CHAPTER 2

LITERATURE SURVEY

A brief literature survey is presented in this chapter. The chapter ends with various objectives of the research work in section 2.2.

2.1 LITERATURE SURVEY

The composite laminates are ideal for structural applications where high strength-to-weight and stiffness-to-weight ratios are of prime importance. Composite materials can be tailored to meet the necessary requirements of stiffness and strength by changing the lay-up and ply orientations. The ability to tailor a composite material to its job is one of the most significant advantages of a composite material over an ordinary material. The research and development of composite materials in the design of aerospace, mechanical and civil structures has risen tremendously in the past couple of decades. It is essential to know the vibration characteristics of these structures, which may be subjected to dynamic loads in complex environmental conditions. If the frequency of the loads variation matches one of the resonance frequencies of the structure, large translation/torsion deflections and internal stresses can occur, which may lead to failure of structure components. A variety of structural components made of composite materials such as aircraft wing, helicopter blade, vehicle axles, and turbine blades can be approximated as laminated composite beams, which call for deeper study and understanding of the vibration of the
composite beams. The practical importance and potential advantages of the composite beams have inspired continuing research interest.

Many structural systems such as wing, fuselage and structures of advanced high speed flight vehicles, helicopter blades, missiles, space structures are modeled as beams and panels. In many applications, design engineering requirements call for the use of thick laminates containing perhaps over hundreds of layers or sandwich structure panels with a thick core between the face sheets of the laminated composite materials. In such thick laminates, the transverse stresses and deformation are not negligible. In addition, thick laminates and sandwich panels often contain adjacent layers whose material properties are drastically different. The response of such structures to external loading is complicated and analysis often requires that the response of each layer be taken into account explicitly.

To predict the response of laminates more accurately, many models have been developed on the basis of various kinematical assumptions. Such models can be roughly divided into three categories: equivalent single layer theories, layer wise theories and zigzag in-plane displacement theories. The equivalent single layer theory is one of the famous and simple theory in which the material properties of all the layers are `smeared` and the laminate is modeled as an equivalent single anisotropic layer. The most popular of these theories is the first order shear-deformation theory given by Allen (FSDT) [1], which assumes that a line originally straight and normal to the reference surface remains straight during deformation but not necessarily perpendicular to the reference surface. In single layer theory the deformation
of the plate is expressed in terms of unknowns at the reference plane, which is usually taken at the middle plane of the plate. This theory yields good prediction of overall laminates behavior, like deflections, natural frequencies, Buckling loads and etc., provided that the beam is thin and the material properties of adjacent layers do not differ significantly. However FSDT does not account for warping of the cross section, which may be significant in laminated composites. In order to reduce the inaccuracies of the FSDT, higher order shear deformation theories (HSDT) were proposed [2, 3]. In these models, it is assumed that the displacements are of higher order polynomial form and are continues through the thickness. However, even though they often can predict well the gross behavior of thin and some moderately thick laminates, all equivalent single layer theories have a common shortcoming that they are unable to account for the severe discontinuities in transverse shear strain that occur at interface between two adjacent layers with drastically different stiffness properties. Therefore, the local deformations and stresses, and sometimes even the overall laminate response, is not predicted accurately.

Based on the first-order shear deformation theory, Allen presented a three-layered model for the analysis of sandwich beams and plates wherein the zigzag deformation pattern was considered. B-spline functions based on FSDT have been used by the Dawe and Wang [4, 5] as trial functions for Rayleigh-Ritz analyses and finite strip analyses. Aiello and Ombres [6] developed a model using FSDT to assess the optimal arrangement of hybrid laminated faces of sandwich panel for local buckling loads. Wang [7] employed the B-Spline Rayleigh-Ritz method based on first order shear deformation theory to study the buckling problem of skew
composite laminates. Sundersan et al. [8] have studied the influence of partial edge compression of composite plates by using a finite element method based on FSDT for analyzing isotropic plates. The structural behavior of sandwich laminated composites cannot be assessed satisfactorily by a FSDT, as the core and face sheets deform in different ways due to wide variation of their material properties, which is identified in the variation of in-plane displacements across the thickness at the interface between the core and face layers. So as to predict the actual parabolic variation of shear stress, this (FSDT) required shear correction factor. This has inspired many researchers to develop a number of refined plate theories.

Discrete layer theories proposed by Cetkovic [9] assumed unique displacement field in each layer and displacement continuity across the layers. In this theory, the number of unknowns increases directly with the increase in the number of layers due to which it required huge computational involvement. Earlier a new triangular element was developed by Chakrabarti and Sheikh [10, 11], for the buckling analysis of composite plate using Reddy’s higher order theory. The effect of partial edge compression was investigated. Frosting [12] obtained the general as well as local buckling loads for sandwich panels consisting of two faces and a soft orthotropic core using HSDT.

Chandrashekhara et al. [13] has found the accurate solutions based on first order shear deformation theory which includes rotary inertia for symmetrically laminated beams. The laminated beams by a systematic reduction of the constitutive relations of the three dimensional anisotropic body and obtained the basic equations of the beam theory, which is based on
the parabolic shear deformation theory represented by Bhimaraddi and Chandrashekhara [14]. A third-order shear deformation theory for static and dynamic analysis of an orthotropic beam incorporating the impact of transverse shear and transverse normal deformations developed by Soldatos and Elishakoff [15]. The exact solutions of symmetric laminated composite beams with 10 different boundary conditions, in which shear deformation and rotary inertia are considered in the analysis developed by Abramovich [16].

Hamilton’s principle is used to calculate the dynamic equations governing the free vibration of laminated composite beams. The impacts of transverse shear deformation and rotary inertia were included, and analytical solutions for unsymmetric laminated beams were obtained by applying the Lagrange multipliers method developed by Krishnaswamy et al. [17]. The free vibration behavior of laminated composite beams by the conventional finite element analysis using a higher-order shears deformation theory, the Poisson effect, coupled extensional with bending deformations and rotary inertia were considered in the formulation studied by Chandrashekhara and Bangera [18].

Abramovich and Livshits [19] presented the free vibration analysis of non-symmetric cross-ply laminated beams based on first-order shear deformation theory. Khdeir and Reddy [20] have evolved the analytical solutions of various beam theories to review the free vibration behavior of cross-ply rectangular beams with arbitrary boundary conditions. Biaxial bending, axial and torsional vibrations using the finite element method and
the first-order shear deformation theory examined by Nabi and Ganesan [21].

The stiffness method for the solution of the purely in-plane free vibration problem of symmetric cross-ply laminated composite beams with axial and transverse shear deformation effects included by the first-order shear deformation theory is developed by Yildirim [22].

Mahapatra et al. [23] presented a spectral element for Bernoulli–Euler composite beams. Ghugal and Shimpi [24] presented a review of displacement and stress-based refined theories for isotropic and anisotropic composite beams and discussed various equivalent single layer and layer wise theories for laminated beams. Higher-order mixed theory for determining the natural frequencies of various composite Simply-Supported beams presented by Rao et. al. [25]. A new refined locking free first-order shear deformable finite element and demonstrated its utility in solving free vibration and wave propagation problems in laminated composite beam structures with symmetric as well as asymmetric ply stacking proposed by Chakraborty et al. [26]. A spectral finite element model, for analysis of axial–flexural–shear coupled wave propagation in thick laminated composite beams and derived an exact dynamic stiffness matrix proposed by Mahapatra and Gopalakrishnan [27].

A new approach combining the state space method and the differential quadrature method for freely vibrating laminated beams based on two-dimensional theory of elasticity proposed by Chen et al. [28]. Chen et al. [29] reported a new method of state space-based differential quadrature for free vibration of generally laminated beams. Ruotolo [30] proposed a
spectral element for anisotropic, laminated composite beams. The axial-bending coupled equations of motion were derived under the assumptions of the first-order shear deformation theory and the spectral element matrix was formulated. A two-noded curved composite beam element, with three degrees-of-freedom per node, for the analysis of laminated beam structures is used. The flexural and extensional deformations together with transverse shear deformation based on first-order shear deformation theories were also incorporated with the poisson effect in the formulation of the beam constitution equation presented [31, 32, 33].

Finite elements have also been developed based on Timoshenko beam theory [34]. Most of the finite element models developed for Timoshenko beams possess a two-node, two degree of freedom structure, based on the requirements of the variation principle for the Timoshenko’s displacement field. A Timoshenko beam element, which shows that the element converged to the exact solution of the elasticity equations for a simply supported beam provided that the correct value of the shear factor was used proposed by Davis et al. [35]. Thomas et al. [36] proposed a new element of two nodes having three degrees of freedom per node, the nodal variables being transverse displacement, shear deformation and rotation of crossection. The rates of convergence of a number of the elements were compared by calculating the natural frequencies of two cantilever beams. Further this paper gave a brief summary of different Timoshenko beam elements. A finite element model was proposed with nodal degrees of freedom which could satisfy all the forced and natural boundary conditions of Timoshenko beam. The element has degrees of freedom as transverse deflection, total slope (slope due to bending and shear deformation), bending
slope and the first derivative of the bending slope presented by Thomas and Abbas [37]. A second-order beam theory requiring two coefficients, one for cross-sectional warping, and other for transverse direct stress, was developed by Levinson [38]. A beam theory for the analysis of the beams with narrow rectangular cross-section and showed that his theory predicted better results when compared with elasticity solution than Timoshenko beam theory. Though this required no shear correction factor, the approach followed by him to derive the governing differential equations was variationally inconsistent developed by Levinson. Later Bickford [39] represented Levinson theory using a variational principle and also discussed how one could obtain the correct and variationally consistent equations using the vectorial approach.

An improved theory in which the in-plane displacement was assumed to be cubic variation in the thickness coordinate of the beam whereas the transverse displacement was assumed to be the sum of two partial deflections, deflection due to bending and deflection due to transverse shear. This theory does not impacts the effect of transverse normal strain and does not satisfy the zero strain/stress conditions at the top and bottom surfaces of the beam reported by Krishna Murty [40]. A higher order beam finite element for bending and vibration problems of the beams. In this formulation, the theory imagines or assumes a cubic variation of the in-plane displacement in thickness coordinate and a parabolic variation of the transverse shear stress across the thickness of the beam. Further the theory satisfies the zero shear strain conditions at the top and bottom surfaces of the beam and neglects the effect of the transverse normal strain developed by Heyliger and Reddy [41]. A C₀ finite element model based on higher order
shear deformation theories including the effect of the transverse shear and
normal strain and the finite element fails to satisfy the zero shear strain
conditions at the top and bottom surfaces of the beam proposed by Kant and
Gupta [42]. The free vibration analysis of the laminated composite beams
using a set of three higher order shear deformation theories and their
corresponding finite elements. These theories also fail to satisfy the zero-
strain conditions at the top and bottom surfaces of the beams. Further the
impacts of the transverse normal strain were not included in the theories
investigated by Marrur and Kant [43]. An analytical solution to the dynamic
analysis of the laminated composite beams using a higher order refined
theory. This model also fails to satisfy the traction- free surface conditions at
the top and bottom surfaces of the beam but has included the effect of
transverse normal strain.

Equivalent Single Layer theories assume the displacements to
follow a global variation across the laminate thickness. These are inadequate
to account for the layer wise distortion of the lines normal to the mid-
surface, and yield inaccurate results for moderately thick laminates and even
for thinner laminates with strong in-homogeneities across the thickness. To
overcome the short comings of the equivalent single layer approach as
mentioned above, the discrete layer (or layer-wise) theories have been
proposed. These theories are based on a unique displacement field for each
layer and enforce inter-laminar continuity of displacements and sometimes
of transverse stresses as well. These theories predict excellent global and
local distributions of in-plane and out-of-plane displacement and stresses.
The major shortcoming in these theories is their large computational
expenses. As the number of layers increases, the number of degrees of
freedom increases proportionally. Discrete layer theories, on the other hand, yield accurate results for thick and thin laminates, but suffer from an excessive number of displacement variables in proportion to the number of discrete layers. To overcome these short coming of the ESL theories and the DLTs, efficient discrete layer theories, also known as zigzag theories have been developed. These are discrete layer theories in which the primary displacement variables are reduced by using the conditions on transverse shear stresses at the top, bottom and layer interfaces. Their number becomes independent of the number of layers.

Di Sciuva in the mid-1980s developed new class of laminate theory [44, 45], called the first order zigzag theory (FZZT). In this theory, in-plane displacements in a laminate are assumed to be piecewise (layer-wise) linear and continuous through the thickness. This is accomplished by analytically satisfying the transverse shear stress continuity conditions at each interface in the laminates. This theory claims to be very accurate for many cases, especially symmetric laminates. On the basis of the concept introduced by Di Sciuva, significant improvements have been made to the FZZT.

The number of unknowns are made independent of the number of layers by introducing the transverse shear stress continuity condition at the layer interfaces of the laminate and the in plane displacements have piece-wise variation across the plate thickness in this theory (ZZT). Averill [46] developed a $C_0$ finite element based on first order zigzag theory and overcome the $C_1$ continuity requirement by incorporating the concepts of independent interpolations and penalty functions.
Averill and Yip [47, 48] and Iqbal and Chakrabarti [49] developed a C_0 finite element based on cubic zigzag theory, using interdependent interpolations for transverse displacement and rotations and penalty function concepts. In some improved version of these theories, the condition of zero transverse shear stresses at the plate/beam top and bottom was also satisfied. The primary improvement was achieved by super imposing a piece wise linear variation of in-plane displacements on a continuous cubic function of the transverse coordinate, creating a displacement field that can better account for the warping that occurs during bending of asymmetric laminates. These latter theories, denoted as higher order zigzag theories (HZZT), also satisfy the homogenous shear traction boundary conditions at the top and bottom surfaces of the laminates. This class of theories appears to have an ideal combination of accuracy and efficiency, making them well suited for use in computational simulations. However, these theories have the unfortunate short coming that the transverse deflection degree of freedom $w_0$ is required to be $C^1$ continuous, so that Hermitian-type interpolation of $w_0$ must be used within the finite element models, which allow only six degrees of freedom (gradients of $w_0$) are present in the finite element models, making it inconvenient, if not impossible to implement the finite element models based on these theories into commercial finite element software packages that allows only six degrees of freedom – three translation and three rotations.

By choosing appropriate shape functions, four shear deformation theories presented by Aydogdu [50], are named as: parabolic shear deformation beam theory, hyperbolic shear deformation beam theory, first
order shear deformation beam theory and exponential shear deformation beam theory, for the buckling analysis of composite beam. Babu and Kant [51] presented two $C_0$ isoparametric finite element formulations, one based on FSDT and other based on HSDT to investigate the effect of skew angle on buckling coefficient. Bambole and Desai [52] and and many more developed a mixed FE approach for accurate analysis of stresses, where the stress components are assumed as the field variables at interface nodes along with displacement field variables.

Dafedar and Desai [53] and have presented a theory based on mixed higher order theory for the buckling analysis of laminated composite structures. The transverse deformation is very significant in case of a laminated sandwich structure having a soft core as there is abrupt change in the values of transverse shear rigidity and thickness of face sheet and the core. As such to achieve sufficient accuracy, unknown transverse displacement fields across the depth in addition to that in the reference plane are essential to represent the variation of transverse deflection. This can be done by using sub-laminate plate theories but the number of unknowns will increase with the increase in the number of sub-laminates. On the other hand, introduction of additional unknowns in the transverse displacement fields invites additional $C_1$ continuity requirements in its finite element implementation by using the zigzag theory as mentioned earlier. However, the application of a $C_1$ continuous finite element is not encouraged in a practical analysis. Karama et.al [54] presented a multilayered laminated composite model by introducing a sine function as a transverse shear stress function where a FE based software package ABAQUS was used to check the efficiency of the model. Matsunaga [55] developed one dimensional
global higher order theory, in which the fundamental equations were derived based on the power series expansions of continuous displacement components to analyze the vibration and buckling problems. Recently Pandit et. al. [56] has presented a FE model for an improved higher order zigzag theory for the static analysis of laminated sandwich plate with soft-core.

Kapuria et.al [57] developed 1D zigzag theory for the analysis of simply supported beams. Kapuria et. al. [57] assessed an efficient zigzag theory of laminated beams by comparison of analytical solution for static patch load natural frequencies, harmonic transverse load with sinusoidal variation and buckling under axial load. The effect of the laminate layup and the thickness to span ratio was investigated. Zigzag models for laminated composite beams were developed by using trigonometric terms to represent the linear displacement field, transverse shear strains and stresses [58]. Cetkovic proposed [59] Discrete layer theories assumed unique displacement field in each layer and displacement continuity across the layers. Zhen and Wanji [60] presented global theories with higher order shear deformation and zigzag theories satisfying continuity of transverse shear stresses at interfaces for analyzing the global response of sandwich laminated beams. The effects of the number of higher order terms in the shear deformation as well as inter-laminar continuity of shear stress on global response of laminated beams and soft core sandwich beams were studied.

Besides that, Benjeddou [61] reviewed finite element modeling of laminated elastic substrate with piezoelectric layers. He concluded that the

An efficient zigzag theory of laminated beams was assessed by comparison of analytical solution for natural frequencies, and buckling under axial load as discussed above. A one dimensional beam finite element with electric degrees of freedom is presented for the dynamic analysis of hybrid piezoelectric using the coupled efficient layer wise (zigzag) theory developed recently by Kapuria [68, 69]. Finite element model is presented for the analysis of hybrid piezoelectric beams under electromechanical load. Using the one-dimensional (1D) coupled zigzag theory was developed in just a decade ago. Two nodded elements are used with cubic Hermite interpolation for deflection and electric potential at the sub layer and with linear interpolation for axial displacement and shear rotation. Exact 2D piezoelectricity solution for buckling of simply supported hybrid piezoelectric beams and panels with solutions serve as useful benchmarks for assessment of various 1D beam theories and approximate 2D numerical solutions such as the solution using the finite element method.
Kapuria and Alam [70] presented efficient layerwise finite element model for dynamic analysis of laminated piezoelectric beams. The novel coupled zigzag theory developed by Kapuria et al. for linear static and dynamic analysis of hybrid beams under electromechanical load are extended to the linear static analysis of hybrid plates under electromechanical load. This theory considers a combination of a third order global variation with a layer wise linear variation across the thickness for the in-plane displacement, a sub-layer wise linear variation for the electric potential and a sub-layer wise quadratic distribution across the thickness for the transverse displacement, which explicitly accounts for the transverse normal strain induced due to the potential fields. It considers all components of the electric field. The condition of absence of shear traction at the top and bottom surfaces and the continuity of the transverse shear stresses at the layer interfaces are imposed to reduce the number of displacement variables to that for the ESL theories, FSDT and TOT. Zigzag theory by Kapuria is that it accounts for the non-uniform layer wise distribution of the transverse displacement due to the piezoelectric transverse normal strain which have been observed to have considerable effect on the response, especially for electric load.

Arya et al. [71] presented a model that uses a sine term to represent the nonlinear displacement field across the thickness as compared to a third order polynomial term in conventional theories. Transverse shear stress and strain are represented by a cosine term as compared to parabolic term. They given analytical results for cross ply laminate. Akhras G. and Li [72] presented spline finite strip method for static analysis of composite
plates using the higher-order zigzag theory for composite plate. Fares and Elmarghany [73] presented refined nonlinear zigzag shear deformation theory of composite laminated plates using a modified mixed variational formulation. Their theory accounts for continuous piecewise layer-by-layer linear variation approximation in the thickness direction for the displacements. Kumari et al. [74] presented a new improved third order theory (ITOT) for hybrid piezoelectric angle-ply flat panels under thermal loading. The ITOT and the existing efficient zigzag theory are assessed for simply-supported angle-ply flat hybrid panels for static loads and for natural frequencies by comparison with 2D solutions. Kapuria and Achary [75] have developed a benchmark 3D solution and assessment of a zigzag theory for free vibration of hybrid plates under initial electro-thermo-mechanical stresses. This theory considers a combination of third order global variation and layerwise linear variation. Kapuria et al. [76] has given a third order zigzag theory based model for layered functionally graded beams in conjunction with the modified rule of mixtures for effective modulus of elasticity is validated through experiments for static and free vibration response.

Composite laminates with some piezoelectric layers, for sensing and actuation to achieve desired control, form a part of new generation of adaptive structures. There have been few exact two–dimensional (2D) elasticity solutions of buckling of composite simply supported beams or plates in cylindrical bending [77,78] under mechanical load. The exact solutions are verified by ABAQUS [79].
Thermal buckling and post buckling of hybrid piezoelectric beams and plates have been presented using classical, first order, third order and layer wise theories [80, 81]. In a review of theories of piezoelectric laminates, Gopinathan et al. [82] pointed out that the usual assumption of linear variation of electric potential across the actuators layer needs to be modified in coupled theories as it violates charge equilibrium equation. In these theories shear continuity condition at the larger interfaces are violated and the number of variables in the layer wise theories depends on the no of layers.

Chandashekhra and Bhatia [83] presented a finite element model for active buckling control of composite plates, with surface bonded or embedded, continuous or segmented, piezoelectric sensors and actuators. Chase and Bhashyam [84] developed optimal design equations to actively stabilize laminated plates loaded beyond the critical buckling load, using a large no of sensors and actuators. Meresssi and Paden [85] showed that the critical load of the beam can be increased using feed back through strain gauges and using piezoelectric actuators. Thompson and Loughlan [86] demonstrated experimentally that the buckling capacity of a column is increased by applying control voltage to the piezoelectric actuators. Wang [87] and Wang Quek [88] have presented coupled 1D classical beam theory for buckling and further analysis of a column with a pair of piezoelectric layers partially or fully covering it. Finite Difference method is used for solution and it is shown that with proper placement of the actuators, the buckling load for statically actuated beams can be significantly increased.
Very recently Ibrahim et. al. [89] has given Finite Element model considering 3-noded beam element for curved isotropic beams. Quadratic interpolation is used for axial displacement and shear rotation and cubic hermite interpolation is used for deflection.

2.2 OBJECTIVES OF PRESENT WORK

After a brief Literature survey as discussed above the following are objectives of present work:

In the current investigation the main objective is to find out the free vibration of generally laminated composite beams based on Zigzag theory derived through the use of Hamilton’s principle. The work presents finite element model based of zigzag theory, for the analysis of composite beams, by extending the work of Kapuria et al. [57] and using some of the important Finite Element model by Kapuria et. al. [69], by Ibrahim et. al. [89], and using interpolation function by Cook [90] for various angle or orientation of laminates of laminated beam of various boundary conditions and for higher noded elements. In general Finite Element Models using 2-noded beam element and 3-noded beam elements are developed for dynamic analysis. The weak form of integration consistent with element mass matrix and stiffness matrices is derived for this purpose. The present 1D- Finite Element model is compared and validated by comparing the dynamic response with 2D-Finite Element model using ABAQUS software.

Cubic Hermite polynomial interpolation is used for deflection, and linear interpolation is used for axial displacement and shear rotation [89, 90]
to meet the continuity. However for three noded beam element 5\(^{th}\) degree polynomial [90] is used for deflection and quadratic interpolation is used for axial displacement and shear rotation to meet the continuity. Natural frequencies and mode shapes for composite (symmetric and asymmetric) and sandwich beams are presented. The present results for cantilever and fixed beams are compared with the 2D-FE results obtained using ABAQUS 6.6 package.

1. (a) To present an efficient zigzag theory based Finite Element model using 2-noded beam element for obtaining the dynamic response of laminated composite beams with various boundary conditions
(b) To develop a general MATLAB program for the same

2. (a) To develop an efficient zigzag theory based Finite Element solution for obtaining Buckling load and critical strain of laminated composite beams with simply supported and cantilever boundary conditions,
(b) To develop a general MATLAB program for the same.

3. (a) To develop a an efficient 1D zigzag theory based Finite Element solution for obtaining the Natural Frequencies using 3-noded beam element.
(b) To develop a general MATLAB program for the same.

4. (a) To present a Zigzag theory based analytical solution for obtaining natural frequency of composite beams with simply supported
boundary conditions. and To develop a general MATLAB program for the same

5. To assess 1D models by comparison with exact 2D solution for buckling and free vibration of initially stressed SS beams.

6. To present a first order shear deformation theory based analytical solution for obtaining Buckling load and critical strain composite beams with simply supported boundary conditions and compare it with zigzag theory.

6. To assess the accuracy of the above mentioned 1D finite element solution for the same beam by comparing with 2D FE (ABAQUS) results.

7. To obtain various flexural mode shapes for cantilever and fixed boundary condition.

8. To find the effect of ply angles and no. of laminates/layers on frequency.
This thesis contains seven chapters including this chapter.

A detailed survey of relevant literature is presented in chapter 2.

In chapter 3 dynamic analysis of laminated composite beam using high order Zigzag theory is presented with Constitutive equations and assumptions. First order shear deformation theory is also presented in brief.

Chapter 4 includes 1 D finite element method formulation for dynamic analysis using 2-noded and 3-noded beam element. It also includes 1 D finite element method formulation for buckling analysis using 2-noded beam element.

Chapter 5 contains results (natural frequencies and mode shapes) of above formulations for common boundary conditions, such as clamped-free, simply supported, clamped-clamped. This chapter including results obtained by MATLAB and ABAQUS 6.6 for the various boundary condition of laminated composite beam.

Finally in chapter 6 important conclusions drawn from the present investigations, reported in chapters 3-5, along with suggestions for further work have been presented. Chapter 7 contains references of important papers etc.
Remark: To conclude, it is clear from the above literature survey that 2noded FE model and 3-noded FE model are necessary to be developed so that vibration and buckling response of the beams other than simply supported Boundary condition could be studied. This is done by extending the work of Kapuria [57], 3-noded beam element FE Model, presented for four degrees of freedom per node, to obtain the values/responses with % of error. The values of frequency may help the researchers in controlling vibration.