Abstract
An Ecological Model

A model is an imperfect and abstract representation of the real world phenomenon. Any such representation in terms of mathematics is said to be a mathematical model. In this thesis, an ecological model means a mathematical model that represents an ecological phenomenon such as the interaction of species to each other and to their environment.

The principal objective of all ecological models (which incorporate relatively a few key parameters) is of uncovering the rules, interactions and constraints that govern the complex systems which they represent. Mathematical models were developed primarily in the physical sciences, physiology and applied sciences such as fisheries and wildlife management. But lately, mathematical models have been quite successfully used to describe and predict the behavior of complex living systems particularly the ecological systems.

Classical Lotka-Volterra Predator-prey and Competition Models

Kolmogoroff [6] (see also Rescigno and Richardson [14]) devised a general model for interacting species formulated by

\[
\frac{dX}{dt} = X F (X, Y) \\
\frac{dY}{dt} = Y G (X, Y)
\]  

(A.1)

Functions F and G in (A.1) denote per capita growth rates of species X and Y respectively. Obviously, classical Lotka-Volterra models are
special cases of (A.1). Some other important and interesting particular cases of (A.1) are due to Leslie [7], Holling [4], Rosenzweig and MacArthur [15], Ayala [2], Barker [3] and May [10].

Inspite of the fact that analytical solutions to most of the classical models have been quite successfully demonstrated, the situations described by them are far from those observed in natural populations. One can consider the case of well-known oscillating hare-lynx populations of Canada. Although these populations exhibit stable limit cycles of a predator-prey system but as May [10] points out these populations can not be taken as an illustration of neutral oscillations of classical Lotka-Volterra model. Furthermore, in many cases the behavior of the laboratory populations grown under strictly controlled conditions has been quite different from that predicted by mathematical models proposed to describe the specific nature [11,13]. These citations show that classical models indeed represent a very restrictive class of situations. Over the course of time, theoretical biologists and applied mathematicians have modified the classical models and models methodologies in many ways.

Various Factors Incorporated in this Thesis Model Systems

The thesis discusses in detail the importance of these factors. One may refer to [1,5,8,9,12] for related literature

- Prey Defensive Switching Behavior
- Age-structure
The present thesis comprises five chapters under the titles:

Chapter-I: Introduction, Synopsis of the work and Methodology
Chapter-II: Coexistence of Species in a Defensive Switching Model
Chapter-III: Evolutionarily Stable Strategies for Prey Defensive Switching
Chapter-IV: A Predator-specific Defense Model with Age-structure
Chapter-V: Oscillations in Exactly Solvable Predator-prey Volterra type Models

In Chapter-II, we consider a mathematical model of a two-predators and one-prey system which has the defensive switching property of predation-avoidance. We assume that the prey remains vigilant against relatively abundant predator species and guards against it by switching to another (relatively rare) predator species. We analyze how the intensity of defensive switching affects the stability of the model system. It is seen that the system generally has a stable three species coexisting equilibrium state. In the special case that the intensity of defensive switching equals one and the two predators have the same mortality rates, it is shown that the system asymptotically settles to a Volterra's oscillation in three-dimensional space. It is observed that a
sufficiently small or sufficiently large value of intensity of defensive switching can make the system unstable. Finally, it is shown that the handling time may have a stabilizing effect on predator-prey systems with defensive switching.

In Chapter-III, we consider a predator-prey system in which two predators share a single prey and the prey defends itself against predators by adopting defensive switching strategy. To illustrate how the defensive switching strategy can evolve under natural selection, dynamical behaviors of two-predators and one-prey system are studied. It is assumed that the defensive strategy of the prey against a predator species is regulated mainly by two parameters: the relative alertness of the prey, \( u \), and the intensity of the defensive switching, \( n \). Assuming that the system is encountered with immigrant prey species with altered defensive switching parameters one after another, we study the conditions of replacement of the native prey by the immigrant prey. It is seen that the system eventually attains an evolutionarily stable state such that no entry of an immigrant prey is possible. It is noticed that unless some trade-off relation exists between \( u \) an \( n \), the evolutionarily stable state is not unique, but it is given by a set of parameters \( \{u,n\} \) which form a curve designated “ESS line”. Therefore, if a number of two-predators and one-prey systems are allowed to evolve independently, each system will reach a different state on the “ESS line” through a different course of mutational events. The strategy leading to optimal (minimum) prey consumption by both predators is also discussed in relation to the “ESS line”.

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Chapter-IV contains a two-predator, one-prey model with prey defensive switching. We introduce delays for the first time in such a model. To this end, we assume that one of the predator species has age-structure that significantly affects its fecundity. The type of the delay we consider in this chapter is that due to age-specific maturation periods. We employ the construction of Liapunov functionals and Krasovskii methods respectively to analyze two special but equally generalized cases of the main model called Model A and Model B. We give sufficient conditions for the asymptotic stability of the positive equilibrium of each model. In contrast to the generally held tenet in population biology that large delays cause instabilities, main results of this chapter suggest that large maturatoin periods can promote the coexistence of two predators and a single prey that exhibits defensive switching property.

In Chapter-V, we consider a Volterra type two species competing system with Gompertz type interactions and compare its stability with the model system of Takeuchi and Adachi [16] that has no Gompertz interactions. We see that although our system predicts a larger region of coexistence of species as compared to [16], it still exhibits non-oscillatory behaviour of species. We show that addition of one or two predators to this system may cause oscillations in species and that now the patterns of species may be (i) coexistence at the globally stable equilibrium and (ii) coexistence in the periodic motion of Hopf-type (limit cycle). We emphasize that though our results seem to be similar to those reported in [16] but in [16] the stability analysis is based on perturbation methods and Hopf bifurcation theory whereas our model systems turn out to be exactly solvable.


