta + 3}}{2} - \frac{t_i^{\beta + 3}}{(\beta + 2)(\beta + 3)} \right) \right\} \right\}
\]

\[
+ \frac{2\alpha t_i^{\beta + 3}}{\beta + 2} - \frac{2\alpha(\beta + 2)t_i^{\beta + 3}}{(\beta + 1)(\beta + 3)} \right\}
\]

\[
F_2 = \frac{1}{T} \left[ p(a - p)T + b(a - p)p \left( \frac{t_i^2}{2} + \frac{b t_i^3}{3} + \frac{\alpha t_i^{\beta + 2}}{\beta + 2} \right) - A \right]
\]

\[
- c_1(a - p) \left( t_i + \frac{b t_i^2}{2} + \frac{\alpha t_i^{\beta + 1}}{\beta + 1} + \frac{1}{\eta} \log \left( 1 + \eta(T - t_i) \right) \right)
\]

\[
- \frac{c_i(a - p)}{\eta^2} \left\{ \eta(T - t_i) - \log (1 + \eta(T - t_i)) \right\} - \frac{c_1(a - p)}{\eta} \left\{ \eta(T - t_i) - \log (1 + \eta(T - t_i)) \right\}
\]

\[
+ \frac{c_i(a - p)}{\eta^2} \left\{ \eta(T - t_i) - \log (1 + \eta(T - t_i)) \right\} - \frac{c_1(a - p)}{\eta} \left\{ \eta(T - t_i) - \log (1 + \eta(T - t_i)) \right\}
\]

\[
16
\]
\[-(a-p) \left( \frac{ht_1^2}{2} + \frac{\delta t_1^3}{6} + \frac{bht_1^3}{6} + \frac{\delta t_1^4}{24} - \frac{b^2ht_1^4}{8} - \frac{b^2\delta t_1^5}{15} + \frac{abt_1^{\beta+2}}{\beta+2} - \frac{\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)} \right) \]

\[+ \frac{3\alpha t_1^{\beta+3}}{2(\beta+3)} - \frac{\alpha bt_1^{\beta+3}}{(\beta+1)} + \frac{\alpha t_1^{\beta+3}}{(2\beta+4)} - \frac{\alpha \delta t_1^{\beta+3}}{2(\beta+2)} - \frac{\alpha \delta t_1^{\beta+4}}{(\beta+1)^2} - \frac{\alpha t_1^{-\delta t_1^{\beta+3}}}{2(\beta+2)(2\beta+3)} \]}

\[-C_1I_e(a-p) \left[ t_1^2 + t_1M^2 - \frac{M^3}{3} - \frac{5t_1^3}{12} - t_1'M + \frac{M^3t_1}{3} + \frac{M^2t_1^2}{3} - \frac{M^4}{4} \right] \]

\[+ \frac{\alpha}{\beta+1} \left( \frac{t_1^{\beta+3}}{(\beta+2)(\beta+3)} - \frac{t_1^{\beta+3}}{(\beta+2)} - \frac{t_1M^{\beta+2}}{(\beta+2)} - \frac{t_1^{2\beta+3}}{2} - \frac{t_1^{\beta+1}M^{\beta+2}}{2} - \frac{M^{\beta+3}}{(\beta+3)} \right) \]

\[+ bMt_1^2 - bt_1M^2 = \frac{bt_1^3}{3} + \frac{bM^4}{\beta+1} + \frac{at_1^{\beta+3}}{\beta+1} \]

\[+C_1I_e \left[ \frac{(a-p)}{2} + b(a-p) \left( \frac{1-b}{3} t_1^3 + \frac{5b}{24} + \frac{\alpha}{\beta+1} \left( \frac{t_1^{\beta+3}}{2} - \frac{t_1^{\beta+3}}{(\beta+2)(\beta+3)} \right) \right] \]

\[+ \frac{2\alpha t_1^{\beta+3}}{\beta+2} - \frac{2\alpha(\beta+2)t_1^{\beta+3}}{\beta+1} \]

(6.21)

\[F_3 = \frac{1}{T} \left[ \left( p(a+\Delta_2-p)T + b(a+\Delta_2-p) \right) p \left( \frac{y^2}{2} + \frac{by^2T^2}{3} + \frac{\alpha y^{\beta+2}T^{\beta+2}}{\beta+2} \right) \right] - (A-\Delta_3) \]

\[-(c_1-\Delta_2)(a+\Delta_2-p)(\gamma T + \frac{by^2T^2}{2} + \frac{\alpha y^{\beta+1}T^{\beta+1}}{\beta+1} + \frac{1}{\eta} \log(1+\eta(T-\gamma T))) \]

\[-(c_2-\Delta_2)(a+\Delta_2-p) \left( \eta(T-\gamma T) - \log(1+\eta(T-\gamma T)) \right) \]

\[-\frac{(c_2-\Delta_2)(a+\Delta_2-p)}{\eta^2} \left( \eta(T-\gamma T) - \log(1+\eta(T-\gamma T)) \right) = \frac{(c_1-\Delta_2)(a+\Delta_2-p)}{\eta} \]

\[\left\{ \eta(T-\gamma T) - \log(1+\eta(T-\gamma T)) \right\} - (a-\Delta_1-p) \left( \frac{(h+\Delta_{12})y^2T^2}{2} + \frac{(\delta+\Delta_{14})y^3T^3}{6} \right) + \frac{b(h+\Delta_{12})y^3T^3}{6} + \frac{b(h+\Delta_{14})y^4T^4}{24} - \frac{b^2(h+\Delta_{12})y^4T^4}{8} - \frac{b^2(\delta+\Delta_{14})y^5T^5}{15} \]

\[+ \frac{\alpha(h+\Delta_{12})y^{\beta+2}T^{\beta+2}}{\beta+2} - \frac{\alpha(h+\Delta_{12})y^{\beta+2}T^{\beta+2}}{(\beta+1)(\beta+2)} \]

\[+ \frac{3\alpha(\delta+\Delta_{14})y^{\beta+3}T^{\beta+3}}{2(\beta+3)} \]

\[- \frac{ab(h+\Delta_{12})y^{\beta+3}T^{\beta+3}}{2(\beta+1)} - \frac{\alpha(\delta+\Delta_{14})y^{\beta+3}T^{\beta+3}}{(\beta+2)} + \frac{b\alpha(\delta+\Delta_{14})y^{\beta+4}T^{\beta+4}}{6(\beta+4)} \]

\[- \frac{\alpha(\delta+\Delta_{14})y^{\beta+4}T^{\beta+4}}{2(\beta+2)} - \frac{\alpha^2(h+\Delta_{12})y^{2\beta+2}T^{2\beta+2}}{2(\beta+1)^2} - \frac{\alpha^2(\delta+\Delta_{14})y^{2\beta+3}T^{2\beta+3}}{(\beta+2)(2\beta+3)} \]
\[-C_{I_r}(a - p)(t_i^2M + t_iM^2 - \frac{M^3}{3} - \frac{5t_i^3}{12} + \frac{5t_i^4}{12} - t_i^3M + \frac{M^3t_i}{3} + \frac{M^2t_i^2}{3} - \frac{M^4}{4}) \]
\[+ \frac{\alpha}{\beta + 1}(t_i^{\beta+3} - \frac{t_i}{\beta + 2}(\beta + 3) - Mt_i^{\beta+2} + \frac{t_i}{\beta + 2}M^{\beta+2} - \frac{t_i^{\beta+2}}{2} + \frac{t_i^{\beta+3}M^{\beta+2}}{2} - \frac{M^{\beta+3}}{\beta + 3}) \]
\[+ bMt_i^2 - bt_iM^2 - \frac{bt_i^3}{3} + \frac{bM^3}{\beta + 1} + \frac{\alpha t_i^{\beta+1}M^{\beta+1}}{\beta + 1} + C_{I_r}[\frac{(a - p)t_i^2}{2} + b(a - p)] \]
\[\frac{(1 - b)t_i^3}{3} + \frac{5b}{24} + \frac{\alpha}{\beta + 1}(t_i^{\beta+3} - \frac{t_i}{\beta + 2}(\beta + 3) - \frac{2\alpha t_i^{\beta+1}M^{\beta+1}}{\beta + 2} - \frac{2\alpha(\beta + 2)t_i^{\beta+3}}{(\beta + 1)(\beta + 3)}) \]
\[(6.22)\]

Now, defuzzified average profit is given by:-
\[\bar{P}(T, p) = \frac{F_1 + 2F_2 + F_3}{4} \quad (6.23)\]

Also, the defuzzified order quantity is Q and is given by:-
\[Q = \left( a + \frac{(\Delta_2 - \Delta_1)}{4} - p \right) \left( t_i + \frac{bt_i^2}{2} + \frac{\alpha t_i^{\beta+1}}{\beta + 1} + \frac{1}{\eta} \log(1 + \eta(T - t_i)) \right) \]

Case II: M > t_i

Now, by the fuzzy triangular rule fuzzy total profit per unit is:-
\[FP(\hat{a}, \hat{A}, \hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{h}, \hat{\delta}) = (F_1, F_2, F_3) \]

And F_1, F_2, F_3 are obtained as:-
\[F_1 = \frac{1}{T} [p(a - \Delta_1 - p)T + b(a - \Delta_1 - p)p \left( \frac{\gamma T^2}{2} + \frac{b\gamma T^3}{3} + \frac{\alpha \gamma T^2}{2} + \frac{\gamma T^3}{\beta + 2} \right) - (A + \Delta_1) \]
\[-(c_i + \Delta_3)(a - \Delta_1 - p)(\gamma T + \frac{b\gamma T^2}{2} + \frac{\alpha \gamma T^2}{\beta + 1} + \frac{1}{\eta} \log(1 + \eta(T - \gamma T))) - \frac{(c_2 + \Delta_3)(a - \Delta_1 - p)}{\eta} \]
\[\frac{\eta(T - \gamma T) - \log(1 + \eta(T - \gamma T))}{\eta} \left\{ \frac{(c_3 + \Delta_3)(a - \Delta_1 - p)}{\eta} \right\} \]
\[-(a + \Delta_2 - p)(\frac{h - \Delta_1}{6})\gamma T^2 + \frac{(\delta - \Delta_3)\gamma T^3}{6} + \frac{b(h - \Delta_1)}{6} \gamma T^3 + \frac{b(\delta - \Delta_3)}{24} \gamma T^4 - \frac{b^2(h - \Delta_1)}{8} \gamma T^4 \]
\[
\frac{-b^2 (\delta - \Delta_{t_1}) \gamma^5 T^5}{15} + \frac{\alpha (h - \Delta_{t_1}) \gamma^{\beta_1 T} T^{\beta_2 + 2}}{\beta + 2} - \frac{\alpha (h - \Delta_{t_1}) \gamma^{\beta_1 T} T^{\beta_2 + 2}}{(\beta + 1)(\beta + 2)} + \frac{3\alpha (\delta - \Delta_{t_1}) \gamma^{\beta_3 T} T^{\beta_3 + 3}}{2(\beta + 3)}
\]

\[
- \frac{\alpha b(h - \Delta_{t_1}) \gamma^{\beta_3 T} T^{\beta_3 + 3}}{2(\beta + 1)} - \frac{\alpha \delta^{\beta_3 T} T^{\beta_3 + 3}}{(\beta + 2)} + \frac{b \alpha (\delta - \Delta_{t_1}) \gamma^{\beta_4 T} T^{\beta_4 + 4}}{6(\beta + 4)} - \frac{\alpha (\delta - \Delta_{t_1}) \gamma^{\beta_4 T} T^{\beta_4 + 4}}{2(\beta + 2)}
\]

\[
- \frac{\alpha^2 (h - \Delta_{t_1}) \gamma^{\beta_2 T} T^{\beta_2 + 2}}{2(\beta + 1)^2} \frac{\alpha^2 (\delta - \Delta_{t_1}) \gamma^{\beta_3 T} T^{\beta_3 + 3}}{(\beta + 2)(2\beta + 3)}
\]

\[
+ c^1 I_c \left[ \frac{(a - p)t^2_1}{2} + b(a - p)\left( \frac{1 - b}{3} \right)t^3_1 \right] + \frac{5b}{24} + \frac{\alpha}{\beta + 1} \left( \frac{t^3_1}{2} - \frac{t^3_1}{(\beta + 2)(\beta + 3)} \right)
\]

\[
+ 2\frac{\alpha t_1^{\beta_3}}{\beta + 2} - \frac{2\alpha (\beta + 2)t_1^{\beta_3}}{(\beta + 1)(\beta + 3)} \right] + (M - t_1)[(a - p)t_1 + b(a - p)\left( \frac{t^2_1}{2} + \frac{bt^3_1}{3} \right)]
\]

\[
+ \frac{\alpha t_1^{\beta_2}}{\beta + 2} - \frac{bt_1^3}{6} - \frac{\alpha t_1^{\beta_2}}{(\beta + 1)(\beta + 2)} \right] \right]
\]

\[
F_2 = \frac{1}{T} [p(a - p)T + b(a - p)p\left( \frac{t^2_1}{2} + \frac{bt^3_1}{3} + \frac{\alpha t_1^{\beta_2}}{\beta + 2} \right)] - A
\]

\[
-c_1(a - p) \left( t_1 + \frac{bt_1^2}{2} + \frac{\alpha t_1^{\beta_1}}{\beta + 1} + \frac{1}{\eta} \log (1 + \eta(T - t_1)) \right)
\]

\[
- \frac{c_2(a - p)}{\eta} \left( \eta(T - t_1) - \log (1 + \eta(T - t_1)) \right) - \frac{c_1(a - p)}{\eta} \left( \eta(T - t_1) - \log (1 + \eta(T - t_1)) \right)
\]

\[
-(a - p) \left[ \frac{b^2 h^4 t^4}{24} + \frac{b^2 h^4 t^4}{8} - \frac{b^2 h^4 t^4}{15} + \frac{\alpha h t_1^{\beta_2}}{(\beta + 2)(\beta + 1)(\beta + 3)} \right]
\]

\[
- \frac{3\alpha \delta^{\beta_3}}{2(\beta + 3)} - \frac{\alpha \delta^{\beta_3}}{2(\beta + 1)} + \frac{\alpha \delta t_1^{\beta_4}}{6(\beta + 4)} - \frac{\alpha \delta t_1^{\beta_4}}{2(\beta + 2)} - \frac{\alpha \delta t_1^{\beta_4}}{(\beta + 2)(2\beta + 3)} \right]
\]

\[
-C_1 I_c (a - p)\left[ t_1^2 M + t_1 M t - M + \frac{h t_1^5 (1 - b)}{(1 - b) t_1^3} + b \left\{ \frac{5t_1^4}{12} - t_1^3 M + \frac{M t_1^5}{3} + \frac{M^2 t_1^2}{3} - \frac{M^4}{4} \right\} \right]
\]

\[
+ \frac{\alpha t_1^{\beta_3}}{\beta + 1} \left( \frac{t_1^{\beta_3}}{\beta + 2} + \frac{t_1 M^{\beta_2}}{(\beta + 2)} - \frac{t_1 M^{\beta_2}}{2} + \frac{t_1^{\beta_3} M^{\beta_2}}{2} - \frac{M^{\beta_3}}{(\beta + 3)} \right)
\]

\[
+ b M t_1^2 - b t_1 M^2 + \frac{b M t_1^2}{3} + \frac{b M t_1^2}{\beta + 1} + \frac{\alpha t_1^{\beta_3}}{\beta + 1} \right]
\]

\[
+C_1 I_c \left[ \frac{(a - p)t_1^2}{2} + b(a - p)\left( \frac{1 - b}{3} \right)t_1^3 \right] + \frac{5b}{24} + \frac{\alpha}{\beta + 1} \left( \frac{t_1^{\beta_3}}{2} - \frac{t_1^{\beta_3}}{(\beta + 2)(\beta + 3)} \right)
\]
\[
\frac{2 \alpha t_1^{\beta+3}}{\beta + 2} - \frac{2 \alpha (\beta + 2) t_1^{\beta+3}}{(\beta+1)(\beta+3)} + (M - t_1)((a - p)t_1 + b(a - p)\left(\frac{t_1^2}{2} + \frac{bt_1^3}{3}\right)
\]
\[
+ \frac{\alpha t_1^{\beta+2}}{\beta + 2} - \frac{bt_1^3}{6} - \frac{\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)}\]

\[F_3 = \frac{1}{T}\left[p(a + \Delta_2 - p)T + b(a + \Delta_2 - p)p\left(\frac{\gamma T^2}{2} + \frac{b\gamma T^3}{3} + \frac{\alpha \gamma^{\beta+2} T^{\beta+2}}{\beta + 2}\right) - (A - \Delta_3)\right]
\]
\[= (c_1 - \Delta_3)(a + \Delta_2 - p)(\gamma T + \frac{b\gamma T^2}{2} + \frac{\alpha \gamma^{\beta+1} T^{\beta+1}}{\beta + 1} + \frac{1}{\eta} \log (1 + \eta(T - \gamma T)))
\]
\[- \frac{(c_2 - \Delta_4)(a + \Delta_2 - p)}{\eta^2} \left[\eta(T - \gamma T) - \log (1 + \eta(T - \gamma T))\right] - \frac{(c_3 - \Delta_5)(a + \Delta_2 - p)}{\eta}
\]
\[\{\eta(T - \gamma T) - \log (1 + \eta(T - \gamma T))\} - (a - \Delta_1 - p)(\frac{(h + \Delta_{12})\gamma T^2}{2} + \frac{(\delta + \Delta_{14})\gamma T^3}{6}
\]
\[+ \frac{b(h + \Delta_{12})\gamma T^3}{6} + \frac{b(\delta + \Delta_{14})\gamma^3 T^4}{24} - \frac{b^2(h + \Delta_{12})\gamma^4 T^4}{8} - \frac{b^2(\delta + \Delta_{14})\gamma^5 T^5}{15}
\]
\[+ \frac{a(h + \Delta_{12})\gamma^{\beta+2} T^{\beta+2}}{\beta + 2} - \frac{a(h + \Delta_{12})\gamma^{\beta+2} T^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{3a(\delta + \Delta_{14})\gamma^{\beta+3} T^{\beta+3}}{2(\beta+3)}
\]
\[+ \frac{ab(h + \Delta_{12})\gamma^{\beta+3} T^{\beta+3}}{2(\beta+1)} - \frac{b(\delta + \Delta_{14})\gamma^{\beta+4} T^{\beta+4}}{(\beta+2)} + \frac{b(a(\delta + \Delta_{14})\gamma^{\beta+4} T^{\beta+4}}{6(\beta+4)}
\]
\[+ \frac{\alpha(\delta + \Delta_{14})\gamma^{\beta+4} T^{\beta+4}}{2(\beta+2)} - \frac{\alpha(\delta + \Delta_{14})\gamma^{\beta+4} T^{\beta+4}}{(\beta+1)^2} - \frac{\alpha^2(\delta + \Delta_{14})\gamma^{2\beta+3} T^{2\beta+3}}{(\beta+2)(2\beta+3)}
\]
\[+ C_f,\left[\frac{(a - p)t_1^2}{2} + b(a - p)\left(\frac{1-b}{2}t_1^3 + \frac{5b}{3} t_1^4 + \frac{\alpha t_1^{\beta+3}}{\beta + 1} - \frac{t_1}{2}\right)\right]
\]
\[+ \frac{2 \alpha t_1^{\beta+3}}{\beta + 2} - \frac{2 \alpha (\beta + 2) t_1^{\beta+3}}{(\beta+1)(\beta+3)} + (M - t_1)((a - p)t_1 + b(a - p)\left(\frac{t_1^2}{2} + \frac{bt_1^3}{3}\right)
\]
\[+ \frac{\alpha t_1^{\beta+2}}{\beta + 2} - \frac{bt_1^3}{6} - \frac{\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)}\]

(6.25)

Now, defuzzified average profit is given by:

\[
\tilde{P}(T, p) = \frac{F_1 + 2F_2 + F_3}{4}
\]

(6.27)
The necessary conditions for maximizing the average profit are

\[ \frac{\partial \tilde{P}(T, p)}{\partial T} = 0 \quad \text{and} \quad \frac{\partial \tilde{P}(T, p)}{\partial p} = 0 \]

Using the software Mathematica-8.1, from the above two equations we can determine the optimum values of \( \tilde{T} \) and \( \tilde{p} \) simultaneously and the optimal value \( \tilde{P}(T, p) \) of the average net profit is determined by (6.18).

- Numerical Example

The data which is taken in this chapter is based on previous study by [13]. All the parameters in the fuzzy environment have the same numerical values as in the crisp environment. The values of fuzzy parameters are as follows: \( \Delta_1 = 9, \Delta_2 = 18, \Delta_3 = 12.5, \Delta_4 = 25, \Delta_5 = 1, \Delta_6 = 2, \Delta_7 = 0.25, \Delta_8 = 0.50, \Delta_9 = 1.25, \Delta_{10} = 2.5, \Delta_{11} = 0.03, \Delta_{12} = 0.06, \Delta_{13} = 0.002, \Delta_{14} = 0.004 \). Based on these input data, the findings are as follows: \( T = 2.5612, \tilde{P}(T, p) = 5634.54 \).

**Table 6.3 Sensitivity analysis of optimal solution w.r.t. system parameters for case I**

<table>
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<tr>
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<tr>
<td>Profit</td>
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<td>-2.6713</td>
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<td>+3.7821</td>
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<tr>
<td>Effect of parameter ‘( \Delta_3 )’</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Profit</td>
<td>-0.0213</td>
<td>-0.0162</td>
<td>+0.0162</td>
<td>+0.0213</td>
</tr>
<tr>
<td>Effect of parameter ‘( \Delta_4 )’</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Profit</td>
<td>+0.0125</td>
<td>+0.0097</td>
<td>-0.0096</td>
<td>-0.0125</td>
</tr>
<tr>
<td>Effect of parameter ‘( \Delta_5 )’</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Profit</td>
<td>+0.0055</td>
<td>+0.0042</td>
<td>-0.0043</td>
<td>-0.0056</td>
</tr>
<tr>
<td>Effect of parameter ‘( \Delta_6 )’</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Profit</td>
<td>-0.0022</td>
<td>-0.0018</td>
<td>+0.0018</td>
<td>+0.0022</td>
</tr>
</tbody>
</table>
Fig. 6.12 Variation in profit w.r.t. $\Delta_1$

Fig. 6.13 Variation in profit w.r.t. $\Delta_2$
Fig. 6.14 Variation in profit w.r.t. $\Delta_3$

Fig. 6.15 Variation in profit w.r.t. $\Delta_4$
Fig. 6.16 Variation in profit w.r.t. $\Delta_5$

Fig. 6.17 Variation in profit w.r.t. $\Delta_6$
6.6 Result Analysis (Fuzzy Mathematical Model)

(i) From Table 6.3 and Figure 6.12, it is observed that when Δ₁ increases, the optimal profit decreases.

(ii) From Table 6.3 and Figure 6.13, it is observed that the optimal profit slightly increases when Δ₂ increases.

(iii) From Table 6.3 and Figure 6.14, it is observed that Δ₃ gives the positive effect on optimal profit.

(iv) From Table 6.3 and Figure 6.15, it is observed that if the value of Δ₄ increases then the optimal profit decreases.

(v) From Table 6.3 and Figure 6.16, it is observed that Δ₅ gives the reverse effect on optimal profit.

(vi) From Table 6.3 and Figure 6.17, it is observed that if the value of Δ₆ increases then the optimal profit increases.

6.7 Conclusion

In this chapter, an order level inventory model for decaying items with multi-variate demand rate and permissible delay in payments is discussed and formulated in which demand rate is dependent on two factors i.e., stock dependent and price dependent. The major outcomes are as :-

(i) The model is proposed in the following two senses: (1) crisp sense and (2) fuzzy sense.

(ii) We have studied the effect of trade credits graphically on both crips and fuzzy sense.

(iii) Numerical examples to validate the results in both the environments are provided and sensitivity analysis with respect to system parameters is also carried out.