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Our study of fixed point theorems has its origin in one of the most fundamental results of functional analysis, the Fixed Point Theorem of metric space. Over the years, there have been many efforts to generalize this theorem for various classes of topological spaces and Banach spaces.

In wider sense, by a fixed point theorem we shall mean a statement which asserts that under certain conditions a mapping $T$ of a space $X$ into a space $Y$ admits one or more points $x$ of $X$ such that $Tx = x$. A good number of researchers have studied this theory due to its usefulness in the existence theory of differential equations, integral equations and its applications in boundary value problems, approximation theory and non-linear analysis.

There is a multitude of metrical fixed point theorems for mappings satisfying certain contraction type conditions. In all these results one considers sequence of iterates, which, due to contraction conditions, becomes a Cauchy sequence and whose limit is a fixed point of the mapping. In case of common fixed point theorems, a joint sequence of iterates is suitable for the purpose.

The present thesis comprises five chapters and each chapter consists of various sections which are numbered in the order in which they occur in the text. Each chapter begins with a brief introduction to its contents.
In Chapter I, we have attempted to give a brief account of the historical development of the subject, preliminary concepts and the important results used throughout the thesis. This chapter is mainly aimed at making the present text as self-contained as possible.

In Chapter II, we have proved certain fixed point theorems in metric spaces employing an extension of Mann iterative process using the contractive conditions alongwith weak commutativity condition and sequence of mappings. Our results generalize the earlier known results of Park and Bae, Ćirić and Rhoades. In the last Section, we have attempted to prove fixed point theorems for single-valued mapping which generalize and extend the earlier results of Banach and Kannan.

In Chapter III, we have studied fixed point theorems for asymptotically regular sequences and asymptotically regular maps. Our main emphasis is to exploit the use of weak commutativity conditions and thus we are able to obtain the fixed point theorems in metric space. Our results generalize the earlier known results of Engl, Browder and Petryshyn, Hardy and Rogers and others.

Chapter IV, is devoted to study the fixed point theorems in metrically convex spaces. In the beginning, we assume a pair of single-valued non-self mappings satisfying the contractive
conditions. Our results are more general and extend an earlier result of Assad for a more general single-valued mapping which is also substantial generalization of Chatterjea. In the last Section, we have obtained some results for metrically convex space which, in turn, generalize earlier results of Assad and Chatterjea.

In the fifth and last Chapter, we have studied certain fixed point theorems in Banach space. The first Section gives brief introduction to the work done in this direction. In the Second Section, we have obtained fixed point theorems for a pair of self-mappings defined on a closed subset in Banach space. Our results improve earlier results of Fisher. In the last Section, we obtained results on fixed point for self-mappings of a convex closed subset in Banach space. In this way our results generalize and extend earlier known results of Fisher and Gregus.

In the end, a Bibliography which can by no means be regarded as exhaustive, is given which contains the references of only those books and papers which have been referred in this exposition.