CHAPTER 8

OPTIMAL RESERVE INVENTORY LEVEL BETWEEN
TWO GRADES IN MANPOWER PLANNING

8.1 INTRODUCTION

An interesting problem in Inventory Control Theory is to find the optimum size of the buffer between two operating systems. Hanssman (1962), in his paper has considered the problem of optimal reserve inventory between two machines and determined the optimal reserve inventory, assuming constant demand rate. Later this result was extended for stochastic demand.

In this chapter, an organisation having two branches B₁ and B₂ are considered. Branch B₁ has grades G₁ and G₂. The Branch B₂ has Grade G₃ which is equivalent to G₂ and it serves as an inventory between G₁ and G₂ in the context of filling up of vacancies that arises in G₂. The vacancies in G₂ are filled up through a promotion process from G₁. In this context, if no suitable person is available in G₁ for promotion to G₂, then personnel working in G₃ are transferred to G₂ under certain conditions. G₁ is said to be in the upstate, if there is atleast one person in G₁ available for promotion to G₂. Otherwise, G₁ is said
to be in the downstate. The idle time for G₂ is the duration in which no personnel is available from G₁ and G₃ to fill up the existing vacancies in G₂. The main objective of this chapter is to find the optimum value of the reserve inventory to be kept in G₃, so as to minimize the total cost incurred. The rest of the chapter is organized as follows: In section 8.2, description of the model is given. In section 8.3, analytical expression for the cost equation is obtained. In section 8.4, the optimum reserve inventory level between branches B₁ and B₂ is obtained for different special cases and the results are illustrated by numerical examples and relevant conclusions are made.

8.2 DESCRIPTION OF THE MODEL

Assumptions:

1. An organisation has two branches B₁ and B₂.
2. B₁ has two Grades G₁ and G₂ and B₂ has Grade G₃.
3. The size of G₃ is S.
4. Personnel from G₁ are promoted to G₂ by a definite promotion process to fill up vacancies in G₂. If G₁ is in the downstate, to fill up vacancies in G₂, atmost S₁ personnel (0 ≤ S₁ < S) from G₃ are transferred to G₂ instantaneously depending upon the need.
Notations:

\[ r_1 \]: rate at which the vacancies arises in \( G_2 \).

\[ h_1 \]: holding cost per unit of \( S_1 \).

\[ d_1 \]: downtime cost for \( G_1 \).

\[ \mu_1 \]: average time between two consecutive start of the downtime duration for \( G_1 \).

\[ g(\tau) \]: probability density function of \( \tau \)

\[ G(.) \]: distribution of \( \tau \)

We assume that \( G_1 \) is in the downstate and \( G_2 \) is in the upstate. Till the grade \( G_1 \) is in upstate the process is going on smoothly. When no one is selected at \( G_1 \) then the Grade 1 comes to downstate, however \( G_2 \) is in upstate as it gets resource from \( B_2 \).

In \( B_2 \), among \( S \) persons only \( S_1 \) persons are available for giving training at \( G_2 \) because there must be a minimum number of people in \( B_2 \) so that \( B_2 \) may not be closed.

When all \( s_1 \) number of persons from \( B_2 \) are sent for training at \( G_2 \), \( G_2 \) has no one to give training either from \( G_1 \) or from \( B_2 \) and idle time for \( G_2 \) starts. Till \( G_1 \) gets the person selected, \( G_2 \) is in idle state. When the persons are available in \( G_1 \) to give training in \( G_2 \), again \( G_1 \) comes to the upstate and the process is going on smoothly, but from \( B_2 \) only one time the persons can be taken for training so as to postpone the idle time in \( G_2 \).
8.3 MAIN RESULT

Assume that the Grade \( G_1 \) is in downstate and Grade \( G_2 \) is in upstate. The cost equation is given as

\[
C(S_1) = h_1 S_1 + \frac{d_1}{\mu_1} \int_{S_1}^{\infty} \left( \tau - \frac{S_1}{r_1} \right) g(\tau) d\tau
\]

(8.3.1)

The first term in (8.3.1) refers to the holding cost per unit of \( S_1 \) and the second term refers to the idle time cost for \( G_2 \). The optimum value of \( S_1 \) which minimizes \( C(S_1) \) can be obtained as follows.

Differentiating (8.3.1) with respect to \( S_1 \) and equating to zero,

\[
\frac{d}{dS_1} (C(S_1)) = 0
\]

(i.e)

\[
h_1 + \frac{d_1}{\mu_1} \int_{S_1}^{\infty} \frac{d}{dS_1} \left( \tau - \frac{S_1}{r_1} \right) g(\tau) d\tau = 0
\]

\[
h_1 + \frac{d_1}{\mu_1} \int_{S_1}^{\infty} \left( - \frac{1}{r_1} \right) g(\tau) d\tau = 0
\]

\[
h_1 - \frac{d_1}{\mu_1 r_1} \int_{S_1}^{\infty} g(\tau) d\tau = 0
\]

\[
h_1 - \frac{d_1}{\mu_1 r_1} G(S_1) = 0
\]

(8.3.2)
Equation (8.3.2) can be solved for $S_1$ when the distribution function $G(.)$ is given.

$C(S_1)$ is minimum for $S_1$ when \( \frac{d^2}{dS_1^2}(C(S_1)) > 0 \)

### 8.4 SPECIAL CASE

In this section for specific distributions cost analysis are made.

**Case (i):** Suppose $G(.)$ follows uniform distribution over $[0,a]$. In this case, the downtime density of $G_1$ is constant and is independent of time.

The holding cost will arise when $a < \frac{S_1}{r_i}$

When $a > \frac{S_1}{r_i}$, from (8.3.2)

\[
h_1 - \left( \frac{d_i}{\mu_i r_i} \right) \left( \frac{1}{a} \right) \left( a - \frac{S_1}{r_i} \right) = 0
\]

\[
S_1 = (ar_i)(1-h_i \mu_i r_i / d_i)
\] (8.4.1)

\[
S_1 = 0 \iff h_i \mu_i r_i = d_i
\] (8.4.2)

\[
S_1 > 0 \iff d_i > h_i \mu_i r_i
\] (8.4.3)
As \( r_1 \) is real, from (8.4.1), we get

\[
S_1 \leq \left( \frac{ad_i}{4h_i \mu_i} \right)
\]

(8.4.4)

**Case (ii)** Suppose \( G(.) \) follows an exponential distribution with parameter \( \lambda \).

From (8.3.2) it follows that,

\[
h_i = \left( \frac{d_i}{\mu_i r_i} \right) e^{-\lambda S_i/\eta} = 0
\]

(8.4.5)

\[
S_1 = \frac{r_i}{\left[ \log(d_i / h_i \mu_i r_i) \right]}
\]

(8.4.6)

\[
S_1 > 0 \iff d_i > h_i \mu_i r_i \quad \text{and} \quad S_1 = 0 \iff h_i \mu_i r_i = d_i
\]

(8.4.7)

In both the cases, if \( S_1 = 0 \), no inventory is necessary between \( G_1 \) and \( G_2 \) and if \( S_1 > 0 \), then a positive inventory is to be kept between \( G_1 \) and \( G_2 \).
**NUMERICAL ILLUSTRATION:**

In this section for the cost analysis made in section 8.4, numerical illustrations are given with relevant conclusions.

**Case (i):** Suppose \( G(.) \) is the uniform distribution over \([0,a]\)

The following Table (8.4.1) gives \( S_1 \) for different combination of the values of \( h_1, \mu_1, r_1, d_1, \) and \( a \).

**Table (8.4.1)**

<table>
<thead>
<tr>
<th>( \mu_1 )</th>
<th>( r_1 )</th>
<th>( h_1 )</th>
<th>( d_1 )</th>
<th>( a )</th>
<th>( S_1 )</th>
</tr>
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<td>5</td>
<td>21</td>
</tr>
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<td>7000</td>
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<td>17</td>
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<td>4000</td>
<td>5</td>
<td>19</td>
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<tr>
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</table>
Case (ii):

$G(.)$ is the exponential distribution with parameter $\lambda$.

The following Table (8.4.2) gives $S_1$ for different combination of the values of $h_1$, $\mu_1$, $r_1$, $d_1$ and $\lambda$.

<table>
<thead>
<tr>
<th>$\mu_1$</th>
<th>$r_1$</th>
<th>$h_1$</th>
<th>$d_1$</th>
<th>$\lambda$</th>
<th>$S_1$</th>
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</tr>
</tbody>
</table>
CONCLUSIONS

(i) From the Table (8.4.1) it is observed that, as $\mu_1$ increases, keeping other parameters fixed, $S_1$ decreases and as $r_1$ increases, keeping other parameters fixed, $S_1$ increases which is realistic.

(ii) From the Table (8.4.2) it is observed that, as $\mu_1$ increases, keeping other parameters fixed, $S_1$ decreases and as $r_1$ increases, keeping other parameters fixed, $S_1$ decreases which is also realistic.