CHAPTER 7

EXPECTED TIME FOR RECRUITMENT IN A TWO-GRADED
MANPOWER SYSTEM HAVING DIFFERENT RENEWAL PROCESS
FOR INTER-DECISION TIMES AND SCBZ PROPERTY FOR
THRESHOLD DISTRIBUTION

7.1 INTRODUCTION

Suresh Kumar, Gopal and Sathiyamoorthy (2006) have obtained the mean and variance of the time to recruitment by assuming that each grade is sustained with the cumulative loss of manpower and the organization goes for recruitment whenever any one grade reaches a breakdown state. Sathiyamoorthi and Parthasarathy (2002), (2003) have discussed about the Recruitment in a two graded marketing organization and they obtained the Expected time to recruitment when the threshold distribution has SCBZ property. In this chapter, an organization with two grades is considered and each grade has its own threshold level with the Setting the Clock Back to Zero (SCBZ) property. Recruitment is made by the organization whenever the threshold crossing takes place in any one of the grades. It is assumed that the policy
decisions are governed by different renewal processes. The objective of this chapter is to obtain the mean time for recruitment in the organization under suitable univariate recruitment policy. The rest of this chapter is organised as follows: In section 7.2, description of the model is given and the analytical expression for meantime to recruitment is obtained. In section 7.3, the results are numerically illustrated and relevant conclusions are made.

7.2 DESCRIPTION AND ANALYSIS OF THE MODEL

Assumptions:

1. An organization having two grades (grade A and grade B) takes policy decisions at random epochs in \([0, \infty)\) and at every decision making epoch, a random number of persons quit the organization.

2. There is an associated loss of manpower to the organization if a person quits and it is linear and cumulative.

3. Each grade has its individual independent threshold. If the total number of persons who leave the organization crosses the minimum of the two thresholds, recruitment becomes necessary. In other words, recruitment is made whenever the organization exceeds a threshold level in any one of the grades.

4. Mobility of manpower from one grade to another is not permitted
Notations:

\(X_{iA}\): continuous random variable representing the random amount of manpower depletion in grade A due to \(i^{th}\) decision making epoch, \(i=1,2...,X_{iA}\)'s are independent and identically distributed random variables

\(X_{iB}\): continuous random variable representing the random amount of manpower depletion in grade B due to \(i^{th}\) decision making epoch, \(i=1,2...,X_{iB}\)'s are independent and identically distributed random variables

\(Y_A, Y_B\): continuous random variables denoting the threshold levels for grades A and B respectively

\(Y\): \(Y=\min(Y_A, Y_B)\)

\(T_A, T_B\): discrete random variable denoting time to recruitment for grades A and B respectively.

\(T\): \(T = \min(T_A, T_B)\), time to recruitment

\(S_{T_A}(t)\): survival function of \(T_A\) for grade A

\(S_{T_B}(t)\): survival function of \(T_B\) for grade B

\(L(t)\): cumulative distribution function of \(T\)

\(T\): \(T = \min(T_A, T_B)\) - time to recruitment in the organization.

\(S_{T_A}(t)\): survival function of \(T_A\) for grade A

\(S_{T_B}(t)\): survival function of \(T_B\) for grade B

\(L(t)\): cumulative distribution function of \(T\)

\(l(t)\): probability density function of \(T\)

\(l^*(s)\): Laplace transform of \(l(t)\).
F(.) :cumulative distribution function of the inter-decision times for grade A
W(.) :cumulative distribution function of the inter-decision times for grade B
H_A(.) : distribution function of Y_A with parameters (θ_1, θ_2, μ_1)
H_B(.) : distribution function of Y_A with parameters (θ_1, θ_2, μ_2)
H(.) : cumulative distribution function of Y
S(t) :Survival function of T and \[ S(t) = S_{T_A}(t) S_{T_B}(t) \]
F_k(.) : k-fold convolution of F(.)
W_k(.) : k-fold convolution of W(.)
g(.) : probability density function of X_iA
g_k(.) : k-fold convolution of g(.)
h(.) : probability density function of X_iB
h_k(.) : k-fold convolution of h(.)
f(.) : probability density function of the inter-decision times of grade A.
w(.) : probability density function of the inter-decision times of grade B.
w_k(.) : k-fold convolution of w(.)
E(T) : meantime for recruitment

**MAIN RESULTS**

In this section an analytical expression for meantime to recruitment is obtained and a special case is discussed.
As in chapter 6, we define

\[ S_{T_A}(t) = P(T_A > t) = \sum_{k=0}^{\infty} \{ \text{probability that there are exactly } k \text{ decisions in } (0, t] \} \text{ and } \{ \text{probability that grade A is sustained with the cumulative loss of manpower} \}. \]

\[ S_{T_B}(t) = P(T_B > t) = \sum_{k=0}^{\infty} \{ \text{probability that there are exactly } k \text{ decisions in } (0, t] \} \text{ and } \{ \text{probability that grade B is sustained with the cumulative loss of manpower} \}. \]

\[ P(T_A > t) = \sum_{k=0}^{\infty} \left[ F_k(t) - F_{k+1}(t) \right] P \left[ \sum_{i=1}^{k} X_{iA} < Y_A \right] \quad (7.2.1) \]

\[ P(T_B > t) = \sum_{k=0}^{\infty} \left[ W_k(t) - W_{k+1}(t) \right] P \left[ \sum_{i=1}^{k} X_{iB} < Y_B \right] \quad (7.2.2) \]

Since \( Y_A \) and \( Y_B \) has SCBZ property with the parameters \((\theta_1, \mu_1)\) and \((\theta_2, \mu_2)\) respectively, from chapter 2, we have

\[ 1 - H_A(x) = P_1 e^{-(\theta_1 + \mu_1)x} + q_1 e^{-\theta_2 x} \quad \text{where} \quad P_1 = \frac{\theta_1 - \theta_2}{\mu_1 + \theta_1 - \theta_2}, q_1 = \frac{\mu_1}{\mu_1 + \theta_1 - \theta_2} \]

and \[ 1 - H_B(x) = P_2 e^{-(\theta_3 + \mu_2)x} + q_2 e^{-\theta_4 x} \quad \text{where} \quad P_2 = \frac{\theta_3 - \theta_4}{\mu_2 + \theta_3 - \theta_4}, q_2 = \frac{\mu_2}{\mu_2 + \theta_3 - \theta_4} \]

Now by the law of total probability
\[
P \left[ \sum_{i=1}^{k} X_{iA} < Y_A \right] = \int_{0}^{\infty} g_k(x) \left[ 1-H_A(x) \right] dx
\]

\[
= \int_{0}^{\infty} g_k(x) \left[ p_t e^{-(\theta_1+\mu_1)x} + q_t e^{-(\theta_2)x} \right] dx
\]

\[
P \left[ \sum_{i=1}^{k} X_{iA} < Y_A \right] = p_k g_k^* (\theta_1 + \mu_1) + q_k g_k^* (\theta_2)
\]  
(7.2.3)

Similarly

\[
P \left[ \sum_{i=1}^{k} X_{iB} < Y_B \right] = p_k h_k^* (\theta_3 + \mu_2) + q_k h_k^* (\theta_4)
\]  
(7.2.4)

Using (7.2.3) in (7.2.1) and (7.2.4) in (7.2.2)

\[
P(T_A > t) = \sum_{k=0}^{\infty} \left[ F_k(t) - F_{k+1}(t) \right] \left[ p_k g_k^* (\theta_1 + \mu_1) + q_k g_k^* (\theta_2) \right]
\]  
(7.2.5)

and

\[
P(T_B > t) = \sum_{k=0}^{\infty} \left[ W_k(t) - W_{k+1}(t) \right] \left[ p_k h_k^* (\theta_3 + \mu_2) + q_k h_k^* (\theta_4) \right]
\]  
(7.2.6)

On simplification of (7.2.5) and (7.2.6) as in chapter 2, we get

\[
P(T_A > t) = \sum_{k=1}^{\infty} F_k(t) \left\{ p_k g_k^* (\theta_1 + \mu_1) \left[ 1 - g_k^* (\theta_1 + \mu_1) \right] + q_k g_k^* (\theta_4) \left[ 1 - g_k^* (\theta_4) \right] \right\}
\]  
(7.2.7)

and

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\[
P(T_B > t) = 1 - \left[ \sum_{k=1}^{\infty} W_k(t) \left\{ p_2 h^*(\theta_3 + \mu_2)[1 - h^*(\theta_3 + \mu_2)] + q_2 h^*(\theta_4)[1 - h^*(\theta_4)] \right\} \right]
\]

From (7.2.7) and (7.2.8)

\[
S(t) = S_{T_T}(t) S_{T_B}(t)
\]

Since

\[
S(t) = p(T_A > t) P(T_B > t)
\]

\[
= 1 - p_2 \sum_{k=1}^{\infty} w_k(t) (h^*(\theta_3 + \mu_2))^{k-1} [1 - h^*(\theta_3 + \lambda \mu)] - q_2 \sum_{k=1}^{\infty} w_k(t) (h^*(\theta_4))^{k-1} [1 - h^*(\theta_4)]
\]

\[
-p_1 \sum_{k=1}^{\infty} F_k(t) (g^*(\theta_1 + \mu_1))^{k-1} [1 - g^*(\theta_1 + \lambda \mu)] - q_1 \sum_{k=1}^{\infty} F_k(t) (g^*(\theta_2))^{k-1} [1 - g^*(\theta_2)]
\]

\[
+p_1 p_2 \sum_{k=1}^{\infty} F_k(t) (g^*(\theta_1 + \mu_1))^{k-1} [1 - g^*(\theta_1 + \lambda \mu)] x \sum_{k=1}^{\infty} w_k(t) (h^*(\theta_3 + \mu_2))^{k-1} [1 - h^*(\theta_3 + \mu_2)]
\]

\[
+p_1 q_2 \sum_{k=1}^{\infty} F_k(t) (g^*(\theta_1 + \mu_1))^{k-1} [1 - g^*(\theta_1 + \lambda \mu)] \sum_{k=1}^{\infty} w_k(t) (h^*(\theta_4))^{k-1} [1 - h^*(\theta_4)]^{k-1}
\]

\[
+q_1 p_2 \sum_{k=1}^{\infty} F_k(t) (g^*(\theta_2))^{k-1} [1 - g^*(\theta_2)] x \sum_{k=1}^{\infty} w_k(t) (h^*(\theta_3 + \mu_2))^{k-1} (1 - h^*(\theta_2 + \mu_2))
\]

\[
+q_1 q_2 \sum_{k=1}^{\infty} F_k(t) (g^*(\theta_2))^{k-1} [1 - g^*(\theta_2)] x \sum_{k=1}^{\infty} w_k(t) (h^*(\theta_4))^{k-1}(1 - h^*(\theta_4))
\]
Now

$L(t) = 1 - S(t)$

\begin{align*}
&= p_1 \sum_{k=1}^{\infty} F_k(t) \left( g^*(\theta_1 + \mu_k) \right)^{k-1} \left[ 1 - g^*(\theta_1 + \mu_k) \right] + q_1 \sum_{k=1}^{\infty} F_k(t) \left( g^*(\theta_2) \right)^{k-1} \left[ 1 - g^*(\theta_2) \right] \\
&\quad + p_2 \sum_{k=1}^{\infty} W_k(t) \left( h^*(\theta_3 + \mu_2) \right)^{k-1} \left[ 1 - h^*(\theta_3 + \mu_2) \right] + q_2 \sum_{k=1}^{\infty} W_k(t) \left( h^*(\theta_4) \right)^{k-1} \left[ 1 - h^*(\theta_4) \right]
\end{align*}

\begin{align*}
&= -p_1 p_2 \sum_{k=1}^{\infty} F_k(t) \left( g^*(\theta_1 + \mu_1) \right)^{k-1} \left[ 1 - g^*(\theta_1 + \mu_1) \right] \sum_{k=1}^{\infty} W_k(t) \left( h^*(\theta_3 + \mu_2) \right)^{k-1} \left[ 1 - h^*(\theta_3 + \mu_2) \right] \\
&\quad - p_1 q_2 \sum_{k=1}^{\infty} F_k(t) \left( g^*(\theta_1 + \mu_1) \right)^{k-1} \left[ 1 - g^*(\theta_1 + \mu_1) \right] \sum_{k=1}^{\infty} W_k(t) \left( h^*(\theta_4) \right)^{k-1} \left[ 1 - h^*(\theta_4) \right] \\
&\quad - p_2 q_2 \sum_{k=1}^{\infty} F_k(t) \left( g^*(\theta_2) \right)^{k-1} \left[ 1 - g^*(\theta_2) \right] \sum_{k=1}^{\infty} W_k(t) \left( h^*(\theta_3 + \mu_2) \right)^{k-1} \left[ 1 - h^*(\theta_3 + \mu_2) \right] \\
&\quad - q_1 q_2 \sum_{k=1}^{\infty} F_k(t) \left( g^*(\theta_2) \right)^{k-1} \left[ 1 - g^*(\theta_2) \right] \sum_{k=1}^{\infty} W_k(t) \left( h^*(\theta_4) \right)^{k-1} \left[ 1 - h^*(\theta_4) \right]
\end{align*}

\begin{align*}
\text{(7.2.10)}
\end{align*}

Differentiating (7.2.10) with respect to ‘t’ we get
\[ l(t) = p_1 \sum_{k=1}^{\infty} f_k(t) \left( g^*(\theta_1 + \mu_i) \right)^{k-1} [1 - g^*(\theta_1 + \mu_i)] + q_1 \sum_{k=1}^{\infty} f_k(t) \left( g^*(\theta_2) \right)^{k-1} (1 - g^*(\theta_2)) \]

\[ + p_2 \sum_{k=1}^{\infty} w_k(t) \left( h^*(\theta_3 + \mu_2) \right)^{k-1} [1 - h^*(\theta_3 + \mu_2)] + q_2 \sum_{k=1}^{\infty} w_k(t) \left( h^*(\theta_4) \right)^{k-1} (1 - h^*(\theta_4)) \]

\[ - p_1 p_2 (1 - g^*(\theta_1 + \mu_i))(1 - h^*(\theta_3 + \mu_2)) \]

\[ \left\{ \sum_{k=1}^{\infty} F_k(t) \left( g^*(\theta_1 + \mu_1) \right)^{k-1} \sum_{k=1}^{\infty} w_k(t) \left( h^*(\theta_3 + \mu_2) \right)^{k-1} \right\} \]

\[ - p_1 q_2 \left[ 1 - g^*(\theta_1 + \mu_i) \right] \left[ 1 - h^*(\theta_3) \right] \left\{ \sum_{k=1}^{\infty} F_k(t) \left( g^*(\theta_1 + \mu_1) \right)^{k-1} \sum_{k=1}^{\infty} w_k(t) \left( h^*(\theta_4) \right)^{k-1} \right\} + \]

\[ + \sum_{k=1}^{\infty} f_k(t) \left( g^*(\theta_1 + \mu_1) \right)^{k-1} \sum_{k=1}^{\infty} w_k(t) \left( h^*(\theta_4) \right)^{k-1} \]
SPECIAL CASE

Suppose \( f(.) \) follows exponential distribution with parameter \( \alpha_1 \) and \( w(.) \) follows exponential distribution with parameter \( \alpha_2 \). Therefore \( f_k(t) \) follows gamma distribution with parameter \( \alpha_1 \) and \( w_k(t) \) follows gamma distribution with parameter \( \alpha_2 \).

\[
\begin{align*}
  f_k(t) &= \frac{\alpha_1^k e^{-\alpha_1 t} t^{k-1}}{(k-1)!} \\
  \text{(i.e)} \\
  w_k(t) &= \frac{\alpha_2^k e^{-\alpha_2 t} t^{k-1}}{(k-1)!} 
\end{align*}
\]

(7.2.12)

\[
\sum_{k=1}^{\infty} f_k(t) (g^*(\theta_2))^{k-1} = \alpha_1 e^{-\alpha_1 t}(1 - g^*(\theta_2))
\]

and

\[
\sum_{k=1}^{\infty} w_k(t) (h^*(\theta_4))^{k-1} = \alpha_2 e^{-\alpha_2 t}(1 - h^*(\theta_4))
\]

(7.2.13)

Now taking Laplace transform on both sides of (7.2.11) and using (7.2.12) and (7.2.13)

\[
l^*(s) = p_1 \left[ 1 - g^*(\theta_1 + \mu_1) \right] \sum_{k=1}^{\infty} f^*(s)^k (g^*(\theta_1 + \mu_1))^{k-1} + q_1 \left[ 1 - g^*(\theta_2) \right] \sum_{k=1}^{\infty} f^*(s)^k (g^*(\theta_2))^{k-1}
\]
\[
\begin{align*}
+ p_2 \left[ 1 - g^*(\theta_3 - \mu_2) \right] \sum_{k=1}^{\infty} w^*(s)^k \left( h^*(\theta_3 + \mu_2) \right)^{k-1} + q_2 \left[ 1 - h^*(\theta_4) \right] \sum_{k=1}^{\infty} w^*(s)^k \left( h^*(\theta_4) \right)^{k-1}
\end{align*}
\]

\[
- p_1 p_2 \left[ 1 - g^*(\theta_1 + \mu_1) \right] \left[ 1 - h^*(\theta_3 + \mu_2) \right] \sum_{k=1}^{\infty} \left( g^*(\theta_1 + \mu_1) \right)^{k-1} \int_{0}^{\infty} F_k(t) \alpha_2 e^{-\alpha_2 t \left( 1 - h^*(\theta_3 + \mu_2) \right)} dt
\]

\[
+ \sum_{k=1}^{\infty} h^*(\theta_3 + \mu_2)^{k-1} \int_{0}^{\infty} W_k(t) \alpha_1 e^{-\alpha_1 t \left( 1 - g^*(\theta_1 + \mu_1) \right)} dt \right\}
\]

\[
- p_1 q_2 \left[ 1 - g^*(\theta_1 + \mu_1) \right] \left[ 1 - h^*(\theta_4) \right] \sum_{k=1}^{\infty} \left( g^*(\theta_1 + \mu_1) \right)^{k-1} \int_{0}^{\infty} F_k(t) \alpha_2 e^{-\alpha_2 t \left( 1 - h^*(\theta_4) \right)} dt
\]

\[
+ \sum_{k=1}^{\infty} h^*(\theta_4)^{k-1} \int_{0}^{\infty} W_k(t) \alpha_1 e^{-\alpha_1 t \left( 1 - g^*(\theta_1 + \mu_1) \right)} dt \right\}
\]

\[
- q_1 p_2 \left[ 1 - g^*(\theta_2) \right] \left[ 1 - h^*(\theta_3 + \mu_2) \right] \sum_{k=1}^{\infty} g^*(\theta_2)^{k-1} \int_{0}^{\infty} F_k(t) \alpha_2 e^{-\alpha_2 t \left( 1 - h^*(\theta_3 + \mu_2) \right)} dt
\]

\[
+ \sum_{k=1}^{\infty} h^*(\theta_3 + \mu_2)^{k-1} \int_{0}^{\infty} W_k(t) \alpha_1 e^{-\alpha_1 t \left( 1 - g^*(\theta_2) \right)} dt \right\}
\]

\[
- q_1 q_2 \left[ 1 - g^*(\theta_2) \right] \left[ 1 - h^*(\theta_4) \right] \sum_{k=1}^{\infty} g^*(\theta_2)^{k-1} \int_{0}^{\infty} F_k(t) \alpha_2 e^{-\alpha_2 t \left( 1 - h^*(\theta_4) \right)} dt
\]

\[
+ \sum_{k=1}^{\infty} g^*(\theta_4)^{k-1} \int_{0}^{\infty} W_k(t) \alpha_1 e^{-\alpha_1 t \left( 1 - g^*(\theta_2) \right)} dt \right\} \quad (7.2.14)
\]

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Using the result \[ 1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x}, \] from (7.2.14), one can write

\[
I*(s) = \frac{p_1(1-a_3)f*(s)}{1-f*(s)a_3} + \frac{q_1(1-a_2)f*(s)}{1-f*(s)a_2} + \frac{p_2(1-a_4)w*(s)}{1-w*(s)a_4} + \frac{q_3(1-a_3)w*(s)}{1-f*(s)a_5}
\]

\[-p_1p_2(1-a_3)(1-a_4) \left\{ \mu_2 \sum_{i=1}^{n} a_4^{k-1} \left( \frac{w*(b_1)}{b_1} \right)^k + \mu_2 \sum_{k=1}^{\infty} a_4^{k-1} \left( \frac{w*(b_1)}{b_1} \right)^k \right\}
\]

\[-p_1q_2(1-a_3)(1-a_5) \left\{ \mu_2 \sum_{k=1}^{\infty} a_3^{k-1} \left( \frac{w*(b_1)}{b_1} \right)^k \right\}
\]

\[-q_1p_2(1-a_2)(1-a_4) \left\{ \mu_2 \sum_{k=1}^{\infty} a_2^{k-1} \left( \frac{w*(b_2)}{b_2} \right)^k \right\}
\]

\[-q_1q_2(1-a_2)(1-a_5) \left\{ \mu_2 \sum_{k=1}^{\infty} a_2^{k-1} \left( \frac{w*(b_2)}{b_2} \right)^k \right\} \quad (7.2.15)
\]

where

\[ c_1 = h^*(\theta_1 + \mu_2), \quad b_1 = h^*(\theta_1 + \mu_2), \quad c_2 = g^*(\theta_2), \quad b_2 = 1 - g^*(\theta_2), \quad c_3 = g^*(\theta_1 + \mu_4), \]

\[ c_4 = h^*(\theta_3 + \mu_2), \quad c_5 = h^*(\theta_4), \quad (7.2.16)
\]

From (7.2.15) and (7.2.16) one can write
\[ l^*(s) = \frac{p(1-c_3)f^*(s)}{1-c_3f^*(s)} + \frac{q(1-c_2)f^*(s)}{1-c_2f^*(s)} + \frac{p(1-c_4)w^*(s)}{1-c_4w^*(s)} + \frac{q(1-c_5)w^*(s)}{1-c_5w^*(s)} \]

\[-p_1p_2(1-c_3)(1-c_4) \left\{ \frac{\alpha f^*(\tilde{c}_1)}{\tilde{c}_1[1-f^*(\tilde{c}_1)c_3]} + \frac{\alpha w^*(\tilde{b}_1)}{\tilde{b}_1[1-w^*(\tilde{b}_1)c_4]} \right\} \]

\[-p_1q_2(1-c_3)(1-c_5) \left\{ \frac{\alpha f^*(\tilde{c}_2)}{\tilde{c}_2[1-f^*(\tilde{c}_2)c_3]} + \frac{\alpha w^*(\tilde{b}_1)}{\tilde{b}_1[1-w^*(\tilde{b}_1)c_5]} \right\} \]

\[-q_1p_2(1-c_2)(1-c_4) \left\{ \frac{\alpha f^*(\tilde{c}_2)}{\tilde{c}_2[1-f^*(\tilde{c}_2)c_2]} + \frac{\alpha w^*(\tilde{b}_2)}{\tilde{b}_2[1-w^*(\tilde{b}_2)c_4]} \right\} \]

\[-q_1q_2(1-c_2)(1-c_5) \left\{ \frac{\alpha f^*(\tilde{c}_2)}{\tilde{c}_2[1-f^*(\tilde{c}_2)c_2]} + \frac{\alpha w^*(\tilde{b}_2)}{\tilde{b}_2[1-w^*(\tilde{b}_2)c_5]} \right\} \]

(7.2.17)

Suppose \( g(.) \) and \( h(.) \) follows exponential distribution with parameters \( \lambda_1 \) and \( \lambda_2 \) respectively. In this case

\[ g(s) = \frac{\lambda_1}{\lambda_1 + s} \text{ and } h(s) = \frac{\lambda_2}{\lambda_2 + s} \]

\[ 1-c_2 = 1-g^*(\theta_2) = \frac{\theta_2}{\lambda_1 + \theta_2}; \quad 1-c_3 = 1-g^*(\theta_1 + \mu_1) = \frac{\theta_1 + \mu_1}{\lambda_1 + \theta_1 + \mu_1} \]

\[ 1-c_4 = 1-h^*(\theta_3 + \mu_2) = \frac{\theta_3 + \mu_2}{\theta_3 + \mu_2 + \lambda_2}; \quad 1-c_5 = 1-h^*(\theta_4) = \frac{\theta_4}{\lambda_2 + \theta_4} \]

(7.2.18)
Since
\[
E(T) = -\frac{d}{ds} \left[ I^*(s) \right]_{s=0} \tag{7.2.19}
\]

Using (7.2.18) in (7.2.19) we get

\[
E(T) = \left( \frac{p_1}{\alpha_1} \right) \left( 1 + \frac{\lambda_1}{\theta_1 + \mu_1} \right) + \left( \frac{q_1}{\alpha_1} \right) \left( 1 - \frac{\lambda_1}{\theta_2} \right) + \left( \frac{p_2}{\alpha_2} \right) \left( 1 + \frac{\lambda_2}{\theta_3 + \mu_2} \right) + \left( \frac{q_2}{\alpha_2} \right) \left( 1 - \frac{\lambda_2}{\theta_4} \right)
\]

\[
\begin{align*}
- p_1 q_2 & \left( \frac{\theta_1 + \mu_1}{\lambda_1 + \mu_1 + \theta_1} \right) \left( \frac{\theta_4}{\lambda_2 + \theta_4} \right)^2 - \alpha_1 \alpha_2 \left( \frac{\theta_1 + \mu_1}{\lambda_1 + \theta_1 + \mu_1} \right) \left( \frac{\theta_3 + \mu_2}{\lambda_2 + \theta_3 + \mu_2} \right) \\
\alpha_1 & \alpha_2 \left( \frac{\theta_2}{\lambda_1 + \theta_2} \right) \left( \frac{\theta_3 + \mu_2}{\lambda_2 + \theta_3 + \mu_2} \right)^2 - \alpha_1 \alpha_2 \left( \frac{\theta_2}{\lambda_1 + \theta_2} \right) \left( \frac{\theta_1 + \mu_2}{\lambda_2 + \theta_1 + \mu_2} \right) \\
- q_1 p_2 & \left( \frac{\theta_2}{\lambda_1 + \theta_2} \right) \left( \frac{\theta_3 + \mu_2}{\lambda_2 + \theta_3 + \mu_2} \right)^2 - \alpha_1 \alpha_2 \left( \frac{\theta_2}{\lambda_1 + \theta_2} \right) \left( \frac{\theta_1 + \mu_2}{\lambda_2 + \theta_1 + \mu_2} \right)
\end{align*}
\]

\[
\begin{align*}
- q_1 q_2 & \left( \frac{\theta_2}{\lambda_1 + \theta_2} \right) \left( \frac{\theta_4}{\lambda_2 + \theta_4} \right)^2 - \alpha_1 \alpha_2 \left( \frac{\theta_2}{\lambda_1 + \theta_2} \right) \left( \frac{\theta_4}{\lambda_2 + \theta_4} \right)
\end{align*}
\]

\[
\alpha_1 \alpha_2 \left( \frac{\theta_2}{\lambda_1 + \theta_2} \right) \left( \frac{\theta_4}{\lambda_2 + \theta_4} \right)^2 - \alpha_1 \alpha_2 \left( \frac{\theta_2}{\lambda_1 + \theta_2} \right) \left( \frac{\theta_4}{\lambda_2 + \theta_4} \right)
\]

\[
(7.2.20)
\]

Equation (7.2.20) gives the mean time for recruitment.
7.3 NUMERICAL ILLUSTRATION

In this section, the analytical expression obtained in (7.2.20) is numerically illustrated and relevant conclusions are made.

Case (i):

Fixing $\theta_1 = 0.6; \theta_2 = 0.7; \theta_3 = 0.8; \theta_4 = 0.9; \mu_1 = 0.5; \mu_2 = 1; \alpha_1 = 0.1; \alpha_2 = 0.2$

and varying $\lambda_1$ and $\lambda_2$, the values of E(T) are computed and tabulated in Table 7.3.1

<table>
<thead>
<tr>
<th>$\lambda_2$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>20.0209</td>
<td>10.8922</td>
<td>8.4613</td>
<td>7.3821</td>
<td>6.7823</td>
</tr>
<tr>
<td>1.0</td>
<td>23.2527</td>
<td>11.7303</td>
<td>8.5370</td>
<td>7.0813</td>
<td>6.2568</td>
</tr>
<tr>
<td>1.5</td>
<td>27.1211</td>
<td>13.2108</td>
<td>9.3235</td>
<td>7.5420</td>
<td>6.5292</td>
</tr>
<tr>
<td>2.0</td>
<td>31.1315</td>
<td>14.8342</td>
<td>10.2670</td>
<td>8.1701</td>
<td>6.9769</td>
</tr>
<tr>
<td>2.5</td>
<td>35.1976</td>
<td>16.5133</td>
<td>11.2715</td>
<td>8.8634</td>
<td>7.4922</td>
</tr>
</tbody>
</table>
CONCLUSION:

From the above table, we make the following observations.

(i) The mean time for recruitment increases as $\lambda_1$ increases, keeping other parameters fixed. In other words, when the average loss of manhours (grade A) increases, the mean time for recruitment increases.

(ii) The mean time for recruitment decreases as $\lambda_2$ increases, keeping other parameters fixed. In other words, when the average loss of manhours (grade B) increases, the mean time for recruitment increases.

(iii) The mean time for recruitment decreases as $\lambda_1$ and $\lambda_2$ increases simultaneously, keeping other parameters fixed.

Case(ii):

Fixing $\theta_1 = 0.6; \theta_2 = 0.7; \theta_3 = 0.8; \theta_4 = 0.4; \mu_1 = 0.5; \mu_2 = 1; \lambda_2 = 0.1; \alpha_2 = 0.2$ and varying $\lambda_1$ and $\alpha_1$ the values of $E(T)$ are computed and tabulated in Table 7.3.2.
Table 7.3.2

\[ E(T) \]

<table>
<thead>
<tr>
<th>( \lambda_i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>9.2382</td>
<td>9.0756</td>
<td>9.4110</td>
<td>9.8952</td>
<td>10.4444</td>
</tr>
<tr>
<td>1.0</td>
<td>12.4734</td>
<td>10.5856</td>
<td>10.0565</td>
<td>9.9196</td>
<td>9.9459</td>
</tr>
<tr>
<td>1.5</td>
<td>18.3014</td>
<td>13.9465</td>
<td>12.5081</td>
<td>11.8642</td>
<td>11.5507</td>
</tr>
<tr>
<td>2.5</td>
<td>35.7339</td>
<td>23.6937</td>
<td>19.6305</td>
<td>17.6460</td>
<td>16.5078</td>
</tr>
</tbody>
</table>

**CONCLUSION:**

From the above table, we make the following observations.

(i) The mean time for recruitment increases as \( \lambda_i \) increases, keeping other parameters fixed. In other words, when the average loss of manhours (grade A) increases, the mean time for recruitment increases.

(ii) The mean time for recruitment increases as \( \alpha_i \) increases, keeping other parameters fixed. In other
words, when the average inter-decision time (grade A) increases, the mean time for recruitment increases.

(iii) The mean time for recruitment increases as $\lambda_i$ and $\alpha_i$ increases simultaneously, keeping other parameters fixed.

Case(iii):

Fixing $\theta_1 = 0.6; \theta_2 = 0.7; \theta_3 = 0.8; \theta_4 = 0.4; \mu_1 = 0.5; \mu_2 = 1; \alpha_1 = 0.2; \lambda_2 = 0.1$

and varying $\lambda_1$ and $\alpha_2$ the values of $E(T)$ are computed and tabulated in Table 7.3.3
Table 7.3.3

$E(T)$

<table>
<thead>
<tr>
<th>$\lambda_1$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>26.2293</td>
<td>28.8410</td>
<td>33.3494</td>
<td>38.3154</td>
<td>43.4631</td>
</tr>
<tr>
<td>1.0</td>
<td>59.5031</td>
<td>66.8563</td>
<td>78.4142</td>
<td>90.9761</td>
<td>103.9346</td>
</tr>
<tr>
<td>1.5</td>
<td>113.3289</td>
<td>129.4382</td>
<td>154.1692</td>
<td>178.7090</td>
<td>204.9610</td>
</tr>
<tr>
<td>2.0</td>
<td>193.2411</td>
<td>222.9520</td>
<td>265.2607</td>
<td>310.5444</td>
<td>356.9964</td>
</tr>
<tr>
<td>2.5</td>
<td>304.5787</td>
<td>353.6675</td>
<td>422.27</td>
<td>495.4627</td>
<td>570.4542</td>
</tr>
</tbody>
</table>

**CONCLUSION:**

From the above table, we make the following observations.

(i) The mean time for recruitment increases as $\lambda_1$ increases, keeping other parameters fixed. In other words, when the average loss of manhours (grade A) increases, the mean time for recruitment increases.

(ii) The mean time for recruitment increases as $\alpha_2$ increases, keeping other parameters fixed. In other words, when the average inter-decision time (grade B) increases, the mean time for recruitment increases.
(iii) The meantime for recruitment increases as $\lambda_1$ and $\alpha_2$ increases simultaneously, keeping other parameters fixed.

**Case (iv):**

Fixing $\theta_1 = 0.6; \theta_2 = 0.7; \theta_3 = 0.8; \theta_4 = 0.4; \mu_1 = 0.5; \mu_2 = 1; \alpha_2 = 0.2; \lambda_4 = 0.3$ and varying $\lambda_2$ and $\alpha_1$, the values of $E(T)$ are computed and tabulated in **Table 7.3.4**

**Table 7.3.4**

<table>
<thead>
<tr>
<th>$\lambda_2$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>7.2166</td>
<td>6.1299</td>
<td>5.6756</td>
<td>5.4268</td>
<td>5.2698</td>
</tr>
<tr>
<td>1.0</td>
<td>8.6780</td>
<td>5.8069</td>
<td>4.6187</td>
<td>3.9704</td>
<td>3.5625</td>
</tr>
<tr>
<td>1.5</td>
<td>13.2541</td>
<td>7.5014</td>
<td>5.1488</td>
<td>3.8723</td>
<td>3.0717</td>
</tr>
<tr>
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<td>20.7433</td>
<td>11.1102</td>
<td>7.1975</td>
<td>5.0817</td>
<td>3.7573</td>
</tr>
<tr>
<td>2.5</td>
<td>31.1359</td>
<td>16.6288</td>
<td>10.7622</td>
<td>7.5966</td>
<td>5.6176</td>
</tr>
</tbody>
</table>
CONCLUSION:

From the above table, we make the following observations.

(i) The mean time for recruitment decreases as $\lambda_2$ increases, keeping other parameters fixed. In other words, when the average loss of manhours increases (grade B), the mean time for recruitment decreases.

(ii) The mean time for recruitment increases as $\alpha_1$ increases, keeping other parameters fixed. In other words, when the average inter-decision time (grade A), the mean time for recruitment increases.

Case(v):

Fixing and varying $\lambda_2$ and $\alpha_2$ the values of $E(T)$ are computed and tabulated in Table 7.3.5

<table>
<thead>
<tr>
<th>$\alpha_2$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>15.3861</td>
<td>11.2003</td>
<td>9.6781</td>
<td>8.9010</td>
<td>8.4316</td>
</tr>
<tr>
<td>1.0</td>
<td>24.1083</td>
<td>15.1027</td>
<td>12.1036</td>
<td>10.6408</td>
<td>9.7817</td>
</tr>
<tr>
<td>1.5</td>
<td>34.4618</td>
<td>19.1910</td>
<td>14.4895</td>
<td>12.2893</td>
<td>11.0298</td>
</tr>
<tr>
<td>2.0</td>
<td>47.2237</td>
<td>23.7824</td>
<td>17.0418</td>
<td>14.0023</td>
<td>12.3022</td>
</tr>
<tr>
<td>2.5</td>
<td>62.8429</td>
<td>28.9946</td>
<td>19.8226</td>
<td>15.8226</td>
<td>13.6321</td>
</tr>
</tbody>
</table>
CONCLUSION:

From the above table, we make the following observations.

(i) The mean time for recruitment decreases as $\lambda_2$ increases, keeping other parameters fixed. In other words, when the average loss of manhours (grade B) increases, the mean time for recruitment decreases.

(ii) The mean time for recruitment increases as $\alpha_2$ increases, keeping other parameters fixed. In other words, when the inter-decision time (grade B) increases on the average, the mean time for recruitment increases.

(iii) The meantime for recruitment decreases as $\alpha_2$ and $\lambda_2$ increases simultaneously, keeping other parameters fixed.

Case(vi):

Fixing $\theta_1 = 0.6; \theta_2 = 0.7; \theta_3 = 0.8; \theta_4 = 0.4; \mu_1 = 0.5; \mu_2 = 1; \lambda_1 = 0.3; \lambda_2 = 0.2$ and varying $\alpha_1$ and $\alpha_2$ the values of $E(T)$ are computed and tabulated in Table 7.3.6
Table 7.3.6

<table>
<thead>
<tr>
<th>( \alpha_2 )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>38.5864</td>
<td>80.9401</td>
<td>143.8968</td>
<td>234.6250</td>
<td>359.8169</td>
</tr>
<tr>
<td>1</td>
<td>51.2270</td>
<td>91.0902</td>
<td>135.4119</td>
<td>185.5977</td>
<td>242.5215</td>
</tr>
<tr>
<td>1.5</td>
<td>81.8885</td>
<td>148.9027</td>
<td>230.3990</td>
<td>331.1265</td>
<td>455.2702</td>
</tr>
<tr>
<td>2.0</td>
<td>126.1658</td>
<td>239.7910</td>
<td>194.8183</td>
<td>605.4131</td>
<td>885.1841</td>
</tr>
<tr>
<td>2.5</td>
<td>183.4545</td>
<td>362.5939</td>
<td>627.1059</td>
<td>1.0068e+003</td>
<td>1.5311e+003</td>
</tr>
</tbody>
</table>

**CONCLUSIONS:**

From the above table, we make the following observations.

(i) The mean time for recruitment increases as \( \alpha_2 \) increases, keeping other parameters fixed. In other words, when the average inter-decision time (grade B) increases, the mean time for recruitment increases.

(ii) The mean time for recruitment increases as \( \alpha_1 \) increases, keeping other parameters fixed. In other words, when the average inter-decision time (grade A) increases on the average, the mean time for recruitment increases.
(iii) The meantime for recruitment increases as $\alpha_2$ and $\alpha_1$ increases simultaneously, keeping other parameters fixed.