Preface

Discrete analytic function theory is concerned with a study of functions defined only at certain lattice points in the complex plane. As it is suitable for treating ordinary difference functions, the lattice of definition is usually taken to be the set of Gaussian integers.

Since its comparatively recent beginnings in 1941, the subject has been extensively developed by numerous writers. The resulting theory has much in common with the theory of analytic functions of a continuous complex variable and of course has many distinguishing features.

The theory of geometric difference functions constitutes an important branch of finite differences. An extension to discrete analytic function theory was carried out in 1972 by C.J. Harman. In this thesis an attempt has been made to extend Harman's theory by taking two bases \( p \) and \( q \) (\( 0 < p < 1, \ 0 < q < 1 \)) as against a single base \( q \) (\( 0 < q < 1 \)) in Harman's theory. For this, a particular lattice is defined and a discrete analytic theory is derived which is applicable to geometric difference functions.

Using \( p \)-difference and \( q \)-difference operators instead of derivatives to define a discrete analytic function, analogues for contour integration, Cauchy integral theorems and elementary functions are established. By defining discrete power series, results are found for the
representation of a discrete analytic function as an
infinite series. Several other results arising from the
theory are obtained.

The thesis contains six chapters. The I Chapter
includes a resume of the research done in the field of
tectom differences together with a review of the origins
and developments of 'discrete analytic' function theory, and
a summary of the basic results of the thesis.

In II Chapter, a particular lattice, suitable for
tometric difference functions in defined A class of
ctions named (p,q) - analytic functions (analogous to
alytic functions) is defined on the lattice and properties
of such functions are examined.

In III Chapter the concept of a discrete line integral
is defined which, in a sense, can be regarded as inverse of
the \( \Delta_{p,x} \), \( \Delta_{q,y} \) operators. Properties studied in this chapter
of these discrete line integrals exhibit a close analogy
with the theory of continuous variables. Analogues for
Green's Formula and Cauchy's Integral formula are also
established.

In IV Chapter, the discrete plane \( Q' \) is extended to
include points on the positive half-axes. If a (p,q) -
analytic function is defined on a subset of \( Q' \) then it has a
unique extension as a (p,q) - analytic function to certain
other points of the discrete plane. An outline of results of
this type is given and a method is devised for the continuation into \( Q' \) of functions defined on the axes. A convolution operator ensues (analogous to multiplication of functions in the continuation theory), and its properties examined.

In V Chapter explicit examples of \((p,q)\)-analytic funtions are considered. Using the continuation operator \( C_y \) and the operator \(*\), discrete analogues of \( z^n \) and of polynomials \( \sum_{n=0}^{m} a_n z^n \) are obtained. Results comparable to classical powers and polynomials are determined and certain fundamental differences are noted.

The discrete power \( z^{(n)} \) and \((z-z_o)^{(n)}\) can be utilized in series of the form

\[
\sum_{j=0}^{M} a_j z^{(j)} \quad \text{and} \quad \sum_{j=0}^{M} a_j (z-z_o)^{(j)}
\]

In the last chapter series of the above type are used to obtain discrete analogues of the exponential function and of the general power \((z-z_o)^{(a)}\). Finally it is shown that \((p,q)\)-analytic function can be represented by a discrete analogue of Maclaurin's series.

In the end an exhaustive and up-to-date list of writings and original papers on the subject matter of this dissertation have been provided in the form of a bibliography.