PREFACE

Special functions in general and hypergeometric functions and polynomials in one or more variables in particular occur frequently in a wide variety of problems in physics, engineering, statistics and operations research. In theory of special functions, generation functions, summations and transformations formula have received some attention of some authors during the last years.

Generating functions, finite sum properties and transformations play a crucial role in the study of special functions.

In view of growing importance of generating functions, this thesis contains certain classes of generating functions which are linear, bilinear, bilateral, double and multiple for certain special functions and polynomials in one, two and several variables. Such generating functions are obtained by using series rearrangement method, integral operator techniques, Nishimoto's fractional calculus and group theoretic method.

Some transformations, fractional derivative formulas, finite sum properties, for certain hypergeometric functions and polynomials are also presented and various special cases are deduced.

In our work many known results of Chatterjea [15], Exton [25], Feldheim [27], Kar and Basu [32], Khan and Shukla [36], Majumdar [45], Maxinar [48], Pathan and Bin Saad [67], Pathan and Khan [64], Srivastava [82, 84, 86] and Srivastava and Manocha [94] are shown as special cases of our main results and many new results are also presented.
The present thesis comprises six chapters. A brief summary of the problems is presented at the beginning of each chapter and then each chapter is divided into a number of sections. Because of the close association of special functions, with generating functions, a brief review of these important topics is presented in the \textit{first chapter}. It provides a systematic introduction to the most of the important special functions that commonly arise in practice and explores many of their salient properties.

This chapter is also intended to make the thesis as much self contained as possible.

\textit{Chapter (2)} deals with the study of Laguerre polynomials of two, three, four and m-variables. Certain new finite sum properties, transformations and generating functions for these polynomials have been obtained in this chapter. A known results given by Exton [25], Srivastava and Manocha [94] and many generating functions involving generalized hypergeometric function \( \text{pFq} \), confluent hypergeometric functions of several variale \( \psi_2^{(s)} \), Kampé de Fériet function of two variable \( F^{A:B';B'}_{C:D;D'}[x,y] \) and generalized Kampé de Fériet of several variables \( {F^{A:B',...,B^{(n)}_{(n)}}}_{C:D',...,D^{(n)}_{(n)}}[q_1,...,q_n] \) are shown here to be special cases of our main results.

\textit{Chapter (3)} devoted to obtain double generating functions for Gauss hypergeometric function \( _2F_1 \), generalized hypergeometric function \( \text{pFq} \) and generalized Kampé de Fériet function of several variables \( {F^{A:B',...,B^{(n)}_{(n)}}}_{C:D',...,D^{(n)}_{(n)}}[q_1,...,q_n] \).
A further double generating relations for Legendre, Laguerre, Jacobi and Rice polynomials are also obtained.

Many known results of Chattajeea [15], Feldeim [27], Khan and Shukla [36], Pathan and Bin Saad [67], Srivastava [84, 86] and Srivastava and Manocha [94] are shown as special cases of these generating functions.

In Chapter (4), we derive some fractional derivatives formulas involving hypergeometric functions of three variables by application of Nishimoto’s fractional calculus (N-Fractional Calculus). Certain generating functions (linear, bilinear, bilateral, double and multiple) for triple hypergeometric function $F^{(3)} [x,y,z]$ and Saran’s functions $F_G, F_N, F_S$ are obtained. Many results (known and new) involving Gauss $F_1$ and Appell’s double hypergeometric functions $F_1, F_2, F_3$ are obtained as special cases in this chapter.

In chapter (5), a new generating functions for triple hypergeometric function $F^{(3)} [x,y,z]$, which are linear and double generating functions is derived by using integral operators. Many special cases involving triple hypergeometric function $F^{(3)} [x,y,z]$, Kampé de Fériet function of two variables $F^{A;B;B'}_{C:D;D'}[x,y]$ and Appell’s double hypergeometric function $F_2$ are obtained. It is also shown how the main results (5.2.4) and (5.4.3) related to a known results Srivastava [82] and Pathan and Khan[68].

Finally, in Chapter (6), we prove a general theorems on a general class of generating relations involving Legendre, Laguerre and Gegenbauer polynomials with the help of group theoretic method. Importance of these theorems lie in the fact that all
particular cases of generating functions can be easily deduced by attributing suitable value to $a_n$ and then making use of known generating functions involving Legendre, Laguerre and Gegenbauer polynomials. Results given by Majumdar [45] and Kar and Basu [32] are shown here to be special cases of our main results.

Articles, definitions and equations have been numbered chapter-wise. The equations are numbered in such a way that, when read as decimals they stand in their proper order.

The first decimal place represents the chapter, the second decimal place represents the section and the last decimal place represent the equation number.

A part of our work has been published/accepted/communicated for publication. A list of papers is given below:

1. On obtaining generating functions by the application of integral operators.
2. Generating relations by means of integral operators
   (Communicated)
3. On generating functions involving laguerre polynomials of several variables
   (Communicated)
4. A note on Laguerre polynomials of several variables
   (Communicated)
5. On generating functions involving Laguerre, Legendre and Gegenbauer polynomials
   (Accepted)
6. On double generating relations involving certain hypergeometric functions
   (Communicated)

7. On obtaining generating functions by the application of Nishimoto’s fractional calculus
   (Communicated)

8. Bilinear generating functions by means of Nishimoto’s fractional calculus
   (Communicated)