CHAPTER- I

INTRODUCTION

1.1 BACKGROUND OF THE STUDY

School is a purposeful affair and there are some definite motives behind which the children are sent to school. The mission of school is not to cover content, but rather to help learners become thoughtful and productive with content. It's not merely helping students to be good at school, but rather to prepare them for the world beyond school—to enable them to apply what they have learnt and solve the issues and problems they will face in the future. The entire school curriculum, instruction and assessment should reflect this central mission, which we call learning for understanding. Learning for understanding requires that curriculum and instruction address three different but interrelated academic goals: helping students (1) acquire important information and skills, (2) make meaning of that content, and (3) effectively transfer their learning to new situations both within school and beyond it.

Unfortunately, the common methods of teaching and testing in schools focus on acquisition at the expense of meaning and transfer. As a result, when confronted with unfamiliar questions or problems many students struggle.
Mathematics is learnt in the school as a part of the curriculum and is practically applied in all spheres of our day-to-day existence. To understand its importance and the difference that would take place if it did not exist, one is also required to look at its roots and place of origin. The process of Mathematics, in Michael Polanyis' words, is “...not quite that of inventing a game, but rather that of the continued invention of a game in the very course of playing the game”. The first step towards such an invention commenced when Egypt and Mesopotamia in 18th Century BC developed a number system which solved practical building and accounting problems. Baudhayana from India used mathematics for astronomy and construction purposes in 9th Century BC. Pythagoras from Greece also worked on relationships of numbers around the same time. Aryabhata I invented zero and decimal system in 100 BC. Aryabhatta II explained size, diameter, rotation and speed of the earth in 499 AD. These mathematicians gave a clear shape to many branches of mathematics. Though mathematics was born out of the felt needs of the man, due to the contributions of the mathematicians across the world, mathematics emerged as the queen of all subjects.

Today, mathematics is used throughout the world as an essential tool in many fields, including natural science, engineering, medicine, and the social sciences such as economics and psychology. Applied
mathematics, the branch of mathematics concerned with application of mathematical knowledge to other fields, inspires and makes use of new mathematical discoveries and sometimes leads to the development of entirely new disciplines.

1.2 HISTORY OF MATHEMATICS EDUCATION

India has a long history of teaching and learning mathematics dating back to the Vedic Age (1500 to 200 BC). Elementary mathematics was part of the education system in most ancient civilizations, including Ancient Greece, the Roman empire, Vedic society and ancient Egypt. In most cases, a formal education was only available to male children with a sufficiently high status, wealth or caste. In the Vedic period, records of mathematical activity are mostly to be found in Vedic texts associated with ritual activities. However, as in many other early agricultural civilizations, the study of arithmetic and geometry was also encouraged by secular considerations. Thus, to some extent early mathematical developments in India mirrored the developments in Egypt, Babylon and China. The system of land grants and agricultural tax assessments required accurate measurement of cultivated areas. As land was redistributed or consolidated, problems of mensuration came up, that required solutions. Tax assessments were based on fixed
proportions of annual or seasonal crop incomes, but could be adjusted upwards or downwards based on a variety of factors. This meant that an understanding of geometry and arithmetic was virtually essential for revenue administrators. Mathematics was thus brought into the service of both the secular and the ritual domains.

During the period AD 200 to 400, several works on astronomy and mathematics were composed, mainly based on indigenous knowledge. Most notable of this period is the contribution of Jaina mathematicians. The Jaina texts prescribed arithmetic as one of the most essential requirements for children's first education. During the period of AD 400 to 1200, a new branch known as Ganita came into existence with three separate components namely, arithmetic, algebra and geometry. But mathematics received prominence as a separate subject only in the 12th century, as referred to in the Leelavati of Bhaskaracharya. The situation with regard to mathematics education remained unchanged after AD 1200 though there had been very significant discoveries. In spite of political instability during the period up to the 18th century, the native system of education maintained its traditional structure up to the advent of British.

In post-independent India, great emphasis has been placed on mathematics teaching and learning. The Education Commission (1964-
recommended mathematics as a compulsory subject for students at school level. The commission seemed to have been influenced by international opinion at that particular time and favored 'new mathematics', which later pervaded secondary education. Thus by the twentieth century mathematics was part of the core curriculum in all developed countries.

1.3 OBJECTIVES OF TEACHING MATHEMATICS

In the twenty-sixth year book of the National Council Of Teachers Of Mathematics (NCTM), a statement of objectives for mathematics has been given and the first thing for the teacher is to be aware of the objectives of teaching mathematics. The objectives should be discussed thoroughly at the teachers’ level and revisited from time to time to evaluate the standard of teaching and make corrective actions to measure up to the intended objectives.

It is envisaged that at the end of secondary school stage of education, the children should be able to fulfil the following objectives due to the efforts of mathematics teachers.

1) Have a knowledge and understanding of mathematical processes, facts and concepts.
2) Have skill in computing with understanding, accuracy and efficiency.

3) Recognize and appreciate the role of mathematics in society.

4) Understand the logical structure of mathematics and the nature of proof.

5) Have the ability to use a general problem-solving technique.

6) Develop reading skill and vocabulary essential for progress in mathematics.

7) Use mathematical concepts and processes to discover new generalizations and applications.

8) Develop study habits for independent progress in mathematics.

9) Demonstrate mental traits such as creativity, imagination, curiosity and visualization.

10) Develop attitudes that lead to appreciation, confidence, respect, initiative and independence. Acquired skills to work with modern technological devices like scientific calculators and computers.

1.4 COMPETENCIES DEVELOPED THROUGH SECONDARY SCHOOL MATHEMATICS

Possessing a mathematical competency means being prepared and able to act mathematically on the basis of knowledge and insight. Mathematical competency is the ability to understand, judge, do and use
mathematics in a variety of intra and extra-mathematical contexts and situations in which mathematics plays a very important role. It is necessary for the students to acquire the competencies stated in the respective grades; otherwise it causes hindrance in understanding and learning mathematics in the next higher classes.

The mathematical competencies that are developed in the secondary school education are given below:

i) **Fluency In Mathematics**

To be capable in mathematics one must have fluency with the basic techniques of mathematics. There are necessary skills and knowledge that students must consistently exercise in order to get mastery. Mathematics is the language of the sciences, and thus fluency in this language is a basic skill. In developing this skill, students first must develop an understanding, and then as they use the skill in different contexts, they gradually sharpen their skills and also reinforce and strengthen their understanding.

ii) **Solving Problems**

Problem-Solving is the essence of mathematics. Problem solving is taught by giving students appropriate experience in solving unfamiliar problems, by then engaging them in a discussion of their
various attempts at solutions, and by reflecting on these processes. Experience in solving problems gives students the confidence and skills to approach new situations creatively, by modifying, adapting and combining their mathematical tools it gives students the determination to refuse to accept an answer until they can explain it.

iii) Analytic Ability And Logic

A classroom full of discussion and interaction that focuses on reasoning is a classroom in which analytic ability and logic are being developed. Therefore the instructional emphasis at all levels should be on a thorough understanding of the subject matter and the development of logical reasoning. This type of instructional approach makes student a more independent and resilient who can analyze and reason well.

iv) Appreciating The Beauty And Fascination Of Mathematics

Students who spend years studying mathematics yet never develop an appreciation of its beauty are cheated of an opportunity to become fascinated by ideas that have engaged individuals and cultures for thousands of years. Applications of mathematics are valuable for motivating students and as mathematics is valuable for more than its utility, an appreciation for the inherent beauty of mathematics should
also be nurtured. Opportunities to enjoy mathematics can make the student more eager to search for patterns, for connections, for answers. This can lead to a deeper mathematical understanding, which also enables the student to use mathematics in a greater variety of applications.

v) Building Confidence

For each student, successful mathematical experiences are self-perpetuating. Genuine success can be built in mathematical inquiry and exploration. Students should find support and reward for being inquisitive, for experimenting, for taking risks, and for being persistent in finding solutions they fully understand. An environment in which this happens is more likely to be an environment in which students generate confidence in their mathematical ability.

vi) Communicating

While solutions to problems are important, so are the processes that lead to the solutions and the reasoning behind the solutions. Students should be able to communicate all of this, but this ability is not quickly developed. Students need extensive experiences in oral and written communication regarding mathematics, and they need constructive, detailed feedback in order to develop these skills.
Mathematics is, among other things, a language, and students should be comfortable using the language of mathematics. The goal is not for students to memorize an extensive mathematical vocabulary, but rather for students to develop an ease in carefully and precisely discussing the mathematics they are learning. However, using appropriate terminology so as to be precise in communicating mathematical meaning is part and parcel of mathematical reasoning.

1.5 CAUSES FOR FAILURE IN MATHEMATICS

The number of subject repeaters in mathematics and number of dropouts from school are frequently due to the deteriorating standards of teaching and learning in mathematics. This trend is due to the following defects in the mathematical education:

1. In our educational set-up, most of the subject teachers are not adequately qualified in the subject concerned and fail to do justice to the subject.

2. The students do not recognize the purpose behind the study of the topics of mathematics.

3. There is serious lack of mathematical apparatus in the schools, without it, the subject becomes abstract.
4. The teacher clings to traditional method, because this offers the path of least resistance. The powers of thinking, understanding and retention are not thus developed in the students.

5. The rules, formulae and set patterns are strictly enforced, when student cannot adjust himself to this rigidity, he starts thinking that he is unfit. Thus discouraged, he does not make much progress.

6. The practical and application aspect of knowledge is not generally emphasized. The subject loses its appeal as it is taught in an abstract, dry and uninteresting manner.

7. Mathematical symbols have their own meanings and have their own significance which the teachers generally fail to bring home to the students.

8. The syllabus does not provide hints and instructions for teacher’s guidance and it is not connected to practical work or project work.

9. The illustrations and problems given in the text books are divorced from actual life and material is made available in a ready-made form which goes against thinking, discovery and originality.
10. The teachers’ unbalanced behavior may also be one of the causes for failure. Some of the feeble minded, nervous students may get scared and discouraged if the teacher is very rigid.

1.6 MYTHS ABOUT MATHEMATICS

Myths about teaching and learning mathematics are also one of the major causes for failure in mathematics. Three widely claimed mathematical myths are:

i) Mathematics Is A Difficult Subject

It is claimed that to many people, mathematics is perceived as a difficult subject to learn and to teach. However, it is also this notion of difficulty in mathematics that attracts some people to mathematics. For them it is a beautiful challenge for the mind, if they succeed in solving the mathematical problems, then there is a strong sense of satisfaction. It is this sense of satisfaction and challenge that can motivate them to go into higher level mathematics. Conversely, if they failed, then this sense of failure might result in low self-esteem.

ii) Mathematics Is Only For The Clever Ones

This is another myth which is closely related to the preconception that mathematics is difficult. Consequently people who excel in school mathematics are highly respected and considered to be
intelligent. For those who fail or perform poorly in school mathematics, it is often assumed that they did not have the so called ‘mathematical ability’.

**iii) Mathematics As A Male Domain**

Put together the above two myths, Issacson (1989) proposes that mathematics has been considered to be a ’hard’ subject, not necessarily in the sense of intellectually difficult, but hard as opposed to soft or feminine. This leads to another widespread mathematics myth that ‘mathematics is a male dominant subject’. Mathematics and science have always been stereotyped as strongly ‘male’ or ‘masculine’. Perhaps traditionally, most mathematics teachers in secondary school and a large majority of mathematicians were found to be men. There is also widespread belief that boys are better in mathematics than girls (Burton, 1989). Various factors have been proposed to contribute to this stereotyped image. Jacobsen (as cited in Burton, 1989) refers this image to the differences in childrearing practices, peer group expectations and social attitudes as the contributing factors. Burton (1989) relates the gender difference in mathematics performance or preference to “bias experiences through patterns of socialization over the period from birth to the end of formal education”.

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1.7 EFFECTIVE CLASSROOM PRACTICES FOR ENHANCING ACHIEVEMENT IN MATHEMATICS

The strongest possibility of improving student learning emerges where schools implement multiple changes in the teaching and learning activities affecting the daily life of students. The quality of the implementation of a teaching practice also greatly influences its impact on student learning. For example: Small group instruction will benefit students only if the teacher knows when and how to use this teaching practice.

Hence, as a teacher implements any of the recommendations, it is essential that he or she constantly monitors and adjusts the way the practice is implemented in order to optimize improvements in quality. There are considerable varieties of the classroom practices that have been found to be effective for enhancing achievement in mathematics:

1. Opportunity To Learn

The relationship between opportunity to learn and student achievement has important implications for teachers. Teachers must ensure that students are given the opportunity to learn important content and skills. If students are to compete effectively in a global, technologically oriented society, they must be taught the mathematical
skills needed to do so. Thus, if problem solving is essential, explicit attention must be given to it on a regular and sustained basis. If we expect students to develop number sense, it is important to attend to mental computation and estimation as part of the curriculum. If proportional reasoning and deductive reasoning are important, attention must be given to them in the curriculum implemented in the classroom.

2. Focus On Meaning

Focusing instruction on the meaningful development of important mathematical ideas increases the level of student learning. Investigations have consistently shown that an emphasis on teaching for meaning has positive effects on student learning, including better initial learning, greater retention and an increased likelihood that the ideas will be used in new situations. This classroom practice can be incorporated in the following ways:

- Create a classroom learning context in which students can construct meaning.

Students can learn important mathematics both in contexts that are closely connected to real life situations and in those that are purely mathematical. The abstractness of a learning environment and how
students relate to it must be carefully regulated, closely monitored and thoughtfully chosen. Consideration should be given to students’ interests and backgrounds. An important factor in teaching for meaning is connecting the new ideas and skills to students’ past knowledge and experience.

- **Make explicit the connections between mathematics and other subjects.**

  For example, instruction could relate data gathering and data-representation skills to public opinion polling in social studies. Or, it could relate the mathematical concept of direct variation to the concept of force in physics to help establish a real-world reference for the idea.

- **Attend to student meanings and student understanding in instruction.**

  Students’ conceptions of the same idea will vary, as will their methods of solving problems and carrying out procedures. Teachers should build on students’ intuitive notions and methods in designing and implementing instruction.
3. Learning New Concepts And Skills While Solving Problems

There is evidence that students can learn new skills and concepts while they are working out solutions to problems. For example, armed with only knowledge of basic addition, students can extend their learning by developing informal algorithms for addition of larger numbers. Similarly, by solving carefully chosen non-routine problems, students can develop an understanding of many important mathematical ideas, such as prime numbers and perimeter/area relations.

Teachers can use students’ informal and intuitive knowledge in other areas to develop other useful procedures. Instruction can begin with an example for which students intuitively know the answer. From there, students are allowed to explore and develop their own algorithm. For instance, most students understand that starting with four pizzas and then eating a half of one pizza will leave three and a half pizzas. Teachers can use this knowledge to help students develop an understanding of subtraction of fractions. Research suggests that it is not necessary for teachers to focus first on skill development and then move on to problem solving. In fact, there is evidence that if students are initially drilled too much on isolated skills, they have a harder time making sense of them later.
4. Opportunities For Both Invention And Practice

Research evidence suggests that students need opportunities for both practice and invention. The findings from a number of research studies show that when students discover mathematical ideas and invent mathematical procedures, they have a stronger conceptual understanding of connections between mathematical ideas.

Students need opportunities to practice what they are learning. To increase opportunities for invention, teachers should frequently use non-routine problems, periodically introduce a lesson involving a new skill by posing it as a problem to be solved, and regularly allow students to build new knowledge based on their intuitive knowledge and informal procedures.

5. Openness To Student Solution Methods And Student Interaction

Research results suggest that teachers should concentrate on providing opportunities for students to interact in problem-rich situations. Besides providing appropriate problem-rich situations, teachers must encourage students to find their own solution methods and give them opportunities to share and compare their solution methods and answers. One way to organize such instruction is to have
students work in small groups initially and then share ideas and solutions in a whole-class discussion.

One useful teaching technique is for teachers to assign an interesting problem for students to solve and then move about the room as they work, keeping track of which students are using which strategies (taking notes if necessary). In a whole class setting, the teacher can then call on students to discuss their solution methods in a pre-determined and carefully considered order, these methods often ranging from the most basic to more formal or sophisticated ones. This teaching structure is used successfully in many Japanese mathematics lessons.

6. Small-Group Learning

Research findings clearly support the use of small groups as part of mathematics instruction. This approach can result in increased student learning as measured by traditional achievement measures, as well as in other important outcomes. When using small groups for mathematics instruction, teachers should:

• Choose tasks that deal with important mathematical concepts and ideas.
• Select tasks that are appropriate for group work.
• Consider having students initially work individually on a task and then follow this with group work where students share and build on their individual ideas and work.

• Give clear instructions to the groups and set clear expectations for each.

• Emphasize both group goals and individual accountability.

• Choose tasks that students find interesting.

• Ensure that there is closure to the group work, where key ideas and methods are brought to the surface either by the teacher or the students, or both.

7. Whole-Class Discussion

It is important that whole-class discussion follow student work on problem-solving activities. The discussion should be a summary of individual work in which key ideas are brought to the surface. This can be accomplished through students presenting and discussing their individual solution methods or through other methods of achieving closure that are led by the teacher, the students, or both. Whole-class discussion can also be an effective diagnostic tool for determining the depth of student understanding and identifying misconceptions. Teachers can identify areas of difficulty for particular students, as well as ascertain areas of student success or progress.
8. Number Sense

‘Number sense’ relates to having an intuitive feel for number size and combinations, as well as the ability to work flexibly with numbers in problem situations in order to make sound decisions and reasonable judgements. It involves being able to use flexibly the processes of mentally computing, estimating, sensing number magnitudes, moving between representation systems for numbers, and judging the numerical results.

As teachers develop strategies to teach number sense, they should strongly consider moving beyond a unit-skills approach (i.e. a focus on single skills in isolation) to a more integrated approach that encourages the development of number sense in all classroom activities, from the development of computational procedures to mathematical problem solving.

9. Concrete Materials

Although successful teaching requires teachers to carefully choose their procedures on the basis of the context in which they will be used, available research suggests that teachers should use manipulative materials in mathematics instruction more regularly in order to give
students hands-on experience that helps them construct useful meanings for the mathematical ideas they are learning.

The use of concrete material should not be limited to demonstrations. It is essential that children use materials in meaningful ways rather than in a rigid and prescribed way that focuses on remembering rather than on thinking. Thus, as Thompson says, ‘before students can make productive use of concrete materials, they must first be committed to making sense of their activities and be committed to expressing their sense in meaningful ways. Further, it is important that students are able to see the two-way relationship between concrete embodiment of a mathematical concept and the notational system used to represent it.’

10. Students’ Use Of Calculators

Research strongly supports the call in Curriculum and evaluation standards for school mathematics, published by the National Council of Teachers of Mathematics, for the use of calculators at all levels of mathematics instruction. Using calculators in carefully planned ways can result in increases in student problem-solving ability and improved affective outcomes without a loss in basic skills.
One valuable use for calculators is as a tool for exploration and discovery in problem-solving situations and when introducing new mathematical content. By reducing computation time and providing immediate feedback, calculators help students focus on understanding their work and justifying their methods and results.

In general, research has found that the use of calculators changes the content, methods and skill requirements in mathematics classrooms. Studies have shown that teachers ask more high-level questions when calculators are present, and students become more actively involved through asking questions, conjecturing and exploring when they use calculators.

The above teaching practices provide a synthesis of the knowledge base for the effective use of practices and improve teaching and learning in mathematics. The teaching practices that are used in classroom may not work in all classrooms at all times. Therefore in education, we need to understand, carefully select, and use combinations of teaching practices that together increase the learning efficiency of students. Therefore teachers should exercise their professional competence, explore promising practices and share information among them, while keeping the focus on the ultimate goal, which is the improvement of student learning.
1.8 THE ROLE OF PROBLEM-SOLVING IN SCHOOL MATHEMATICS


Stanic and Kilpatrick (1989) identify three general themes that have historically characterized the role of problem solving in school mathematics: problem solving as context, problem solving as skill, and problem solving as art.

**Problem Solving As Context:** When problem solving is used as context for mathematics, the emphasis is on finding interesting and engaging tasks or problems that help illuminate a mathematical concept or procedure. By providing this problem-solving context, the teacher’s goals are multiple: to create opportunities for students to make discoveries about the concepts using a familiar and desirable medium
(motivation); to help make the concepts more concrete (practice); and to offer a rationale for learning (justification).

Problem Solving As A Skill: Advocates of this view teach problem solving skills as a separate topic in the curriculum, rather than throughout as a means for developing conceptual understanding and basic skills. Here students are taught a set of general procedures (or rules of thumb) for solving problems—such as drawing a picture, working backwards, or making a list—and give them practice in using these procedures to solve routine problems. When problem solving is viewed as a collection of skills, however, the skills are often placed in a hierarchy in which students are expected to first master the ability to solve routine problems before attempting non routine problems.

Problem Solving As Art: In his classic book, How To Solve It, George Polya (1945) introduced the idea that problem solving could be taught as a practical art, like playing the piano or swimming. He encouraged presenting mathematics not as a finished set of facts and rules, but as an experimental and inductive science. The aim of teaching problem solving as art is to develop students’ abilities to become skillful and enthusiastic problem solvers; to be independent thinkers who are capable of dealing with open-ended, ill-defined problems.
TEACHING THE KEY TRAITS OF PROBLEM SOLVING

According to current research and literature on problem solving, there are some key traits like conceptual understanding, strategies and reasoning, communication, computation and execution, and mathematical insights that students will exhibit when they are performing at a high level of problem solving.

The traits are described below, followed by prompts that teachers can give students to support their problem-solving attempts. These prompts help to develop problem-solving skills.

**Conceptual Understanding:** Students show good conceptual understanding of the mathematics in a problem when they choose appropriate representations, use relevant information, use mathematical terms precisely, and select applicable mathematical procedures (Hiebert et al., 1997; NCTM, 1989, 2000; Schoenfeld, 1989, 1992; Greenwood, 1993).

Prompts a teacher might give to help students to understand the mathematical concepts and interpret the problem are:

- What is the problem about? Rewrite the problem in your own words.
- What is the problem asking you to find?
Strategies And Reasoning—Students demonstrate their ability to use strategies and reasoning by investigating and selecting appropriate problem-solving strategies and conducting a logical, well-planned, and supported process that leads to a reasonable solution. All forms of representations are consistent and integrated into their solution, progress is self-monitored and adjustments are made as needed, and work is verified or a proof of its correctness is provided (Hiebert et al., 1997; NCTM, 1989, 2000; Schoenfeld, 1989, 1992; Greenwood, 1993; Becker & Shimada, 1997; Polya, 1945, 1962-65; Stacey & Groves, 1985).

Prompts to help students get started and think about their solution might include:

- Would drawing a picture or diagram or making a model be helpful in solving this problem? Or would it be helpful to organize your information in a chart or table or in an organized list?
- Would guessing, checking, and adjusting be helpful for solving this type of problem?
- Should you be looking for patterns in your information?
- Would it help to first change the problem using simpler numbers?
- Can you work backwards from where you want to end up to where you want to begin?
- Is the strategy you used efficient? If not, could you find a more efficient way to solve the problem?
- Can you give examples to support your solution?
- Do you understand your plan/strategy well enough to explain it to someone else?
- Are there other ways to approach this problem that might work?
- Can you apply what you’ve learned to other problems?

**Communication**—Students who reflect on what they do and communicate with others about it are in the best position to build useful connections in mathematics (Hiebert et al., 1997). Good communication depends on having a well-organized and clearly articulated solution plan or strategy. Having students explain their strategies orally before writing them can be helpful in developing skill in the trait of communication. (Hiebert et al., 1997; NCTM, 1989, 2000; Schoenfeld,
Prompts to help students communicate their thinking might include:

- Should you use tables, graphs, pictures, words, or a combination of these in explaining or expanding your thinking?
- What did you do first? Why? What did you do next? How did that help you toward your goal?
- How did you figure out ___? What did you learn from doing this problem?
- Did you show how you verified your answer?
- Read your explanation to someone else to make sure it explains your process clearly and is easy to understand.

**Computation and Execution**—Basic skills and conceptual understanding should develop together. To learn skills so that one remembers them, can apply them when needed, and can adjust them to solve new problems, one must learn them with understanding. If students are asked to develop their own procedures for calculating answers to arithmetic problems and to share their procedures with others, their mathematical understanding will be fostered through
executing, discussing, and reflecting on each other’s ideas (Hiebert et al., 1997; NCTM, 1989, 2000; Schoenfeld, 1989, 1992).

Prompts to aid students in improving their computational skills might include:

- Did you double-check your calculations as you went?
- Did you show the rule or formula you used?
- Did you verify your answer was correct by solving the problem in a different way?
- If applicable, did you check your graphs and charts to make sure they are properly labeled?

**Mathematical Insights**—Students show insight into a problem when they recognize the significance of the problem in its relationship to other problems or in its connections to other disciplines or “real-world” applications. By recognizing patterns embedded in the problem, discovering multiple approaches and/or solutions, or creating a general rule or formula, students demonstrate insight into the underlying structure of the problem (Hiebert et al., 1997; NCTM, 1989, 2000; Schoenfeld, 1989, 1992; Becker & Shimada, 1997; Dirkes, 1993; Polya, 1945, 1962-65).

Prompts to improve insights might include:
- How is this problem similar to other problems you have seen or to situations in real life?
- Did you discover any patterns while you were solving the problem?
- What assumptions did you make in solving the problem?
- Is your solution the only one that will work for this problem?
- Can you find a process (or formula) that could be used to solve all forms of this problem?

1.9 ROLE OF MATHEMATICAL CREATIVITY IN SCHOOL MATHEMATICS

Creativity is a topic which is often neglected within mathematics teaching. Usually teachers think that it is logic that is needed in mathematics in the first place, and that creativity is not important in learning mathematics. On the other hand, if we consider a mathematician who develops new results in mathematics, we cannot overlook his/her use of the creative potential.

Commonly, people think that creativity and mathematics have nothing to do with each other. But the mathematicians disagree strongly. For example, Kiesswetter (1983) states that, in his own experience, flexible thinking which is one component of creativity is one
of the most important abilities – perhaps the most important – which a successful problem-solver ought to have. According to Bishop (1981), one needs two very different complementary modes of thinking in mathematics: Creative thinking, for which “intuition” is typical, and analytic thinking, for which “logic” is typical. Verbality, which is always one dimensional, is connected to logic, and visuality which is usually two- or three-dimensional, to intuition. The same idea is put forward by Wachsmuth (1981), who speaks about a “logic mode” and a “relax mode” in thinking. If we observe the performance of a mathematician (or a scientist in any other discipline) when he encounters a new task, we can surely note that he is experimenting at first. These first experimentations are random, but they gradually settle in one direction as an idea of the possible solution is awakening in the mind. Based on the experimentations, the mathematician may set a hypothesis which he tries to prove. Thus, we see that creative performance is an essential part of doing mathematics.

**Development Of Mathematical Creativity**

Mathematical creativity is difficult to develop if one is limited to rule-based applications without recognizing the essence of the problem to be solved. The visionary classrooms described by leaders in the National Council of Teachers of Mathematics (NCTM) (2000) enable
students to confidently engage in complex mathematical tasks...draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. (NCTM, 2000, p.3)

5Cropley (1997) in his review on promoting creativity in the classroom identified ten cognitive aspects of creativity that teachers should strive to promote in students:

1. Possession of a fund of general knowledge.
2. Knowledge of one or more special fields.
3. An active imagination.
4. Ability to recover, discover or invent problems.
5. Skill at seeing connections, overlaps, similarities and logical implications.
7. Ability to think up many ways to solve problems.
9. Ability and willingness to evaluate their own work.
10. Ability to communicate their results to other people.
The idea of encouraging mathematical creativity implicitly through these aims is clearly attractive to curriculum developers and teachers.

Köhler (1997) discussed an experiment by Hollenstein in which one group of children worked on a mathematics exercise presented in the traditional method. This method is described by Romberg and Kaput (1999) as a three-segment lesson: correction of the previous day's homework, teacher presentation of new material and student practice. A second group was given the conditions on which the first group's exercise was based and asked to develop and answer problems that could be solved using calculations. The open-ended nature of the task given to the second group did not limit them to a set of problems. This group created and answered more questions than were posed to the first group, calculated more accurately and arrived at more correct results.

Researchers at Japan’s National Institute for Educational Research conducted a six-year research study that evaluated higher-order mathematical thinking using open-ended problems (problems with multiple correct answers). In a round-table review of the study, Sugiyama from Tokyo Gakugei University affirmed this approach as a
means to allow students to experience the first stages of mathematical creativity (Becker & Shimada, 1997).

Doing what mathematicians do as a means of developing mathematical creativity (as opposed to replication and practice) is consistent with the work at The National Research Center on the Gifted and Talented (Reis, Gentry & Maxfield, 1998; Renzulli 1997; Renzulli, Gentry & Reis, 2004, 2003).

Emphasis is placed on creating authentic learning situations where students are thinking, feeling, and doing what practicing professionals do (Renzulli, Leppien & Hays, 2000; Tomlinson et al., 2001). The fundamental nature of such authentic high-end learning creates an environment in which students apply relevant knowledge and skills to the solving of real problems (Renzulli, Gentry & Reis, 2004).

The solving of real problems also entails problem finding as well as problem solving. Balka (1974) provided participants with mathematical situations from which they were to develop problems. Mathematical creativity was measured by the flexibility, fluency and originality of the problems the participants constructed. By working
with these types of mathematical situations, students are encouraged to use their knowledge flexibly in new applications.

Devlin (2000) identifies four faces of mathematics as (1) computational, formal reasoning and problem solving, (2) a way of knowing, (3) a creative medium, and (4) applications. Of these four, he states that current educational practices in elementary and secondary education focus on the first and touch on the fourth, ignoring the other two.

Pehkonen (1997) suggested that the constant emphasis on sequential rules and algorithms may prevent the development of creativity, problem solving skills and spatial ability. If the instruction focuses on memorization rather than meaning, then the student will correctly learn how to follow a procedure, and will view the procedure as a symbol pushing operation that obeys arbitrary constraints.

Creativity needs time to develop and thrives on experience. Drawing from contemporary research, Silver (1997) suggested, “creativity is closely related to deep, flexible knowledge in content domains; is often associated with long periods of work and reflection rather than rapid, exceptional insight; and is susceptible to instructional
and experiential influences” Whitcombe (1988) described an impoverished mathematics experience as one in which instruction only focuses on utilitarian aspects of mathematics and is without appropriate interest-stimulating material and time to reflect. Such experiences deny creativity the time and opportunities needed to develop.

1.10 GENESIS OF THE PROBLEM

As reported in the three-day national conference of Association of Mathematics Teachers of India (AMTI) in December 2008, the rate of failure of students in mathematics is considerably higher as compared to other subjects and more so of girls in general. Reason for this is the way mathematics is presented to school children; due to this many students find it a difficult and dry subject.

In view of the scope of mathematics, and its distinctive role in solving day to day life problems, mathematics has been considered as one of the core curriculum at the secondary school level. As the secondary school mathematics is loaded with abstract concepts compared to other subjects, parent and pupils at large consider mathematics as a difficult subject and this result in more number of failures in this subject. The rate of failure in mathematics is higher than
in other subjects. The pass percentage in mathematics at the SSLC Examination (for the year April 2006, 2007 and 2008) conducted by Karnataka Secondary Education Board is given below:
Table 1.1: Showing Achievement In Mathematics In Matriculation Examination In Karnataka – Year 2006-2008.

<table>
<thead>
<tr>
<th>Sl No.</th>
<th>Subject</th>
<th>2006 Pass %</th>
<th>2007 Pass %</th>
<th>2008 Pass %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Boys</td>
<td>Girls</td>
<td>Total</td>
</tr>
<tr>
<td>1</td>
<td>First Lang</td>
<td>89.10</td>
<td>94.16</td>
<td>91.47</td>
</tr>
<tr>
<td>2</td>
<td>Second Lang</td>
<td>91.88</td>
<td>92.92</td>
<td>92.37</td>
</tr>
<tr>
<td>3</td>
<td>Third Lang</td>
<td>97.34</td>
<td>98.17</td>
<td>97.73</td>
</tr>
<tr>
<td>4</td>
<td>Maths</td>
<td>76.59</td>
<td>78.60</td>
<td>77.53</td>
</tr>
<tr>
<td>5</td>
<td>Science</td>
<td>82.85</td>
<td>84.67</td>
<td>83.70</td>
</tr>
<tr>
<td>6</td>
<td>S.Science</td>
<td>92.97</td>
<td>93.41</td>
<td>93.18</td>
</tr>
</tbody>
</table>
From the above table it is evident that pass percentage in mathematics is less than other subjects. The number of failures is more in mathematics compared to other subjects. Though there is slight improvement in the achievement index of mathematics in the year 2007, again there is huge fall in the pass percentage in the year 2008. This indicates that the efforts made in enhancing the achievement index are not successful. Since ‘n’ number of factors contributes to the achievement in mathematics, it is the need of the hour to find the cause for the failures and rectify it.

The National Council of Teachers of Mathematics in *Curriculum and Evaluation Standards for School Mathematics* (NCTM Standards) proposed national standards defining what all children should know and do in their school development. NCTM identified the central goal of mathematics education to be developing students’ mathematical power: “an individual’s ability to explore, make assumptions, reason logically, as well as use a variety of mathematical methods effectively to solve non routine problems.” This clearly indicates that a child’s growth in mathematics involves more than just mastering computational skills. Mathematical talent is most often measured by speed and accuracy of a student’s computation with little emphasis on problem solving and pattern finding and no opportunities for students
to work on rich mathematical tasks that require divergent thinking. Such an approach limits the use of creativity in the classroom and reduces mathematics to a set of skills to master and rules to memorize. Doing so causes many children’s natural curiosity and enthusiasm for mathematics to disappear as they get older. Keeping students interested and engaged in mathematics by recognizing and valuing their mathematical creativity may reverse this tendency.

In the present study, the researcher has considered some of the factors like mathematical creativity; intelligence and problem-solving ability which said to have positive influence on achievement in mathematics and intended to study the interaction effect of these factors on achievement in mathematics.

1.11 NEED AND IMPORTANCE OF STUDY

Teaching children mathematics is a central element in educational systems across nations and within countries. Speaking on importance of education Dr. APJ Abdul Kalam opines that, “for India to get transformed into a developed nation by 2020, education is an important component.” Education plays important role in developing the human resource and something in the personality of the child will
certainly remain unrealized in the absence of the study of mathematics. Progress in mathematics is the key factor for any developed nation. It is evident that the more developed the nation is in science and technology the more advanced the mathematical studies there. Since mathematics acts as a catalyst for the progress in science and technology more stress should be given on the effective teaching and learning of mathematics.

When we look at the present classroom teaching especially in the field of mathematics, children often do not know why they are learning mathematics. To some extent they know the use and application of arithmetic concepts, but most of the students do not know why they are learning algebra, geometrical theorems, trigonometry, calculus etc. A report presented in the International Panel on Policy and practice in Mathematics Education: 2001(IPME 2001) on teaching practice says that in India, the classroom teaching is influenced by examinations to a great extent. More stress is given to the content that appears mostly in the examinations. Children do lot of drill work and finally get ready for the examinations with formulae, steps, theorems and their proofs without actually knowing the practical aspect of it. These children suffer drastically when they are encountered with problems in day-to-day life. Though they solve the
problems given in the text book, they would not have developed the problem-solving skills, which fail them miserably in the higher classes.

There are ‘n’ numbers of factors which contribute significantly to the achievement of mathematics. Without understanding the interrelationship between these factors when the subject is taught, it fails to reach maximum number of students.

As all of us know each child is different, its learning style is also different. With only one teaching style when teacher teaches the concepts it does not caters to all the students and they withdraw and start developing a negative attitude towards the subject. When the child is not aware of the importance of the subject matter it is learning, it stops showing interest in the subject. Even in the text books there is a limited opportunity for application of the concepts. Though in some text books importance is given to application again it is not related to the immediate surrounding of the child where it can make use of the concept to understand the things which it sees, feels everyday in a better way. Therefore teacher should be a skilled person to handle 30 or 40 individuals in the class and teach right thing in a right way. Teacher should know the depth and breadth of the concept he/she is teaching and should be able to relate the concept to the day-to-day activities of the child.
The processes of mathematical creativity should be given greater importance as interesting, ingenious and puzzling problems can be created and solved. When children learn to play with numbers, they do not feel mathematics as a boring subject, instead they start developing curiosity about the subject. The richness of mathematics lies in its almost unlimited variety of new problems at all levels, and teacher should help students to derive pleasure through a sense of achievement by solving these problems successfully.

As the problem-solving skills stresses more on learning with understanding, mathematical creativity acts as a catalyst to change the attitude towards learning mathematics and intelligence is very closely associated with problem-solving ability and creativity, investigator felt the need to find interaction effect of these three variables on achievement in mathematics.
1.12 ORGANIZATION OF THE STUDY

The present study comprises of five chapters:

The first chapter includes – Background of the study, Genesis of the problem, Need and importance of the study and Organization of the study.

In the second chapter, the researcher presents- Review of related literature which covers the reviews related to achievement in mathematics; achievement in mathematics and intelligence; achievement in mathematics and mathematical creativity; achievement in mathematics and problem-solving ability; and achievement in mathematics and gender.

The Third chapter deals with Methodology which includes statement of the problem, operational definition of technical terms, variables used in the study, discussion of the variables, objectives of the study, hypotheses of the study, tools used for the collection of the data, description of the tools, sampling procedure and statistical techniques used for the analysis of the data.

In the fourth chapter, the researcher presents the analysis and interpretation of data.
In the fifth chapter, the researcher presents, summary of the research, findings of the study based on the interpretation of the data, limitations of the study, suggestions for further research and Educational Implications.