CHAPTER 6

EXPECTED TIME FOR RECRUITMENT IN A TWO-GRADED MANPOWER SYSTEM HAVING DIFFERENT RENEWAL PROCESS FOR INTER-DECISION TIMES

6.1 INTRODUCTION

For a two graded manpower system, Sathiymoorthy and Parthasarathy (2002) have obtained the mean and variance of the time for recruitment when (i) loss of manpower is a continuous random variable (ii) threshold for loss of manpower for the two grades are continuous random variable and the inter-decision times for both the grades form the same renewal process. Kasthurri and Srinivasan (2005(a)) have obtained results for different forms of the threshold when the inter-decision times are constantly correlated exchangeable exponential random variables.

In this chapter, an organization with two grades is considered and each grade has its own threshold level. Recruitment is made by the organization whenever the threshold crossing takes place in any one of the grades.
It is assumed that the policy decisions are governed by different renewal processes. The objective of this chapter is to obtain the mean and variance of time to recruitment in an organization having two grades when (i) the loss of manpower and threshold are discrete random variables and (ii) the inter-decision times for the two grades form two different renewal process. This chapter is organised as follows: In section 6.2, description of the model is given and the analytical expression for mean time to recruitment is obtained. In section 6.3, the results are numerically illustrated and relevant conclusions are made.

6.2 DESCRIPTION AND ANALYSIS OF THE MODEL

In this section a two graded manpower system is considered and the description of the model is given below:

Assumptions:

1. An organization having two grades (grades A and B) takes policy decisions at random epochs in $[0, \infty)$ and at every decision making epoch, a random number of persons quit the organization.

2. There is an associated loss of manpower to the organization if a person quits and it is linear and cumulative.
3. Each grade has its individual independent threshold. If the total number of persons who leave the organization crosses the minimum of the two thresholds, recruitment becomes necessary. In other words, recruitment is made whenever the loss of manhours in the two grades crosses any one of the thresholds.

4. Mobility of manpower from one grade to the other grade is not allowed.

**Notations:**

- $X_{iA}$: continuous random variable representing the random amount of manpower depletion in grade A due to $i^{th}$ decision making epoch, $i=1,2,...$. $X_{iA}$’s are independent and identically distributed random variables.
- $X_{iB}$: continuous random variable representing the random amount of manpower depletion in grade B due to $i^{th}$ decision making epoch, $i=1,2,...$. $X_{iB}$’s are independent and identically distributed random variables.
- $Y_A, Y_B$: discrete random variables denoting the threshold levels for grades A and B respectively, $Y_A$ and $Y_B$ follows geometric distributions with parameters $\theta_1$ and $\theta_2$ respectively.
- $Y$: $Y=\min (Y_A, Y_B)$
- $T_A, T_B$: discrete random variable denoting time to recruitment for grades A and B respectively.
- $T$: $T = \min (T_A, T_B)$, time to recruitment.
\( S_{r_a}(t) \) : survival function of \( T_A \) for grade A

\( S_{r_b}(t) \) : survival function of \( T_B \) for grade B

\( L(t) \) : cumulative distribution function of \( T \)

\( F(.) \) : cumulative distribution function of the inter-decision times for grade A

\( W(.) \) : cumulative distribution function of the inter-decision times for grade B

\( H(.) \) : cumulative distribution function of \( Y = \min (Y_A, Y_B) \)

\( S(t) \) : Survival function of \( T \) and \( S(t) = S_{r_a}(t) S_{r_b}(t) \)

\( F_k(.) \) : \( k \)-fold convolution of \( F(.) \)

\( W_k(.) \) : \( k \)-fold convolution of \( W(.) \)

\( l(t) \) : probability density function of \( T \)

\( l^*(s) \) : Laplace transform of \( l(t) \).

\( g(.) \) : probability density function of \( X_{iA} \)

\( g_k(.) \) : \( k \)-fold convolution of \( g(.) \)

\( h(.) \) : probability density function of \( X_{iB} \)

\( h_k(.) \) : \( k \)-fold convolution of \( h(.) \)

\( f(.) \) : probability density function of the inter-decision times for grade A.

\( w(.) \) : probability density function of the inter-decision times for grade B

\( f_k(.) \) : \( k \)-fold convolution of \( f(.) \)

\( w_k(.) \) : \( k \)-fold convolution of \( w(.) \)

\( E(T) \) : mean time for recruitment
**MAIN RESULTS**

In this subsection an analytical expression for the meantime to recruitment is obtained.

\[ S_{T_A}(t) = P(T_A > t) = \sum_{k=0}^{\infty} \{\text{probability that there are exactly } k \text{ decisions in } (0, t] \} \times \{\text{probability that grade A is sustained with the cumulative loss of manpower} \} \]

\[ S_{T_B}(t) = P(T_B > t) = \sum_{k=0}^{\infty} \{\text{probability that there are exactly } k \text{ decisions in } (0, t] \} \times \{\text{probability that grade B is sustained with the cumulative loss of manpower} \} \]

\[ (i.e) P(T_A > t) = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] P\left[ \sum_{i=1}^{k} X_{iA} < Y_A \right] \]  \hspace{1cm} (6.2.1)

\[ P(T_B > t) = \sum_{k=0}^{\infty} [W_k(t) - W_{k+1}(t)] P\left[ \sum_{i=1}^{k} X_{iB} < Y_B \right] \]  \hspace{1cm} (6.2.2)

Since \( Y_A \) and \( Y_B \) follows geometric distributions with the parameters \( \theta_1 \) and \( \theta_2 \) respectively, the distribution functions are given by

\[ H_A(x) = \theta_1 \bar{\theta}_i, \hspace{1cm} \bar{\theta}_i = 1 - \theta_i \]

and
\[ H_B(x) = \theta_2 \bar{\theta}_2, \quad \bar{\theta}_2 = 1 - \theta_2 \] (6.2.3)

respectively.

We now obtain \( L_t \)

\[
P\{\text{threshold level is not crossed in the first } k \text{ decisions}\}
= P\left[ \sum_{i=1}^{k} X_i < Y \right]

Using the law of total probability, we get

\[
P\left[ \sum_{i=1}^{k} X_i < Y \right] = \sum_{n=0}^{\infty} P\left( Y_A > n / \sum_{i=1}^{k} X_{iA} = n \right) \left( \sum_{i=1}^{k} X_{iA} = n \right) P(\sum_{i=1}^{k} X_{iA} = n)
= \sum_{n=0}^{\infty} P\left( \sum_{i=1}^{k} X_{iA} = n \right) P(\sum_{i=1}^{k} X_{iA} = n)
= \sum_{n=0}^{\infty} P\left( \sum_{i=1}^{k} X_{iA} = n \right) (\bar{\theta})^n
\]

(i.e) \[
P\left[ X_{iA} < Y_A \right] = g(\bar{\theta}_i) \] (6.2.4)

\[
\therefore P\left[ \sum_{i=1}^{k} X_{iA} < Y_A \right] = g(\bar{\theta}_i)^k \] (6.2.5)

Similarly it can be shown that

\[
P\left[ \sum_{i=1}^{k} X_{iB} < Y_B \right] = h(\bar{\theta}_2)^k \] (6.2.6)
Now

\[ S_{r_2}(t) = \sum_{k=0}^{\infty} V_k(t)g(\theta_1)^k \]

\[ = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]g(\theta_1)^k \]

\[ = 1 - \sum_{k=1}^{\infty} [F_k(t)]g(\theta_1)^{k-1}\{1-g(\theta_1)\} \]  
(6.2.7)

and

\[ S_{r_2}(t) = 1 - \sum_{k=1}^{\infty} [W_k(t)]^k h(\theta_2)^{k-1}\{1-h(\theta_2)\} \]  
(6.2.8)

Since

\[ S(t) = S_{r_1}(t) - S_{r_2}(t) \]  we have

\[ S(t) = \left[ 1 - \sum_{k=1}^{\infty} [F_k(t)]g(\theta_1)^{k-1}\{1-g(\theta_1)\} \right] \left[ 1 - \sum_{k=1}^{\infty} [W_k(t)]^k h(\theta_2)^{k-1}\{1-h(\theta_2)\} \right] \]

\[ = 1 - \sum_{k=1}^{\infty} [F_k(t)]g(\theta_1)^{k-1}\{1-g(\theta_1)\} - \sum_{k=1}^{\infty} [W_k(t)]^k h(\theta_2)^{k-1}\{1-h(\theta_2)\} + \{1-g(\theta_1)\}\{1-h(\theta_2)\}\sum_{k=1}^{\infty} [F_k(t)]g(\theta_1)^{k-1}\sum_{k=1}^{\infty} [W_k(t)]^k h(\theta_2)^{k-1} \]  
(6.2.9)

\[ L(t) = 1 - S(t) \]
\[
L(t) = \sum_{k=1}^{\infty} \left[ F_k(t) g(\bar{\theta}_1)^k \right] \left\{ 1 - g(\bar{\theta}_1) \right\} + \sum_{k=1}^{\infty} \left[ W_k(t) h(\bar{\theta}_2)^k \right] \left\{ 1 - h(\bar{\theta}_2) \right\} \\
- \left\{ 1 - g(\bar{\theta}_1) \right\} \left\{ 1 - h(\bar{\theta}_2) \right\} \left[ \sum_{k=1}^{\infty} \left[ F_k(t) g(\bar{\theta}_1)^k \right] \right] \left[ \sum_{k=1}^{\infty} \left[ W_k(t) h(\bar{\theta}_2)^k \right] \right]
\]  
(6.2.10)

Differentiating (6.2.10) with respect to ‘t’ we get

\[
l(t) = \left\{ 1 - g(\bar{\theta}_1) \right\} \left[ \sum_{k=1}^{\infty} \left[ F_k(t) g(\bar{\theta}_1)^k \right] \right] \left\{ 1 - g(\bar{\theta}_1) \right\} + \left\{ 1 - h(\bar{\theta}_2) \right\} \left[ \sum_{k=1}^{\infty} \left[ W_k(t) h(\bar{\theta}_2)^k \right] \right] \\
- \left\{ 1 - g(\bar{\theta}_1) \right\} \left\{ 1 - h(\bar{\theta}_2) \right\} \left[ \sum_{k=1}^{\infty} \left[ F_k(t) g(\bar{\theta}_1)^k \right] \right] \left[ \sum_{k=1}^{\infty} \left[ W_k(t) h(\bar{\theta}_2)^k \right] \right] \\
- \left\{ 1 - g(\bar{\theta}_1) \right\} \left\{ 1 - h(\bar{\theta}_2) \right\} \left[ \sum_{k=1}^{\infty} \left[ F_k(t) g(\bar{\theta}_1)^k \right] \right] \left[ \sum_{k=1}^{\infty} \left[ W_k(t) h(\bar{\theta}_2)^k \right] \right] \\
\]  
(6.2.11)

By taking Laplace transform for (6.2.11) we get \( l*(s) \)

Clearly \[
E(T) = \frac{-d}{ds} \left[ l*(s) \right]_{s=0},
\]  
(6.2.12)

Using \( l*(s) \) in (6.2.12), mean time for recruitment can be obtained.

**SPECIAL CASE**

Suppose \( f(.) \) follows exponential distribution with parameter \( \alpha_1 \) and \( w(.) \) follows exponential distribution with parameter \( \alpha_2 \).

\[
\therefore f_k(t) \text{ follows gamma distribution with parameter } \alpha_1 \text{ and } w_k(t) \text{ follows gamma distribution with parameter } \alpha_2.
\]
\[ f_k(t) = \frac{\alpha_t^{k}e^{-\alpha_t t}k^{-1}}{(k-1)!} \]

and
\[ w_k(t) = \frac{\alpha_2^{k}e^{-\alpha_2 t}k^{-1}}{(k-1)!} \]  \hspace{1cm} (6.2.13)

\[ \sum_{k=1}^{\infty} f_k(t) (g(\bar{\theta}_1))^{k-1} = \sum_{k=1}^{\infty} \frac{\alpha_t^{k}e^{-\alpha_t t}k^{-1}}{(k-1)!} (g(\bar{\theta}_1))^{k-1} \]

\[ = \alpha_t e^{-\alpha_t (1-g(\bar{\theta}_1))} \]  \hspace{1cm} (6.2.14)

and
\[ \sum_{k=1}^{\infty} w_k(t) (h(\bar{\theta}_2))^{k-1} = \alpha_2 e^{-\alpha_2 (1-h(\bar{\theta}_2))} \]  \hspace{1cm} (6.2.15)

Suppose we write

\[ c_1 = 1 - g(\bar{\theta}_1) \quad \text{and} \quad c_2 = 1 - h(\bar{\theta}_2) \]

\[ \therefore \text{from}(6.2.11) \]

\[ l(t) = c_1 \alpha_1 e^{-\alpha_1 c_1 t} + c_2 \alpha_2 e^{-\alpha_2 c_2 t} - c_1 c_2 \left[ \sum_{k=1}^{\infty} W_k(t) (h(\bar{\theta}_2))^{k-1} \alpha_t e^{-\alpha_t t} + \sum_{k=1}^{\infty} F_k(t) (g(\bar{\theta}_1))^{k-1} \alpha_2 e^{-\alpha_2 t} \right] \]

\[ \text{(6.2.16)} \]
In (6.2.16) consider the term

$$\sum_{k=1}^{\infty} [W_k(t)h(\theta)]^k \alpha e^{-\alpha t}$$

Now

$$L\left[\sum_{k=1}^{\infty} [W_k(t)h(\theta)]^k \alpha e^{-\alpha t}\right] = \sum_{k=1}^{\infty} \alpha (h(\theta))^k \int_0^{\infty} W_k(t)e^{-st}e^{-\alpha t} dt$$

$$= \sum_{k=1}^{\infty} \alpha (h(\theta))^k \int_0^{\infty} W_k(t)e^{-(s+\alpha c_1)} dt$$

(6.2.17)

From Laplace transformation one can write

$$\int_0^{\infty} \{W_k(t)e^{-(s+\alpha c_1)}\} dt = \frac{w_k^*(s + \alpha c_1)}{s + \alpha c_1}$$

$$= \left(\frac{\alpha_2}{\alpha_2 + s}\right)^k$$

(6.2.18)

$$\therefore L\left[\sum_{k=1}^{\infty} [W_k(t)h(\theta)]^k \alpha e^{-\alpha t}\right] = \sum_{k=1}^{\infty} \alpha (h(\theta))^k \frac{w_k^*(s + \alpha c_1)}{s + \alpha c_1}$$

$$= \alpha \sum_{k=1}^{\infty} (h(\theta))^k \left[\frac{\alpha_2}{\alpha_2 + s + \alpha c_1}\right]^k$$

$$= \frac{\alpha_1 \alpha_2}{\{\alpha_2 + s + \alpha c_1\}\{s + \alpha c_1\}} \sum_{k=1}^{\infty} h(\theta)^{k-1}$$

$$= \frac{\alpha_1 \alpha_2}{\{\alpha_2 + s + \alpha c_1\}\{s + \alpha c_1\}} \sum_{k=1}^{\infty} h(\theta)^{k-1}$$

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\[
\begin{align*}
= \frac{\alpha_1 \alpha_2}{\{\alpha_2 + s + \alpha_1 \alpha_1\}} \left[ 1 - \frac{h(\theta_2) \alpha_2}{\alpha_2 + s + \alpha_1 \alpha_1} \right]^{-1} \\
= \frac{\alpha_1 \alpha_2}{\{s + \alpha_1 \alpha_1\}} \left[ \frac{\alpha_2 + s + \alpha_1 \alpha_1 - h(\theta_2) \alpha_2}{\alpha_2 + s + \alpha_1 \alpha_1} \right]^{-1} \\
= \frac{\alpha_1 \alpha_2}{\{s + \alpha_1 \alpha_1\}} \left[ \frac{1}{\alpha_2 + s + \alpha_1 \alpha_1 - h(\theta_2) \alpha_2} \right]
\end{align*}
\]

(i.e) 
\[
L \left[ \sum_{k=1}^{\infty} [W_k(t) h(\theta_2)^{k-1} \alpha_1 e^{-\alpha_1 t}] \right] = \frac{\alpha_1 \alpha_2}{\{s + \alpha_1 \alpha_1\}} \left[ \frac{1}{s + \alpha_1 \alpha_1 + \alpha_2 \alpha_2} \right]
\]

By similar computation for other terms in (6.2.16) we get

\[
L \left[ \sum_{k=1}^{\infty} [F_k(t) g(\theta_2)^{k-1} \alpha_2 e^{-\alpha_2 t}] \right] = \frac{\alpha_1 \alpha_2 c_1 c_2}{\{s + \alpha_1 \alpha_1 + \alpha_2 \alpha_2\}} \left( \frac{1}{\{s + \alpha_2 \alpha_2\}} \right)
\]

\[
L(c_1 \alpha_1 e^{-\alpha_1 t}) = \frac{\alpha_1 c_1}{\alpha_1 \alpha_1 + s} \quad \text{and} \quad L(c_2 \alpha_2 e^{-\alpha_2 t}) = \frac{\alpha_2 c_2}{\alpha_2 \alpha_2 + s} \quad (6.2.19)
\]

From (6.2.16) and (6.2.19) it can be shown that

\[
I^*(s) = \frac{\alpha_1 c_1}{\alpha_1 \alpha_1 + s} + \frac{\alpha_2 c_2}{\alpha_2 \alpha_2 + s} - \frac{\alpha_1 \alpha_2 c_1 c_2}{\{s + \alpha_1 \alpha_1 + \alpha_2 \alpha_2\}} \left[ \frac{1}{\{s + \alpha_2 \alpha_2\}} + \frac{1}{\{s + \alpha_1 \alpha_1\}} \right]
\]

(6.2.20)

(6.2.20) gives the value of \( I^*(s) \)
Suppose $g(.)$ and $h(.)$ follows exponential distribution with parameters $\lambda_1$ and $\lambda_2$ respectively. In this case, we have

$$g(s) = \frac{\lambda_1}{\lambda_1 + s} \quad \text{and} \quad h(s) = \frac{\lambda_2}{\lambda_2 + s}$$ \quad (6.2.21)

$$c_1 = 1 - g^*(\bar{\theta}_1) = \frac{-\theta_1}{\lambda_1 + \theta_1}; \quad c_2 = 1 - h^*(\bar{\theta}_2) = \frac{-\theta_2}{\lambda_2 + \theta_2}$$ \quad (6.2.22)

Now

$$\frac{d}{ds}(I'(s)) = \left( \frac{-\alpha c_1}{(\alpha c_1 + s)^2} \right) + \left( \frac{-\alpha c_2}{(\alpha c_2 + s)^2} \right) - \alpha\alpha c_1 c_2 \left\{ \frac{1}{s + \alpha c_1 + \alpha c_2} \left[ \frac{-1}{(s + \alpha c_2)^2} + \frac{-1}{(s + \alpha c_1)^2} \right] \right\}$$

From (6.2.12)

$$E(T) = \frac{1}{\alpha c_1} + \frac{1}{\alpha c_2} - \frac{\alpha\alpha c_1 c_2}{\alpha c_1 + \alpha c_2} \left\{ \frac{1}{(\alpha c_2^3) + \frac{1}{(\alpha c_1^3)} \right\} +$$

$$= \frac{\alpha c_1 + \alpha c_2}{\alpha c_1 \alpha c_2} - \frac{\alpha\alpha c_1 c_2}{\alpha c_1 + \alpha c_2} \left\{ \frac{(\alpha c_1)^2 + (\alpha c_2)^2}{(\alpha c_1)\alpha c_2^2} \right\} +$$

$$= \frac{(\alpha c_1 + \alpha c_2)^2}{(\alpha c_1 \alpha c_2)(\alpha c_1 + \alpha c_2)} - \frac{(\alpha c_1)^2 + (\alpha c_2)^2 + \alpha c_1 \alpha c_2}{(\alpha c_1 \alpha c_2)(\alpha c_1 + \alpha c_2)}$$
Using (6.2.22) in (6.2.23) we get

\[
E(T) = \frac{1}{(\alpha_1 c_1 + \alpha_2 c_2)}
\]

Equation (6.2.23) gives the meantime for recruitment for the special case.

### 6.3 NUMERICAL ILLUSTRATION

In this section, the analytical expression obtained in (6.2.23) is numerically illustrated and relevant conclusions are made.

**Case (i):** Fixing \( \theta_1 = 0.8; \theta_2 = 0.7; \alpha_1 = 0.4; \alpha_2 = 0.5 \) and varying \( \lambda_1 \) and \( \lambda_2 \), the values of \( E(T) \) are computed and tabulated in **Table 6.3.1**
Table 6.3.1

<table>
<thead>
<tr>
<th>E(T)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
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<td>( \lambda_2 )</td>
<td>( \lambda_1 )</td>
<td>( \lambda_2 )</td>
<td>( \lambda_1 )</td>
<td>( \lambda_2 )</td>
</tr>
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<td>6.5899</td>
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<tr>
<td>1.5</td>
<td>6.6667</td>
<td>8.3544</td>
<td>9.2308</td>
<td>9.7674</td>
<td>10.1299</td>
</tr>
<tr>
<td>2.0</td>
<td>7.5824</td>
<td>9.8444</td>
<td>11.0843</td>
<td>11.8673</td>
<td>12.4066</td>
</tr>
<tr>
<td>2.5</td>
<td>8.3168</td>
<td>11.1191</td>
<td>12.7273</td>
<td>13.7705</td>
<td>14.5020</td>
</tr>
</tbody>
</table>

CONCLUSION:

From the above table, we make the following observations.

(i) The mean time for recruitment increases as \( \lambda_1 \) increases, keeping other parameters fixed. In other words, when the average loss of manhours (grade A) increases, the mean time for recruitment increases.

(ii) The mean time for recruitment increases as \( \lambda_2 \) increases, keeping other parameters fixed. In other words, when the average loss of manhours (grade A) increases, the mean time for recruitment increases.
(iii) The meantime for recruitment increases as $\lambda_1$ and $\lambda_2$ increases simultaneously, keeping other parameters fixed.

**Case (ii):** Fixing $\theta_1 = 0.8; \theta_2 = 0.7; \alpha_2 = 0.4; \lambda_2 = 0.5$ and varying $\lambda_1$ and $\alpha_1$ the values of $E(T)$ are computed and tabulated in **Table 6.3.2**

**Table 6.3.2**

<table>
<thead>
<tr>
<th>$\lambda_1$</th>
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<th>4</th>
<th>5</th>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>0.5</td>
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</tr>
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<td>3.2653</td>
<td>3.7168</td>
<td>4.0625</td>
</tr>
</tbody>
</table>
CONCLUSION:

From the above table, we make the following observations.

(i) The mean time for recruitment increases as $\lambda_i$ increases, keeping other parameters fixed. In other words, when the average loss of manhours increases, the mean time for recruitment increases.

(ii) The mean time for recruitment decreases as $\alpha_i$ increases, keeping other parameters fixed. In other words, when the loss of manhours increases on the average, the mean time for recruitment increases.

(iii) The mean time for recruitment decreases as $\lambda_i$ and $\alpha_i$ increases simultaneously, keeping other parameters fixed.

**Case (iii):** Fixing $\theta_1=0.8; \theta_2=0.7; \alpha_i=0.6; \lambda_2=0.5$; and varying $\lambda_i$ and $\alpha_2$, the values of $E(T)$ are computed and tabulated in Table 6.3.3
### Table 6.3.3

<table>
<thead>
<tr>
<th>$\lambda_1$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
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<td>2.4779</td>
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<tr>
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<td>1.2935</td>
</tr>
<tr>
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<td>1.0080</td>
<td>1.0256</td>
<td>1.0351</td>
<td>1.0410</td>
</tr>
</tbody>
</table>

**CONCLUSION:**

From the above table, we make the following observations.

(i) The mean time for recruitment increases as $\lambda_1$ increases, keeping other parameters fixed. In other words, when the average loss of manhours increases (grade A), the mean time for recruitment increases.
(ii) The mean time for recruitment decreases as $a_2$ increases, keeping other parameters fixed. In other words, when the inter-decision time (grade B) increases on the average, the mean time for recruitment decreases.

(iii) The meantime for recruitment decreases as $\lambda_1$ and $a_2$ increases simultaneously, keeping other parameters fixed.

Case (iv): Fixing $\theta_1 = 0.8; \theta_2 = 0.7; \lambda_1 = 0.5; a_2 = 0.6$ and varying $\lambda_2$ and $a_1$ the values of $E(T)$ are computed and tabulated in Table 6.3.4

<table>
<thead>
<tr>
<th>$\lambda_2$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1 = 0.5$</td>
<td>3.5547</td>
<td>4.5225</td>
<td>5.0658</td>
<td>5.4137</td>
<td>5.6555</td>
</tr>
<tr>
<td>$a_1 = 1.0$</td>
<td>2.3575</td>
<td>2.7474</td>
<td>2.9389</td>
<td>3.0527</td>
<td>3.1282</td>
</tr>
<tr>
<td>$a_1 = 1.5$</td>
<td>1.7636</td>
<td>1.9730</td>
<td>2.0699</td>
<td>2.1257</td>
<td>2.1620</td>
</tr>
<tr>
<td>$a_1 = 2.0$</td>
<td>1.4087</td>
<td>1.5392</td>
<td>1.5975</td>
<td>1.6306</td>
<td>1.6518</td>
</tr>
<tr>
<td>$a_1 = 2.5$</td>
<td>1.1727</td>
<td>1.2618</td>
<td>1.3007</td>
<td>1.3225</td>
<td>1.3365</td>
</tr>
</tbody>
</table>
CONCLUSION:

From the above table, we make the following observations.

(i) The mean time for recruitment increases as $\lambda_2$ increases, keeping other parameters fixed. In other words, when the loss of manhours increases (grade B) on the average, the mean time for recruitment increases.

(ii) The mean time for recruitment decreases as $\alpha_1$ increases, keeping other parameters fixed. In other words, when the inter-decision time (grade A) increases on the average, the mean time for recruitment decreases.

(iii) The meantime for recruitment decreases as $\alpha_1$ and $\lambda_2$ increases simultaneously, keeping other parameters fixed.

Case (v): Fixing $\theta_1 = 0.8; \theta_2 = 0.6; \alpha_1 = 0.7; \lambda_1 = 0.9$ and varying $\lambda_2$ and $\alpha_2$ the values of $E(T)$ are computed and tabulated in Table 6.3.5
Table 6.3.5

<table>
<thead>
<tr>
<th>$\lambda_2$</th>
<th>$\alpha_2$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td></td>
<td>3.7019</td>
<td>4.7482</td>
<td>5.3736</td>
<td>5.7895</td>
<td>6.0861</td>
</tr>
<tr>
<td>1.0</td>
<td></td>
<td>2.4214</td>
<td>3.4021</td>
<td>4.0830</td>
<td>4.5833</td>
<td>4.9666</td>
</tr>
<tr>
<td>1.5</td>
<td></td>
<td>1.7991</td>
<td>2.6506</td>
<td>3.2923</td>
<td>3.7931</td>
<td>4.1949</td>
</tr>
<tr>
<td>2.0</td>
<td></td>
<td>1.4312</td>
<td>2.1711</td>
<td>2.7581</td>
<td>3.2353</td>
<td>3.6308</td>
</tr>
<tr>
<td>2.5</td>
<td></td>
<td>1.1883</td>
<td>1.8384</td>
<td>2.3731</td>
<td>2.8205</td>
<td>3.2004</td>
</tr>
</tbody>
</table>

CONCLUSION:

From the above table, we make the following observations.

(i) The mean time for recruitment increases as $\lambda_2$ increases, keeping other parameters fixed. In other words, when the average loss of manhours increases, the mean time for recruitment increases.

(ii) The mean time for recruitment decreases as $\alpha_2$ increases, keeping other parameters fixed. In other words, when the inter-decision time increases on the average, the mean time for recruitment decreases.
(iii) The meantime for recruitment decreases as $\alpha_2$ and $\lambda_2$ increases simultaneously, keeping other parameters fixed.

**Case (vi):** Fixing $\theta_1 = 0.8; \theta_2 = 0.6; \lambda_1 = 0.9; \lambda_2 = 0.7$ and varying $\alpha_1$ and $\alpha_2$ the values of $E(T)$ are computed and tabulated in **Table 6.3.6**

**Table 6.3.6**

<table>
<thead>
<tr>
<th>$\alpha_2$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>0.5</td>
<td>2.2000</td>
<td>1.2222</td>
<td>0.8462</td>
<td>0.6471</td>
</tr>
<tr>
<td>1.0</td>
<td>1.8333</td>
<td>1.1000</td>
<td>0.7857</td>
<td>0.6111</td>
<td>0.5000</td>
</tr>
<tr>
<td>1.5</td>
<td>1.5714</td>
<td>1.0000</td>
<td>0.7333</td>
<td>0.5789</td>
<td>0.4783</td>
</tr>
<tr>
<td>2.0</td>
<td>1.3750</td>
<td>0.9167</td>
<td>0.6875</td>
<td>0.5500</td>
<td>0.4583</td>
</tr>
<tr>
<td>2.5</td>
<td>1.2222</td>
<td>0.8462</td>
<td>0.6471</td>
<td>0.5238</td>
<td>0.4400</td>
</tr>
</tbody>
</table>
CONCLUSION:

From the above table, we make the following observations.

(i) The mean time for recruitment decreases as $a_2$ increases, keeping other parameters fixed. In other words, when the loss of manhours increases on the average, the mean time for recruitment decreases.

(ii) The mean time for recruitment decreases as $a_1$ increases, keeping other parameters fixed. In other words, when the inter-decision time increases on the average, the mean time for recruitment decreases.

(iii) The mean time for recruitment decreases as $a_2$ and $a_1$ increases simultaneously, keeping other parameters fixed.