7.1 Modelling

In this work at wheel rail interfacing assume single point contact. Figure –7.1 and figure– 7.2 show the model for wheel rail interaction. It consist of a beam represent the rail, which supported by elastic foundation, represent the track. The rail as well as the track loaded by moving oscillator represent the part of the train vehicle and travelling with velocity $V$ along the rail. In oscillator model, unsprung axle mass and the stiffness of hertzian
contact and primary suspension are taken into account. Vertical rail geometry is described by $z(x)$. The rail is continuously supported [42, 49, 50].

The above model is based on following assumption, irregularity at interface is short with respect to the ration of train velocity to the bogie motion Eigen frequency. Frequency is higher than approximately 250 Hz, the influence of modelling on contact force magnitude of wheel rail is negligible[30]. Now from figure –7.1 and figure –7.2, problem simplification from moving wheel to stationary wheel base in the following assumptions,

- Damping Mechanism are not accommodated
- At contact point, due to interfacial irregularity the dynamic contact force is generated is energy input in rail. That energy is dissipated in rail pad, ballast etc. [45, 46].
- However measuring the wheel rail dynamic contact force occur at some points of irregularities is relatively short time scale. It play a role with increase in time scale.

So, for wheel rail dynamic interaction for high frequency regim, these approach is used. The stationary wheel model is shown in figure –7.2, in which excitations as Hertzian spring contact roll over rail geometry $z(x)$ at the velocity $V$ change into excitation $z(t)$ of Hertzian spring contact at $t = 0$ as function of time.

The model is consider in frequency domain. Some restriction in frequency domain are

- Model does not allow contact loss
- Nonlinearity in the contact cannot be accounted and individual sleepers are not modelled

Frequency domain has two main advantages

i. Result is very simple, and analytical closed form expression for the main involved parameters in frequency domains which is equal to integral expressions in time domains allowing for fast computations for a large number of simulations.

ii. Frequency domain approach provides insight in occurring mechanisms, which become manifest in the time domain. Time domain
models or even much more accurate Finite Elements models do not provide such insight. This makes the model very suitable for a qualitative investigation of characteristic features of dynamic interfacial irregularity types. This will prove advantages especially when dealing with wheel flats.

The following mathematical is valid for model as shown in figure –7.2 [31],

a. The rail motion equation, which is model as a Euler Bernoulli beam on the elastic foundation is given by (t > 0)

\[ E I \frac{\partial^4 w(x, t)}{\partial x^4} + \rho A \frac{\partial^2 w(x, t)}{\partial t^2} + k_f w(x, t) = -F(t) \delta(x) \]  \hspace{1cm} (7.1)

b. For wheel system, motion equation is

\[ m_w \frac{d^2 u(t)}{dt^2} + k_1 u(t) = F(t) \] \hspace{1cm} (7.2)

c. At Contact point, the Wheel rail contact force is

\[ F(t) = k_H (-u(t) + w(0, t) + z(t)) \] \hspace{1cm} (7.3)

d. Assume damping in the system is infinity but non zero. So, the rail must be undisturbed at \( x = -\infty \) at any moments

e. Assume for all starting condition are zero.

\[ u(0) = \dot{u}(0) = 0 \]
\[ w(x, 0) = \dot{w}(x, 0) = 0 \]

Model is linear and load is statics. Result in field of rail displacement is steady state which is not included. Dynamic field deflection and dynamic force is consider. So, analytical approach of a linearized Hertzian spring is allowin frequency domains.

7.2 Solution

The beam motion equation (7.1), in the t and x domains can be given by

\[ \frac{\partial^4 w(x, t)}{\partial x^4} + \frac{1}{a^2} \frac{\partial^2 w(x, t)}{\partial t^2} + b^2 w(x, t) = -\frac{1}{E I} F(t) \delta(x) \]

\[ a = \sqrt{\frac{E I}{\rho A}}, \quad b = \sqrt{\frac{k_f}{E I}} \] \hspace{1cm} (7.4)
A system subjected to periodic excitation has two components of motion, the transient and the steady state. In most of such causes the transient part is not important as it dies out soon, and the steady state part is one that persists. However, where the excitation is of a periodic nature like a shock pulse or a transient excitation, the response of the system is purely transient. After the duration of excitation, the system undergoes vibration with its natural frequency with an amplitude depending upon the type and duration of excitation. It is in such case transient vibration is importance. The practical example of shock excited transient vibration are rock explosion, punching operation, automobiles at high speed passing over pits or curbs on the road etc. it means use of Laplace transform is for the analysis of systems subjected to shock pulses.

Laplace transform, these expression with respects to time and $w(x,0) = \dot{w}(x,0) = 0$,

$$\frac{\partial^4 w(x,s)}{\partial x^4} + \left( \frac{s^2}{a^2} + b^2 \right) \tilde{w}(x,s) = -\frac{1}{E_1} \tilde{F}(s), \delta(x) \quad \text{--------- (7.5)}$$

In equation (7.5), transform variables are define by a tilde. Any time moment, since rail is undistributed at $t = \pm \infty$, then the governing variables will be find out by Fourier transforming with respects to $x$. The Fourier transforms is define by

$$\tilde{a}(k_x) = \int_{-\infty}^{\infty} a(x) e^{ikx} dx \quad \text{--------- (7.6)}$$

The Fourier transform of expression (7.5) is written by

$$\tilde{w}(k,s) \left( k^4 + \frac{s^2}{a^2} + b^2 \right) = -\frac{1}{E_1} \tilde{F}(s) \quad \text{--------- (7.7)}$$

In equation (7.7), Fourier transform variables are define by bar [32]. From expression (7.7)

$$\tilde{w}(k,s) = -\frac{1}{E_1} \cdot \frac{\tilde{F}(s)}{k^4 + \frac{s^2}{a^2} + b^2} \quad \text{--------- (7.8)}$$

For this expression, the inverse Fourier transformed in $x$ and $s$ domains is given by

$$\hat{w}(x,s) = -\frac{\tilde{F}(s)}{2\pi E_1} \int_{-\infty}^{\infty} \frac{e^{-ikx}}{k^4 + \frac{s^2}{a^2} + b^2} dk \quad \text{--------- (7.9)}$$
These integrations can be determined which add semicircular joining \(-\infty\) to \(\infty\) and theory of Cauchy’s residue. The integrand pole are equivalent to the equation’s solutions

\[ k^4 + \frac{s^2}{a^2} + b^2 = 0 \quad \text{-------- (7.10)} \]

And solutions are given by,

\[ k_{1,2,3,4} = (\pm 1, \pm i)k_0, \quad \text{-------- (7.11)} \]

\[ k_0 = \frac{1}{\sqrt{2}} \frac{s}{\sqrt{a^2 + b^2}} \]

With the choice \(l_m(k_0) < 0, Re(k_0) > 0\), the individuals poles maybe given as,

\[ k_1 = (1 + i)k_0, \]
\[ k_2 = (-1 + i)k_0 \]
\[ k_3 = (-1 - i)k_0 \]
\[ k_4 = (1 - i)k_0 \quad \text{-------- (7.12)} \]

\(I_m(k_1) > 0, I_m(k_2) > 0, I_m(k_3) < 0, I_m(k_4) < 0\). The pole \(k_1\) and \(k_2\) and situated in the upper half and \(k_3\) and \(k_4\) in the lower half plane of the complex k plane.

By Jordan’s lemma formula

\[ \lim_{R \to \infty} \int f(k)e^{-ikx}dx = 0, \ x > 0, \]

Then close contour c is in 3rd and 4th quadrants and \(f(k)\) tend uniform zero for increase in radius R of the contours. And \(c < 0\), yield a close contours in upper half plane of the complex k. Both poles and closed contours are shown in figure –7.3.
Figure 7.3 - Outlines of \( x < 0 \), and \( x > 0 \) respectively

Since integration is single value. So, theory of Cauchy’s residue can apply. It is apply for 1st contours \( x < 0 \), two pole enclosed by contour.

\[
\int_{-\infty}^{\infty} \frac{e^{-ikx}}{k^4 + \frac{s^2}{a^2} + b^2} \, dk =
\]

\[
2\pi i \sum_{k_1, k_2} \text{Residue of } \frac{e^{-ikx}}{k^4 + \frac{s^2}{a^2} + b^2} \text{ at } k_{1,2} = 2\pi i \sum_{k_1, k_2} \frac{e^{-ikx}}{4k^3} \text{ --------- (7.13)}
\]

Substitute the valued of \( k_1 \) and \( k_2 \) from equation (7.12) in to equation (7.13),

\[
\int_{-\infty}^{\infty} \frac{e^{-ikx}}{k^4 + \frac{s^2}{a^2} + b^2} \, dk = 2\pi i \sum_{k_{1,2}} \frac{e^{-ikx}}{4k^3} = 2\pi i \left( \frac{e^{-i(1+i)k_0x}}{4(1+i)k_0^3} + \frac{e^{-i(-1+i)k_0x}}{4((-1+i)k_0^3)} \right)
\]

\[
= \frac{\pi}{8k_0} \left( (1-i)e^{(1-i)k_0x} + (1+i)e^{(1+i)k_0x} \right) \text{ --------- (7.14)}
\]

Now, theory of Cauchy’s residue can be apply to second contour \( x > 0 \). The contour is negligible. So, negative sign is added.

\[
\int_{-\infty}^{\infty} \frac{e^{-ikx}}{k^4 + \frac{s^2}{a^2} + b^2} \, dk =
\]

\[
-2\pi i \sum_{k_3, k_4} \frac{e^{-ikx}}{4k^3} = -2\pi i \left( \frac{e^{-i(-1-i)k_0x}}{4((-1-i)k_0^3)} + \frac{e^{-i(1-i)k_0x}}{4((1-i)k_0^3)} \right)
\]

\[
= \frac{\pi}{8k_0} \left( (1-i)e^{(-1+i)k_0x} + (1+i)e^{(-1-i)k_0x} \right) \text{ --------- (7.15)}
\]
If $x < 0$, then $x = -|x|$ and for $x > 0$, then $x = |x|$. Then substituting in equation (7.14) and equation (7.15), can be written as,

\[
\int_{-\infty}^{\infty} \frac{e^{-ikx}}{k^4 + \frac{z^2}{a^2} + b^2} \, dk = \frac{\pi}{8k_0^4} \left( (1 - i)e^{(-1+i)k_0|x|} + (1 + i)e^{(-1-i)k_0|x|} \right)\quad \text{----- (7.16)}
\]

Now, substituting equation (7.16) in to equation (7.9),

\[
\tilde{w}(x, s) = -\frac{F(s)}{16EIk_0^2} \left( (1 - i)e^{(-1+i)k_0|x|} + (1 + i)e^{(-1-i)k_0|x|} \right) \quad \text{---------- (7.17)}
\]

Where, $k_0 = \frac{1}{\sqrt{2}} \cdot \frac{4}{\sqrt{a^2 + b^2}}$.

Now, expression for oscillator, for the wheel mass,

\[
m_w \frac{d^2u(t)}{dt^2} + k_1 u(t) = F(t)
\]

Now Laplace transform of above expression with respect to time with $u(0) = \dot{u}(0) = 0$, and transform variables are define by tilde. Above equation may be written as

\[
(m_w s^2 + k_1)\tilde{u}(s) = \tilde{F}(s)
\]

\[
\tilde{u}(s) = \frac{\tilde{F}(s)}{(m_w s^2 + k_1)} \quad \text{------- (7.18)}
\]

Now, equation (7.17) and equation (7.18) substituting in equation (7.3) is establish the interaction force in time domains. Laplace transform of equation (7.3)

\[
\tilde{F}(s) = k_H (-\tilde{u}(s) + \tilde{w}(0, s) + \tilde{z}(s)) \quad \text{--------- (7.19)}
\]

The contact force final equation,

\[
\tilde{F}(s) = \frac{\tilde{z}(s)}{k_H \frac{1}{(m_w s^2 + k_1) \frac{1}{(EI\tilde{R})}}} \quad \text{--------- (7.20)}
\]

Where, $k_0 = \frac{1}{\sqrt{2}} \cdot \frac{4}{\sqrt{EI}} \left( \rho As^2 + k_f \right)$.  

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Equation (7.18), shows the wheel displacement. Substitute equation (7.20) in to equation (7.18),

\[ \ddot{u}(s) = \frac{\ddot{z}(s)}{(m_w s^2 + k_1) \left( \frac{1}{k_B} + \frac{1}{(m_w s^2 + k_1) + \left( \frac{1}{8EIk_B} \right)} \right)} \]

Now, for finding out rail displacement at contact point, substitute equation (7.18) and equation (7.20) in to equation (7.19),

\[ \tilde{w}(0,s) = \ddot{z}(s) \left( 2 - \frac{1}{8EIk_B} \left( \frac{1}{k_H} + \frac{1}{(m_w s^2 + k_1) + 1} \right) \right) \]

The equation (7.20) to equation (7.22) has be transform to time domains. The inverse Laplace transformation is integration over frequency is calculated by numerical. Because integration is not a single value, the analytical inverse transforms by mean of contour integrations. Inverse Laplace transforms of \( \ddot{F}(s) \) is given by

\[ F(t) = \frac{1}{2\pi i} \int_{\sigma-i\omega}^{\sigma+i\omega} \tilde{F}(s) e^{st} ds \]

Substitute \( s = \sigma + i\omega \), where, \( \sigma \) = some small positive real value and \( ds = id\omega, -\infty < \omega < \infty \). Now, equation (7.23) can written as,

\[ F(t) = \frac{e^{\sigma t}}{2\pi} \int_{-\infty}^{\infty} \tilde{F}(\sigma + i\omega) e^{i\omega t} d\omega \]

From equation (7.24),

\[ \tilde{F}, e^{i\omega t} = \left( \text{Re}(\tilde{F}) + i \text{Im}(\tilde{F}) \right) \cdot (\cos \omega t + i \sin \omega t) \]

\[ = \left( \text{Re}(\tilde{F}) \cos \omega t - \text{Im}(\tilde{F}) \sin \omega t \right) + i \left( \text{Re}(\tilde{F}) \sin \omega t + \text{Im}(\tilde{F}) \cos \omega t \right) \]

Where, force \( F(t) \) is real when imaginary part is add up to zero. Means With respect to \( \omega \), \( \sin \omega t \) and \( \cos \omega t \) is anti-symmetric and symmetric respectively. \( \text{Re}(\tilde{F}) \) is an anti-symmetric with respect \( \omega \). So, equation (7.24) may be given as
In Laplace domains, Equation (7.26), is used for integration by numerically. Integral form for wheel rail dynamic contact force, displacement of wheel and rail at contact points are as functions of time for an excitation function \( z(t) \) with Laplace image \( z(s) \). From equation (7.20), equation (7.21) and equation (7.22) can be given as,

\[
F(t) = \frac{e^{\sigma t}}{\pi} \int_{0}^{\infty} \text{Re}\left( \frac{z(s)}{k_H + \frac{1}{m_w s^2 + k_1}} \right) e^{iw \omega} d\omega \quad \text{(7.26)}
\]

\[
F(t) = \frac{e^{\sigma t}}{\pi} \int_{0}^{\infty} \text{Re}\left( \frac{z(s)}{\frac{1}{k_H + \frac{1}{(m_w s^2 + k_1) + 1}}} \right) e^{iw \omega} d\omega \quad \text{(7.26)}
\]

\[
u(t) = \frac{e^{\sigma t}}{\pi} \int_{0}^{\infty} \text{Re}\left( \frac{\tilde{z}(s)}{1 + (m_w s^2 + k_1)} \right) e^{iw \omega} d\omega \quad \text{(7.27)}
\]

\[
\tilde{w}(0, s) = \frac{e^{\sigma t}}{\pi} \int_{0}^{\infty} \text{Re}\left( \tilde{z}(s) \left( 2 - \frac{1}{8Eik_0 \left( \frac{1}{k_H + \frac{1}{(m_w s^2 + k_1) + 1}} \right) + 1} \right) \right) e^{iw \omega} d\omega \quad \text{(7.28)}
\]

Where, \( k_0 = \frac{1}{\sqrt{2}} \sqrt{\frac{1}{Ei} (\rho As^2 + k_f)}, s = \sigma + iw \).

7.3 Model for Wheel Rail Interface Irregularity

![Figure 7.4–Single Linear irregularity for Wheel and Rail Contact Surface in Longitudinal direction](image-url)
Figure 7.5 - Longitudinal Wheel Rail Contact Surface multi linear irregularity

Figure 7.4, shows the elementary model, which contains only one non zero slop, on a variable basis. The rail surface is horizontal except for an interval $\Delta x$, where the rail geometry has a constant slop $\theta$. The slop is start in the origin. Now, wheel rail interface irregularity of arbitrary shape $z(x)$ is given and wheel passed at a constant velocity $V$ with respect to time a wheel overpassing an irregularity yield a transient excitation and response state.

Longitudinal functions coordinates $x \geq 0$ in vertical rail surface is given by

$$z(x) = \theta x H(\Delta x - x) + \theta \Delta x H(x - \Delta x) \quad \text{-------- (7.30)}$$

Where, $H =$ Heaviside Step Function

For a contact point moving at speed $V$, the equation in time domains become

$$z(t) = \theta . V . t . H \left( \frac{\Delta x}{V} - t \right) + \theta . \Delta x . H \left( t - \frac{\Delta x}{V} \right), t \geq 0 \quad \text{-------- (7.31)}$$

The Laplace image of this equation is given by

$$\tilde{z}(s) = \frac{\theta . V}{s^2} \left( 1 - e^{-s \Delta x / V} \right) \quad \text{-------- (7.32)}$$

Figure 7.5 shows the more extended model for short irregularity with an arbitrary geometry. The model consists of a sequence of four discrete slop corresponding to five discrete points. But it can be extended to an arbitrary number $N$ slopes. By Laplace transformed linearity, slope sequence can be present by summation of all term.

Vertical surface as functions of longitudinal coordinates $x \geq 0$ is given by
\[ z(x) = \theta_1 \cdot x \cdot H(\Delta x - x) + ((\theta_1 - \theta_2) \cdot \Delta x + \theta_2 x) \cdot H(x - \Delta x) \cdot H(2\Delta x - x) + ((\theta_1 + \theta_2 - 2\theta_3) \cdot \Delta x + \theta_3 x) \cdot H(x - 2\Delta x) \cdot H(3\Delta x - x) + ((\theta_1 + \theta_2 + \theta_3 - 3\theta_4) \cdot \Delta x + \theta_4 x) \cdot H(x - 3\Delta x) \cdot H(4\Delta x - x) \]  

(7.33)

This equation in the time domain is given by,

\[ z(x) = \theta_1 \cdot V \cdot t \cdot H\left(\frac{\Delta x}{V} - t\right) + ((\theta_1 - \theta_2) \cdot \Delta x + \theta_2 \cdot V \cdot t) \cdot H\left(t - \frac{\Delta x}{V}\right) \cdot H\left(\frac{2\Delta x}{V} - t\right) + ((\theta_1 + \theta_2 - 2\theta_3) \cdot \Delta x + \theta_3 \cdot V \cdot t) \cdot H\left(t - \frac{2\Delta x}{V}\right) \cdot H\left(\frac{3\Delta x}{V} - t\right) + ((\theta_1 + \theta_2 + \theta_3 - 3\theta_4) \cdot \Delta x + \theta_4 \cdot V \cdot t) \cdot H\left(t - \frac{3\Delta x}{V}\right) \cdot H\left(\frac{4\Delta x}{V} - t\right) \]  

(7.34)

For the four-slope, the Laplace transform becomes,

\[ \tilde{z}(s) = \frac{V}{s^2} \left(1 - e^{-s\Delta x / V}\right) \cdot \left(\theta_1 + \theta_2 \cdot e^{-s\Delta x / V} + \theta_3 \cdot e^{-2s\Delta x / V} + \theta_4 \cdot e^{-3s\Delta x / V}\right) \]  

(7.35)

This equation can be easily extended for a larger number of slopes. The Laplace image for \( N \) discrete slopes becomes

\[ \tilde{z}(s) = \frac{V}{s^2} \left(1 - e^{-s\Delta x / V}\right) \sum_{n=1}^{N} \theta_n \cdot e^{-\left(n-1\right)s\Delta x / V} \]  

(7.36)

### 7.4 Energy Input at Irregularity Track

The energy input into the track due to an arbitrary track irregularity with length \( l \) follows from integration of the power input over the period \( T \) of passing this irregularity.

\[ E_m = \int_0^T e^{l / v} F(t) \cdot \dot{w}(0, t) dt \]  

(7.37)

Input energy and power input into the track must be minimize, for that balance requirements between force and displacement. The work performed by the contact load should be minimum. For low stiffness, the force level is low and displacement is large and vice versa. So, the stiffness must be optimize. Since this stiffness is dynamic and function of frequency and depend on modelling which may consider for or disregard wave propagation. Also, wave length depend on spectrum must be optimized. However this cannot be
done in general sense, since this spectrum optimization can only be perform for given geometry. So, the optimum track stiffness spectrum does not exist.

According to equation (7.37), the power into the track is computed as the product of contact force and resulting displacement,

\[ P(t) = F(t)\dot{w}(0, t) \]

\[ L\{P(t)\} = L\{F(t)\dot{w}(0, t)\} \quad \text{-------- (7.38)} \]

Where, \( L \) denoted the Laplace transform. Since multiplication in time domains which are equal to loop frequency domains using convolution theorem in a general form,

\[ L\{f(t)g(t)\} = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \tilde{f}(p)g(s-p)dp \quad \text{-------- (7.39)} \]

So, power input in Laplace domain

\[ \tilde{P}(s) = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \tilde{F}(p)s\tilde{w}(0, s-p)dp \quad \text{-------- (7.40)} \]

Using the Laplace domain equation for the contact force and the rail deflection can be transformed to the time domain. Equation (7.27) and equation (7.29) may be written alternatively in the time domain as,

\[ F(t) = \frac{1}{\pi} \int_0^\infty Re \left( \frac{\tilde{z}(s) e^{st}}{\frac{1}{k_H} + \frac{1}{(m_\omega s^2 + k_1)} + \frac{1}{(8EIk_2^2)}} \right) d\omega \]

\[ \dot{w}(0, s) = \frac{1}{\pi} \int_0^\infty Re \left( \tilde{z}(s) \left( 2 - \frac{1}{8EIk_2^2} \frac{1}{\frac{1}{k_H} + \frac{1}{(m_\omega s^2 + k_1)} + 1} \right) s e^{st} \right) d\omega \quad \text{-------- (7.41)} \]

The power added to the track during the passage of the irregularity can be written as,

\[ P_{in}(t) = \frac{1}{\pi} \int_0^\infty Re \left( \frac{\tilde{z}(s) e^{st}}{\frac{1}{k_H} + \frac{1}{(m_\omega s^2 + k_1)} + \frac{1}{(8EIk_2^2)}} \right) d\omega \cdot \int_0^\infty Re \left( \tilde{z}(s) \left( 2 - \frac{1}{8EIk_2^2} \frac{1}{\frac{1}{k_H} + \frac{1}{(m_\omega s^2 + k_1)} + 1} \right) s e^{st} \right) d\omega \quad \text{-------- (7.42)} \]

Where, \( \tilde{z}(s) = \frac{V}{s^2} \left( 1 - e^{-s\Delta x/V} \right) \sum_{n=1}^{N} \theta_n e^{-(n-1)s\Delta x/V} \). So, the equation (7.37) is allow for calculating the energy input.
7.5 Parameters of Model

The rail properties for freight used UIC 60. The static Axle load 40 T, No damage is there. It expect negligible effect in contacts, which controlled in wheel rail interface is negligible.

Table – 7.1 Parameters of Model

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Static wheel load ( F_{\text{stat}} )</td>
<td>( 200 \times 10^3 ) N for 40 T Axle Load</td>
</tr>
<tr>
<td>2</td>
<td>Wheel Radius ( R_{\text{wheel}} )</td>
<td>0.45 m</td>
</tr>
<tr>
<td>3</td>
<td>Transversal railhead radius ( R_{\text{railhead}} )</td>
<td>0.3 m</td>
</tr>
<tr>
<td>4</td>
<td>Wheel mass ( m_w )</td>
<td>900 kg</td>
</tr>
<tr>
<td>5</td>
<td>Primary suspension Stiffness ( k_1 ) [34]</td>
<td>( 4.9 \times 10^6 ) N/m</td>
</tr>
<tr>
<td>6</td>
<td>Rail bending stiffness for UIC 60 ( EI )</td>
<td>( 6.110 \times 10^6 ) N.m(^2)</td>
</tr>
<tr>
<td>7</td>
<td>Rail distributed mass for UIC 60 ( pA )</td>
<td>60.34 kg/m</td>
</tr>
<tr>
<td>8</td>
<td>Rail Supported Stiffness ( k_f ) [34]</td>
<td>( 400 \times 10^6 ) N/m</td>
</tr>
</tbody>
</table>

The wheel rail contact stiffness according to Hertzian contact theory is nonlinear quantity. A linearization of this stiffness has been used to enable frequency domain approach in equation (7.3) and equation (7.20). This linearization is applied at level of static wheel load yielding a tangent stiffness. Assume a circular contact area have a radius equals to geometrical mean of the elliptical radii and an infinite radius of the transverse wheel profile, an approximate equation of nonlinear Hertzian contact stiffness is given by

\[
k_H = \frac{3E^2F_{\text{stat}}\sqrt{R_w.R_r}}{\sqrt{2(1-\varrho^2)^2}}
\]

Applied all parameters from the above table and take \( E = 2.1 \times 10^{11} \) N/m\(^2\) and \( \varrho = 0.3 \)

\[
k_H = \sqrt[3]{2.93 \times 10^{22}.F_{\text{stat}}} \quad (7.43)
\]

The resulting linear stiffness after substitute the value of \( F_{\text{stat}} \) conduct to sound result for dynamic load fluctuation around these level. Simplify
equation (7.43), the Hertzian stiffness has been conducted by third roots of wheel load. Therefore, model result is considered to be trusty, when dynamic amplifications factor (DAF) does not exceed value 2.

\[
k_H(2.F_{stat}) = \sqrt[3]{2} \cdot k_H(F_{stat})
\]

\[
= 1.26 \cdot k_H(F_{stat})
\]

So, the lower the value for DAF, higher is expected contact force accuracy.