2.1. INTRODUCTION

The analysis of structures, subjected to static and dynamic loads producing large displacements and large plastic strains, is nowadays a routine task of structural analysts for which they employ finite element method and other numerical techniques as well as the rigid-plastic methods of analysis. However, regardless of the method used, the prediction of failure is usually difficult. The problem gets further complicated when the load is impact load which may arise due to the associated large displacements, strains, interaction of various modes of failure, and strain rate effects on material properties. The mechanics of material deformation and ultimate failure have so far, not been understood properly. The behaviour of such structures which are subjected to impact loads producing large inelastic strains is important for a broad class of engineering problems. An extensive literature on theoretical and experimental studies in this particular field is now available which is presented and critically reviewed in this Chapter.

A detailed study of available literature pertaining to the different aspects of the problem of impact on beams, plates and cylindrical shells has been made and the significant works are discussed in this Chapter. The shortcomings, if any, in experimental as well as the analytical studies are highlighted and efforts have been made to overcome the same in subsequent Chapters.

2.2. STRAIN RATE EFFECT

The properties of many materials under dynamic loading conditions are different from the corresponding static properties. In particular, the stress-strain relations are sensitive to the
speed of a test, a phenomenon which is known as strain rate sensitivity, or viscoplasticity. Material strain-rate-sensitivity is a highly non-linear phenomenon.

The influence of strain-rate-sensitivity on the strain at fracture is different for different materials depending on the lattice structure of the material. The stress-strain characteristics of the materials at high strain are important, since these will affect the strain at fracture. The difference in the material strain-rate-sensitivity between the initial stage of plastic flow and that at larger strains causes some difficulty in developing a consistent dynamic constitutive relationship. However, it is possible to model the dynamic behaviour in the initial stage at large plastic strains separately. The behaviour of large plastic strains would be relevant for a failure analysis.

Manjoine [49] reported his studies in 1944 on some tensile tests which he conducted on low-carbon steel using a high-speed tension machine. His experimental results indicated that the lower yield stress and the ultimate tensile stress increases with increase in strain rate, the increase being more significant for the lower yield stress.

Campbell and Cooper [50] have examined the dynamic tensile behaviour of low-carbon mild steel specimens up to fracture. The upper and lower yield stresses increase with increase in strain rate, as observed by Manjoine [49]. However, the ultimate tensile stress also increase, but more slowly. Thus, the reduction in the importance of material strain hardening with increase in strain rate for dynamic compression is also found in mild steel with large tensile strains and large strain rates. Indeed, it appears, apart from the upper yield stress, the material behaves as a perfectly plastic material with little or no strain hardening at high strain rates.

Many authors have conducted dynamic tensile tests since the early experiments of Manjoine [49]. Symonds [51] has gathered the dynamic lower yield or flow stresses for mild steel which have been recorded over a two-decade period in a number of laboratories. These results reveal a trend of increasing flow stress with increase of strain rate over a wide range of strain rates. The data have considerable scatter which is,
undoubtedly, related to the range of different mild steels having different grain sizes and heat treatments, and variety of testing machines and data recording equipment.

The influence of material strain-rate-sensitivity manifests itself as a strengthening effect in a structure. As discussed by Bodner and Symonds [52], sometimes a structural mode change occurs which causes larger and not smaller associated permanent deformations. This might suggest that it is a beneficial phenomenon since it provides an additional safety factor.

It is interesting to note from the results of Campbell and Cooper [50] that the associated fracture strain might decrease with increase in strain rate. In other words, the material becomes more brittle at higher strain rates. However, Perrone [53] remarked that energy absorption systems for enhancing the structural crashworthiness of vehicles, for example, could impart unacceptable forces on the human body which might otherwise be acceptable for an identical material (and structure) but with strain-rate-independent material properties. Material strain-rate-sensitivity is a material effect and is independent of the structural geometry.

Many different constitutive equations for the strain-rate-sensitive behaviour of materials have been proposed in the literature [20, 54]. Careful experimental work is required in order to generate the various coefficients in these constitutive equations. Many authors have studied the characteristics of constitutive equations which is indispensable for guiding experimental test programs.

Vaughan [55] observed that the axial force which resists motion in a rigid-plastic cylindrical shell is constant when the effects of material strain hardening, material strain rate sensitivity and thickness changes are neglected. This gives rise to a constant deceleration of the striking mass and, therefore, a linear decrease with time of the axial velocity and the other velocity dependent quantities, such as strain rates. Florence and Abrahamson [56] have examined the influence of material strain rate sensitivity on the magnitude of the critical collapse velocity which were presented for various specified
amplifications of the initial radial displacement imperfections. Unfortunately, no experimental results are available to assess the accuracy of the theoretical predictions for cylindrical shell made from strain rate sensitive or strain rate insensitive material [57]. Moreover, this phenomenon has not been explored for any other structural problems.

According to the Cowper-Symonds constitutive relationship, the dynamic flow stress \( \sigma_d \) at a uniaxial plastic strain rate \( \dot{\varepsilon}_m \) for a rigid, perfectly plastic material is given by [58]:

\[
\sigma_d = \sigma_0 \left[ 1 + \left( \frac{\dot{\varepsilon}_m}{D} \right)^q \right]^{1/q}
\]  

(2.1)

where, \( \sigma_0 \) is the static flow stress in a static uniaxial tensile test, and, \( D \) and \( q \) are material constants whose values for different materials are given in Table 2.1.

<table>
<thead>
<tr>
<th>Material</th>
<th>( D ) (s(^{-1}))</th>
<th>( q )</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild steel</td>
<td>40.4</td>
<td>5</td>
<td>Cowper and Symonds [58]</td>
</tr>
<tr>
<td>Aluminium alloy</td>
<td>6500</td>
<td>4</td>
<td>Bodner and Symonds [52]</td>
</tr>
<tr>
<td>( \alpha )-Titanium (Ti 50 A)</td>
<td>120</td>
<td>9</td>
<td>Symonds and Chon [62]</td>
</tr>
<tr>
<td>Stainless steel 304</td>
<td>100</td>
<td>10</td>
<td>Forrestal and Sagartz [63]</td>
</tr>
</tbody>
</table>

The mean uniaxial strain rate \( \dot{\varepsilon}_m \) is estimated with the help of the Perrone and Bhadra’s approximation [59] which is further simplified by Jones [44] as \( \dot{\varepsilon}_m = \frac{w_m V}{3\sqrt{2}L^2} \) for beams, where \( w_m \) is the maximum permanent deflection and \( L \) is the half-length of a beam struck by a projectile at velocity, \( V \).
Shen [60] modified the above equation for circular plates as 
\[ \dot{\varepsilon}_m = \frac{w_m V}{3\sqrt{2} R_p^2} \], where 
\( R_p \) is the radius of circular plate. For the rectangular plate [61], the above equation may be modified as:

\[ \dot{\varepsilon}_m = \frac{w_m V}{3\sqrt{2}(L_p - S_w)B_p} \]  

(2.2)

where, \( L_p, B_p \) and \( S_w \) are half-length, width half-width of plate and half-length of wedge striker, respectively.

### 2.3. IMPACT ON BEAMS

Beams are defined as structural members having a length which is large compared with the corresponding width and depth. It is observed in this circumstance that the lateral, or transverse shear stresses are small compared with the axial, or longitudinal, stresses.

The first theoretical and experimental consideration of the behaviour of a long beam due to transverse impact by a concentrated load which induces plastic deformation is that by Duwez, Clark and Bohnenblust reported in 1950. They extended a classical analysis of nearly a century old due to Boussinesq and discussed the case of an infinitely long beam, which after impact at its centre moved with constant speed. They showed that the strain is not propagated along a beam at a uniform speed, as is the case for longitudinal impact; bending was shown to be of primary importance and shear and tension secondary. The strain was shown to depend on the ratio of the square of the distance from the point of impact to the time from first contact.

Menkes and Opat [64] conducted an experimental investigation into the dynamic inelastic failure of metallic beams subjected to impulsive loads. Three major failure modes were identified: viz., large inelastic deformation, tensile tearing and transverse shear failure at
the supports. This kind of problem has been examined analytically by Jones [65] using a simple rigid, perfectly-plastic method and by Shen and Jones [66] who utilized an energy criterion which caters for the interactions between the bending moments, transverse shear forces and membrane forces.

The deflection and failure of clamped metal beams struck transversely by a mass was systematically examined by Liu and Jones [67-69] and Yu and Jones [70, 71]. It was found [68] that, for the range of impact conditions examined, the location and type of failure for a dynamically loaded beam were similar to those for a beam loaded statically. Two types of beam failure were observed and classified as tensile tearing and shear failure. The onset of these failure modes are related to the uniaxial rupture strain of the materials, the location of the impact point and the clamping conditions. In order to obtain a more general failure criterion, a method was developed in Refs. [69] and [72] for computing the maximum strain by employing a plastic hinge whose length was estimated using slip-line theory.

Liu and Jones [68] in their experimental tests on clamped metal beam struck transversely by a mass at a distance varying from the mid span of the beam to the immediate vicinity of a support, found two types of beam failure which were classified as tensile tearing and shear failure. The onset of these failure modes depends on uniaxial rupture strain of the materials, the location of impact point and the clamping condition. It was found that the transverse shear effect may dominate the dynamic plastic response when the impact point is close to support. The membrane force plays an important role in the dynamic response of the beam when the transverse deformations are sufficiently large. It was observed that the energy absorbing capacity of beams decreased sharply when the beam was struck close to support.

In an experimental investigation, Yu and Jones [71] studied the failure of clamped beams of aluminium alloy and mild steel under impact loads and tried to draw complete engineering and true stress strain curves at strain rate up to 140 1/s and studied the dependence of rupture strain on strain rate for further numerical and theoretical studies.
involving dynamic inelastic failure. They found that the mild steel used in experiment was less strain rate sensitive at large plastic strain than small plastic strain.

Wen, Reddy and Reid [73] studied the phenomenon of deformation and failure of clamped beams under low speed impact loading (quasi static) at any point on the span by a heavy mass to estimate the dynamic plastic response and failure of clamped beams. Based on the principle of virtual work they obtained the load displacement relationship. For the rupture of beams they suggested the influence of axial and shear strain. They found the effective strain rate failure criteria to overcome the inconsistency and their theoretical prediction produced similar results as the experimental results for critical impact energy when material strain rate sensitivity is taken into account on mass ratio and on the type of structural problem.

Alves and Jones [36] studied the phenomenon of failure of beams made from a perfectly plastic material which retains strain effect and subjected to impact load theoretically based on ductile damage mechanics. They employed continuum damage mechanics (CDM), for predicting analytically the failure of material in simple structure. The CDM idealizes a material as a continuum using usual concept of stress and strain. A simple ductile damage model for perfectly plastic and linear strain hardening material requires strain and strain rate value, which were obtained by defining a plastic hinge length.

Yu and Chen [74] re-examined the plastic shear failure (Mode III) of impulsively loaded clamped beam and focused on two effects: (a) the interaction between the shear force and bending moment; and (b) the weakening of sliding section during the process of failure. A dimensional analysis was first performed to obtain a general form of threshold impulsive velocity. The elementary failure criterion is then modified to incorporate the sliding section weakening effect. The interaction between shear force and bending moment at supporting end is considered by using circular curve, Hodge's curve of yield condition based on slip line solution by taking into account the shear force and bending moment over the failing cross-section, the plastic deformation and the failure process of
the beams are traced and the ratio of plastic shear dissipation to the total plastic dissipation is thus calculated.

Li and Jones [75] proposed an analytical model for studying the material failure in shear hinges, which develops during the dynamic plastic response of beams under impulsive pressure loadings. They used a rigid perfectly plastic theory, to model the structure away from the shear hinges. However within shear hinge, strain hardening, strain rate and temperature effects were considered by them in order to obtain the condition for an adiabatic shear failure. They obtained a critical impulsive velocity for Menkas ND Opat's beam problem, which describe a transition from a transverse shear failure mode (Mode III) to an adiabatic shear-bending mode with increase of impulse.

Sperling and Partom [76] performed numerical calculations for simply supported, axially restrained beams under suddenly applied uniform pressures, and an axially restrained, clamped beam with central mass that is impacted by projectile. They considered large elastic plastic deformation and used method of finite difference. They assumed two different constitutive equations: the elastic perfectly plastic relation, and a special elastic viscoplastic, strain hardening model. Analysis of the results included examining the interaction between the bending moment and axial force, the variation of axial force, bending moment and deflections with time, and propagation velocities of the various phenomenon during motion. Experiments were carried out in which a rifle projectile hit a central mass, which had been fastened to a clamped beam. The theoretical and experimental dynamic deflections were compared.

Sharma and DasGupta [77] studied the problem of bending of axially constrained beams on non-linear Winkler type elastic foundation and obtained the solution by an iteration method using Green’s functions. Numerical results presented by them are for uniformly loaded beam hinged at both ends. Axial forces and deformations given by them in terms of three dimensionless parameters, which depend on the load intensity, foundation characteristics and properties of the beam.
Ranganath and Clifton [78] studied finite deflection dynamics of elastic beams. They obtained the solution for the problem of infinite elastic beam subjected to essentially constant velocity boundary conditions at one point of the beam and included the effects of finite deflections, normal force, rotary inertia and shear deformation. The equations of the problem were converted into non-dimensional form and perturbation approach was used to obtain a constant approximation. They obtained the numerical solution for the bending moment; shear force and normal force for different velocities of impact. The solution to the problem was dependent on a combined geometrical and material parameter, which was found to vary significantly for the compact sections and a loading parameter, which determined the amplitude of the response.

2.4. IMPACT ON PLATES

The phenomena of projectile impact on thin plates has long been a subject of great interest in a wide range of situations including the design of lightweight body armour, crashworthiness of vehicles and protection of occupants. The early work in the field includes the development of formulae for the prediction of depth of penetration into semi-infinite targets when struck normally by a projectile. The advent of battleship armour in the 19th century led to the development of equations predicting the depth of penetration of finite thickness armour plating. Even to this day these formulae and others like them are being used extensively by engineers. A comprehensive survey involving impact of projectile on plates and shells has been published by Backman and Goldsmith [23], and Corbett et al. [79] other papers covering the major experimental and analytical works done in the field.

While a number of books have been written on the subject of interior and exterior ballistics, there are far fewer monographs devoted exclusively to terminal ballistic. The earliest compilation in 1873 [80] consists of an analysis of test results using an empirical relation known as the Poncelet equation [81]. Shortly thereafter, the classical treatise of Helie [82] devoted four chapters to experimental data and their description by a variety of empirical formulas, followed by a similar exposition for the perforation of armour plate
Reference [84] summarizes experimental results obtained earlier separating the targets according to the class of material employed (concrete, steel, wood, earth, and sand), and surveys proposed semi-empirical and empirical force laws as well as mechanisms of penetration. Activities conducted in this area in the United States during World War II were reported [85]. In addition to the basic concepts and variables involved in target failure, this source lists the relative effects of different projectile types; that of normal versus oblique impact, and that of target parameters. The subject is subdivided by target material (here separated into armour plate, concrete, plastic coverings, and soil) with much of the information presented in qualitative, phenomenological, or empirical form. Detailed references are given to individual investigations. Another monograph on the entire subject of ballistic [86] contains a substantial section on terminals effects, primarily based on empirical or quasi-empirical force-penetration relations that include the topics of oblique incidence and blunt and conical nose shapes. Force laws, post-perforation velocities and mass loss predictions for frangible bullets were presented for various physical models. Some aspects of the mechanics of penetration were also mentioned in two summary reports [87, 88].

2.4.1. Perforation Studies

As the science of ballistic modeling has advanced, it has provided considerable insight to the overall understanding of penetration mechanics. The impact energy of a projectile is absorbed in global deformation of plate and local indentation or penetration. The global deformation may include bending, membrane and shear deformation. Their contributions in energy absorption depend mainly on the thickness of the target plate and the impact velocity. Membrane deformation decreases with the increase of plate thickness and bending deformation may reach a maximum at a certain plate thickness. Generally, local and shear deformations become more important with the increases of plate thickness or impact velocity. For a thick target, the perforation is dominated by the local penetration although the failure mechanism of the final perforation also influences the ballistic limit of a thick target, which depends on the target material, target dimensions, projectile nose and impact velocity. The failure mechanism leading to the final perforation of a target
plate could be ductile hole enlargement or petalling for a sharp nose, fragmentation for high impact velocity and brittle target, plugging or adiabatic shearing for a blunt projectile [89].

From the experimental study [90] it was found that the nose shape of the projectile severely affected both the energy absorption and the failure mode of the target plate during penetration. Hemispherical and conical projectiles penetrated the target mainly by indentation and ductile hole enlargement. In this failure mode, the material in front of the moving projectile is pushed away laterally. Blunt projectiles, on the other hand, caused failure by plugging which is dominated by shear banding. However, at high impact velocities, conical projectiles were found to be the most effective penetrator.

Some investigations have studied as to what way different nose shapes affect the ballistic behaviour of the target material, but the results are to some extent incompatible. From the experimental study [90] it was found that the projectile severely affected both the energy absorption and the failure mode of the target plate during penetration. Experimental results by Ipson and Recht [91] and Grabarek [92] found higher ballistic limit for conical projectiles as compared to blunt projectiles as long as the target thickness was moderate. However, for thin and thick targets it was just opposite. This is in some conflict with the results by Corran et al. [93], which showed that the critical impact energy depends on the projectile nose radius. They found that the energy reached a maximum when there was a change in failure mode from shear plugging to tensile stretching. Johnson et al. [94] reported similar results in quasi-static punching tests of metal plates. Wingrove [95] observed similar behaviour in perforation tests of aluminum alloy targets. He showed that blunt projectiles penetrated the target with the least resistance, followed by hemispherical and ogival penetrators in that order, as long as the target thickness to projectile diameter ratio was less than one. Othe et al. [96] found that the critical perforation energies for blunt and hemispherical projectiles were similar, while less energy was required for conical projectiles. They also found a distinct drop in the perforation resistance of the target as the nose angle of the sharp projectile was decreased. This decrease in critical energy was attributed to the decrease in the effective contact area for the projectiles.
Wilkins [97] found that sharp projectiles gave lower ballistic limits than blunt projectiles when fired into thick metal plates, while the opposite was found for thin targets.

With the increase of the plate thickness, the impact velocity and the bluntness of the projectile, shear plugging becomes a likely failure mode of the final perforation of a plate. Based on the conservation of momentum and energy, Recht and Ipson [98] proposed a shear plugging model to predict the residual velocity according to a given impact velocity while a ballistic limit velocity was obtained from a dimensional analysis, which completely ignored the structural response for relatively thin plates and the local penetration for relatively thick plates.

The so-called structural model may be considered as a further development of Recht and Ipson's model [98], but it is actually an application of rigid-plastic structural dynamics [41] in perforation studies. A structural model, which neglects the local penetration, has given good prediction for thin plate perforation under blunt projectile impact, as shown in Refs. [99-101].

The shear plugging in a structural model is based on the shear hinge concept in a rigid-plastic model, which may be triggered at the early response stage for a relatively thick structural member [44, 102-104]. It is still a challenge to obtain a general structural model of perforation analysis, which incorporate the local failure analysis in the rigid-plastic model because different local response and failure modes, such as dishing, petalling or penetration, may appear for different plate thicknesses and impact velocity [79]. Meanwhile, it is difficult to get simple formulations for a general structural model, which is not in favour of engineering applications.

Goldsmith and Finnegan [105] reported a series of tests involving the normal impact of non-deformable steel spheres on aluminium and mild steel plates at 150 to 2700 m/s. It was found that the velocity drop experienced by the projectile decreases from the ballistic
limit to a minimum value and then increases monotonically with initial velocity for all projectile-target systems. It was observed that the amount of global target deformation of the plate is at a maximum at the ballistic limit and then decreases as the impact velocity increases.

Goldsmith and Finnegan [106] carried out an experimental programme of normal impact of cylindro-conical and blunt cylindrical projectiles of 12.7 mm diameter, on aluminium and steel plates at the velocity range of 20 to 1025 m/s. Projectile of hard steel and soft aluminium were used in the study. Velocity drop during perforation was measured and metallurgical examination of the target damage was carried out on selected plates.

With the increase of plate thickness, the local effect becomes more important, accompanied by a reduced effect of structural response on the plate perforation. Multi-stage models have been proposed to study the perforation of relatively thick plates when structural response is negligible. Awerbuch [107] divided the penetration of a plate target into two stages. In the first stage, only inertia and compressive forces are introduced to decelerate the effective mass of the projectile. The second stage is initiated when a shear plug of the target material is formed, during which the compressive resistance is replaced by the surrounding shear force. Goldsmith and Finnegan [105] improved the model of Awerbuch by considering the reduction of the shear force in the second stage. Awerbuch's two-stage model was extended to a three-stage model for the perforation of a plate by a non-deformable projectile [108]. The projectile is subjected to the inertia and the compressive resistance of the target material as well as the shear resistance around the plug in the middle stage. This model has been further modified by Ravid and Bodner [109], whose two-dimensional model assumes five stages of plate penetration, namely dynamic plastic penetration, bulge formation, bulge advancement, plug formation and projectile exit. The first, principal stage involves the determination of an approximate plastic flow (velocity) field surrounding the projectile at each increment of penetration and includes the development of a “lip” on the entry surface of the target. Material strain rate dependence was considered in obtaining the associated stresses and the work rate terms. Certain parameters that characterize the field of stage one are determined by
utilizing a modification of the upper bound theorem of plasticity to include dynamic effects. These five stages are continuously coupled during penetration and the model predicts not only the exit velocities of the projectile and the plug, but also the bulge and plug shapes as well as the force-time history of the process. This analytical model was further generalized by Ravid et al. [110] to include various projectile nose shapes, change in the plastic flow field due to deep penetrations and thermal softening effects. Liss et al. [111] proposed another five-stage interactive model for the penetration and perforation process, where plastic wave propagation in both the thickness and the radial directions of the plate is considered. A numerical procedure is necessary to solve equations of motion of the projectile in these five stages.

In the five-stage perforation model of Liss et al. [111], the shear wave propagation in the target plate outside the plug interface was considered. However, the structural response outside the shear plate is generally neglected in most multi-stage models. Shadbolt et al. [112] hybridized the multi-stage model and the structural model. It was shown that the analytical prediction is improved when Reissner's plate theory is used as a structural model to replace the simplified plastic membrane and bending model. However, uncertainties of some empirical data and the involvement of numerical procedures increase the difficulty for their practical application.

Experimental studies on the perforation of metallic plates have been summarized in Refs. [23, 79]. Experimental results were presented either by the variation of ballistic limit with plate thickness and the hardness of the plate material, or by the variation of the residual velocity of the projectile with impact velocity, which have revealed several interesting phenomena. First, it was shown that the perforation ballistic limit may decrease with the increase of plate thickness in a range of plate thicknesses [93]. Most of the analytical models cannot predict such a local drop of the ballistic limit except the rigid-plastic plate model of Liu and Stronge [100], which considered all possible structural responses in a perforation analysis. However, their model ignored local penetration and require a numerical algorithm to solve non-linear differential equation based on a selected velocity field, which is not favorable to practical applications. Second, a residual velocity jump at
the ballistic limit was noted by Forrestal and Hanchak [113] for the perforation of HY-100 steel plates. They showed that the residual velocity may jump to a finite value at the ballistic limit in some cases. Neither Recht and Ipson's model [98] nor other structural and multi-stage models can predict the velocity jump at the ballistic limit. They used a rigid-plastic beam model to illustrate this phenomenon. Third, a local drop of ballistic limit with the increase of target material hardness was observed by Sangoy et al. [114]. This observation is in contradiction with the previous design philosophy, i.e., the higher the hardness, the better the perforation performance. Further experimental evidence for this phenomenon was given by Pereira and Lerch [115]. It is believed that this local drop is due to the initiation of adiabatic shearing regime [79, 114, 115] although the increased brittleness of the hardened material may also contribute to an easy failure of the target. The transition from shear failure to adiabatic shear failure in a beam was studied by Li and Jones [116]. However, authors have not noted any analytical model to study this problem for a plate.

Plastic deformation of plates is large especially at low velocities when the perforation does not take place. Beynet and Plunket [117] studied the plastic deformation of thin plates when the plate is impacted by a blunt projectile at velocities below the ballistic limit. Their analysis for transverse deformation of thin plates agrees with the experimental values except near the elastic plastic boundaries and before the commencement of elastic recovery. The technique of Moire displacement fringes was applied for the measurement of transverse deformation of plates.

Calder and Goldsmith [118] in the same year studied the plastic deformation of thin plates (1.25 mm) resulting from the impact of 12.5 mm diameter spherical or cylindro-conical projectiles at a velocity range of 25 to 300 m/s. They proposed a simplified model for central deflection of rigid-plastic linear work hardening material. They found that the simplifications made for modeling of perforation phenomena, introduced
incorrectness of results at low velocity impact, whereas, the results matched the experimental data at higher impact velocities.

Othe et al. [96] estimated the critical fracture energy of carbon steel plates of thicknesses, ranging from 7 to 38 mm when impacted normally by heavy blunt, hemispherical and conical nose stainless steel projectiles of diameters varying from 66 to 160 mm. The ratio of diameter of projectile to the thickness of target in these experiments varied from 1.7 to 23. The mass of the projectiles varied from 3 to 50 Kg, and the tests were carried out in the velocity range of 25 to 170 m/s. They derived a formula for the evaluation of critical fracture energy, based on the experimental results for three kinds of nose shapes of the projectiles.

Mode of deformation and ballistic limit of mild steel tubes and plates were studied by Palomby and Stronge [119]. Blunt and round nose projectiles of 12.7 mm diameter were impacted on tubes plates, which were 1 to 2.5 mm thick, at 100 to 250 m/s impact velocities. They observed that failure mechanism in both cases were plugging accompanied by dishing of the target.

Petalling is the dominant phenomenon in thin aluminium plates when struck by ogival-nosed or cylindro-conical projectile at sub-ordnance velocities. Landkof and Goldsmith [120] carried out a theoretical and experimental investigation of petalling of thin metallic plates impacted by cylindro-conical projectiles. A model considering initial crack propagation, plastic hinge motion up to the root of crack, and petal bending due to hinge rotation was presented. The experimental programme was carried out for verification of the model wherein 12.7 mm diameter cylindro-conical projectiles were impacted on 3.175 mm thick aluminium plates both with and without hole at their centre. Good correlation was found between the experimental and theoretical residual velocities except near ballistic limited to and beyond the position of projectile passage.

Corren et al. [93] investigated the effect of projectile mass, nose shape and hardness on penetration of steel and aluminium alloy plates. Blunt and cylindro-conical projectiles of
12.5 mm diameter, were impacted on plates of 1.3 to 5.9 mm thickness in the velocity range of 50 to 250 m/s. The mass of the projectile was varied from 15 to 100 g. They carried out some experiments on layered plates as well. They observed that ballistic limit of the plate changes with change of projectile mass and nose shape. In multilayer targets, they observed that the order of unequal plate thicknesses is important and there is an advantage in placing the layers in contact.

Levy and Goldsmith [121] used instrumented projectiles having piezoelectric quartz disc inside the projectile body. Interchangeable heads of 6.35 and 12.7 mm diameters and three nose shapes viz. blunt, spherical and conical, were fitted onto the same instrumented projectile body. A detailed experimental study was made in the velocity range of 20 to 300 m/s, and on 1.27 and 3.175 mm thick aluminium plates and 1.2 mm thick mild steel plates. Impact load, permanent deflection and strain were measured with the help of instrumented projectile, optical profile projector and strain gauges respectively. Most of the observations were recorded at just below and just above the ballistic limit. The measured peak force and plate deflection of aluminium plate below the ballistic limit shows a good correlation with the values predicted by the model [119] but in the case of mild steel plates, the results show discrepancy.

Virostek et al. [122] employed instrumented projectiles, similar to those used by Levy and Goldsmith [121], for recording force histories of normal impact of conical and hemispherical nosed projectiles on aluminium plates. The tests were carried out at impact velocities between 45 to 170 m/s. The effects of initial velocity and nose shape on the peak have been studied. Simple models have also been developed to predict the peak force.

Liu and Stronge [100] presented three rigid plastic models, based on different assumptions for yield, for deformation of circular plate impacted by blunt projectiles. Bending and stretching of the plate and shearing of a plug at the centre of the plate is taken care of in the modeling. Computed results, based on these models, are compared with some experimental results available in literature.
Wen and Jones [123] proposed a semi-empirical equation using dimensional analysis, for the perforation energy in the case of mild steel plates struck by blunt projectiles. Energy absorbed by global structural response, including the effect of bending and stretching and energy absorbed in shearing out a plug equal to projectile diameter were taken care of in the formulation. The results from this equation compared well with the impact test results of mild steel plates at low impact velocity.

Dey et al. [124] tried to observe the effect of target strength on the perforation of steel plates using three different projectile nose shapes. Using three steel alloys, viz, Wedox 460 E, Weldox 700 E and Weldox 900 E, perforation tests were carried out in 12 mm thick plate with blunt, conical and ogival nose projectile. A compressed air gun was used to launch projectile within the velocity range from 150 to 350 m/s. Their work mainly concentrated on the ballistic capacity of the targets, where the varying factors were the target strength in relation to impact velocity and nose shape of the projectile. Although most empirical models indicated that there is monotonic increase in capacity for increasing target hardness. But some scientists have also found that as the hardness of the plate in increased the ballistic limit increases, but only up a certain hardness level. Beyond, this limit when the failure is dominated by the adiabatic shear the ballistic limit velocity decreases for increasing hardness. The projectile used by Dey et al. [124] was having mass of 197 g and diameter 20 mm. In order to keep a constant mass, the length of the projectile varied slightly depending on the nose shape. The target plate was having a free span of 500 mm diameter and it was clamped in a circular frame. In those tests in which perforation occurred, the residual velocity of the projectile and possible plugs were measured using a laser curtain before they were captured in a rag box filled with graded plywood. The results clearly show that the ballistic limit velocity decreased for increasing yield strength when using blunt projectile while the opposite trend is seen for tests with conical and ogival projectiles. They performed some tests and collected data for the ballistic limit velocity and residual velocity. A curve through the data point was fitted to an analytical model.
Goldsmith [125] reported some valuable experiment involving the oblique impact of spherical steel projectile against thin plates of mild steel (SAW 1010), 2024-T3 and 6061-T6 aluminium targets. The tests were conducted at sub-ordinance, ordinance and ultra-ordinance and hypervelocity range. First he did analysis and compared it with the experimental values.

Gupta and Sagar [126] observed the penetration mechanism of 8.5 mm steel sphere on to mild steel plates of 1.6 mm and 3.0 mm thickness by doing some experimental tests. The projectiles were launched between striking velocities of 10 m/s to 900 m/s with the help of smooth bore gun. The effect of striking velocity on various mechanisms like plate bending, petalling, projectile deformation and plug fusion etc were observed. During these trials various parameters influencing the penetration mechanism like crater dimensions, post perforation velocity, mass of debris, plug and projectile mass after perforation were recorded. In their study an attempt was been made to establish a correlation between striking velocity and crater volume, striking velocity and crater diameter and energy absorbed during penetration as a function of crater diameter.

An experimental investigation involving normal and oblique impact of a spinning armour piercing projectile of core diameter 6.2 mm, on aluminium, mild steel and RHA steel plates of 6 to 40 mm thicknesses was carried out by Gupta and Madhu [127, 128]. Residual velocity, velocity drop and crater dimensions were measured. Simple models based on the experimental results were proposed for residual velocity, velocity drop, ballistic limit and critical ricochet angle.

In most cases of ballistic perforation, there is deformation and flattening of the projectile due to the high resisting forces. This change of geometry generally causes further increase in those forces, and the interaction between projectile deformation and penetration is an important aspect in the development of analytical models. In Awerbuch and Bodner [120], for example, the cross-sectional area of the projectile is taken to be an empirical function of the longitudinal co-ordinate. It is usually sufficient in practical use of that model to consider a mean cavity diameter based on averaging the entry and exit
holes. For a given projectile and target plate, it is generally possible to extrapolate limited test results for the cavity size and shear zone extent over a wider range of impact velocities and target thicknesses. Dehn [129, 130] also accounts for mushrooming in a phenomenological manner. Furthermore, he treats finite thickness effects in a similar phenomenological manner [129], where an average parameter, interpreted as the target thickness at the moment of failure, is used analogous to an average presented area (i.e. 'mushrooming'). However, these empirical or semi-empirical approaches are limiting, and it is desirable to formulate ballistic models that are as deterministic as possible.

Nurick and Conoly [131] have reported their experiments on the response of single and double stiffened clamped rectangular plates. A plate of $113 \times 70$ mm with thickness of 16 mm was placed on a steel plate and was clamped using eight 10 mm diameter bolts. The beam was placed in the centre for one case across the late width and in the next case they were unequally spaced across the plate width but were equally spaced along the length. For these two cases, explosives with different configurations and at different positions were used. For the single stiffener arrangement, the largest deflection occurred at midway between the beam and clamped boundary. For the second case, the deflections occurred at the plate mid point while the deflection of the plate at a position of the beam and clamped boundary was also larger than the deflection at the stiffener point.

Nurick and Crawcour [132] investigated the ballistic limit of single and layered fiberglass plates. Plates of 190 mm diameter and 1 to 5 mm thickness were subjected to projectile impact in the velocity range of 20 to 70 m/s. Single plates and layered plates of 1 mm thickness were impacted by blunt or conical nose projectiles of 12.6 mm diameter. Results were compared with models of metallic plates with slight modifications and found to match reasonably well for monolithic as well as layered plates.

A commonly used measure of a target's ability to withstand projectile impact is its "Ballistic Limit Velocity" simply known as "Ballistic Limit" and much work has been carried out by researchers to enable estimates of this parameter. Another useful term is
"Ballistic Limit Thickness", which is the minimum thickness of plate required for a projectile of known weight and velocity to prevent any perforation.

In addition to the analytical studies, empirical relations for predicting the plate response have also been developed by researchers in the past. Two fairly simple formulae which are being used for mild steel were developed by Stanford Research Institute (SRI) and the Ballistic Research Laboratories (BRL). SRI formula provides an estimate of the perforation energy of the plate for a particular target projectile system.

The perforation analysis of ductile plates under the impact of projectile is a complex problem and there are three reasonably distinct approaches for these investigations: (i) development of empirical formulae based on experimental data; (ii) derivation of analytical models by applying the relevant equations of motion and material behaviour, and (iii) numerical solution based on solving the governing equation over a spatial grid at successive time increments.

2.4.2: Growth of Hole Studies

The hypervelocity domain is of concern to the ballisticians because of the application of shaped charges and of the comparable effects produced by high velocity penetrators. It is also of fundamental scientific interest in the study of impacts produced by meteors and meteorites; it permits the understanding of system behavior in a regime not ordinarily experienced. The representation of the diameter of hole in the shield/plate or some other component of a spacecraft as a function of various projectile and target parameters helps in the assessment of its damage and hence its performance and functioning. The diameter of hole in thin targets may also give an idea about the characteristics of the projectile. When a meteoroid strikes the shield, it breaks the meteoroid thus reducing the impact on the body of the spacecraft.

The larger stresses occurring in hypervelocity impact permit neglect of rigidity and compressibility of the striking bodies, and the impact is viewed as fluid flow. The
description of material properties is then greatly simplified and correction terms are added to the original Bernoulli equation to account for target and projectile strength. Some such fluid mechanics based analytical solutions for predicting hole diameter under hypervelocity impact are available in literature [133, 134] but these models also involve empirical constants. The influence of body curvature of the target on the penetration hole size was included in Ref. [135] but its effect was found to be of the order of other uncertainties and was thus negligible. For the impact of spheres on plates, the morphology of lip structure was found to be dependant on the ratio of plate thickness to sphere diameter especially when this ratio is large. Many empirical relations are available in literature [136-138] but most of them are applicable for the data for which these have been developed.

The velocity range of interest to spacecraft industry is up to 18 km/s because the anticipated average impact velocity of orbital debris for spacecraft in low earth orbit is about 11 km/s and of micro-meteoroids is about 18 km/s. In the beginning, the velocity range in experimental studies was upto 1 km/s using powder guns which later in 1960’s increased to 3 km/s using light-gas gun. It was, however, only just the beginning to enter into the truly hypervelocity regime for most targets and projectile materials. Several researchers [139-143] carried out experimental investigations involving the impact of spherical projectiles on thin plates of the same or different material at impact velocity varying from 1 km/s to 15 km/s using the light-gas guns.

When a projectile strikes a target plate, it forms crater and if sufficient kinetic energy is there, it perforates the target. It has been observed in experiments that the perforation may cause formation of lips or it may be a clean hole with fine lips, if any, depending upon the values of impact velocity and characteristics of projectile and the target. If the target plate is perforated by projectile of circular cross-section, the hole is circular for normal strike and elliptical for oblique strike and the diameter of hole is usually greater than the projectile diameter. A large number of models mostly empirical and some semi-empirical based on mechanics have been developed for the prediction of the hole-diameter in a thin target plate impacted by spherical projectiles at hypervelocity.
Maiden et al. (1963) [144] conducted tests using 3.2 mm diameter Aluminum spheres of an unspecified alloy and 2024-T3 Aluminum plates and developed the following model for the prediction of hole diameter,

\[
\frac{D_h}{D_p} = 0.45 \left( \frac{T_i}{D_p} \right)^{2/3} V + 0.9
\]  \hspace{1cm} (2.3)

Rolsten (1964) developed a mechanics-based model for spherical projectile which involves only the projectile and target material densities [145]:

\[
\frac{D_h}{D_p} = \sqrt{2 + \frac{\rho_i}{\sqrt{\rho_p}}}
\]  \hspace{1cm} (2.4)

If Rolsten's model is used for Aluminum projectile and target, the ratio, \( \frac{D_h}{D_p} \), is constant equal to \( \sqrt{3} \). Rolsten's model does not follow the experimental data and thus it could not find any favour.

Sawle [146] derived a model from Ames penetration depth equation for the prediction of the diameter of hole,

\[
\frac{D_h}{D_p} = C_1 \left[ \left( \frac{\rho_p}{\rho_i} \right) \left( \frac{V}{c} \right) ^{0.22} \left( \frac{T_i}{D_p} \right)^{2/3} \right] + 1.0
\]  \hspace{1cm} (2.5)

The theoretical value of coefficient, \( C_1 \), in the above model was obtained as 2.6 but on the basis of regression analysis of data, its value was suggested as 3.2. Taking a lead from the penetration analysis of Sorenson [147], Sawle (1969) modified [146] the above model by incorporating the target shear strength, as given below:
where, \( \sigma_{ts} \) is the shear strength of the target material. Nysmith & Denardo (1969) [148] used the following equation to describe the results of tests that used 3.2 mm diameter, 2017-T4 Aluminum spheres and 2024-T3 Aluminum plates

\[
\frac{D_h}{D_p} = \left( \frac{\rho_p}{\rho_i} \right)^{0.05} \left[ \frac{\rho_p}{\sigma_{ts}} \frac{V^2}{T_i} \left( \frac{T_i}{D_p} \right)^{2/3} + 1.0 \right]
\]  

(2.6)

Carey et al. (1985) [149] developed the following equation to describe the diameter of hole produced by small iron projectiles impacting Aluminum foils at velocities ranging from 2 to 20 km/s.

\[
\frac{D_h}{D_p} = 1.47 \left( \frac{T_i}{D_p} \right)^{0.45} V^{0.5}
\]  

(2.7)

When a spherical projectile strikes the target at an angle, \( \theta \), with the normal, the hole in the target plate has been found to be elliptical for which Schonberg (1990) [150] developed empirical equations for major and minor diameters of ellipse through multivariable linear regression of data obtained from tests conducted at the Marshall Space Flight Center (MSFC) and Space Debris Impact Facility (SDIF) as given below:

\[
\frac{D_h}{D_p} = 2.825 \left( \frac{V}{c} \right)^{1.043} \left( \frac{T_i}{D_p} \right)^{0.782} \cos^{0.283} \theta + 1.01
\]  

(2.10)
Piekutowski (1996) [143] developed a simple model for predicting hole diameters in thin Aluminum plates when impacted by 2017-T4 Aluminum projectiles at impacted velocities of approximately 6.7 km/s. This relationship is given as:

\[
\frac{D_h}{D_p} = 4.5 \left( \frac{T_t}{D_p} \right) + 1.0
\]  
(2.12)

Piekutowski stated that this relationship is not valid for ratios of target thickness to projectile diameter less than 0.08 because of the nonlinear nature of the relationship below this level. This model is independent of the velocity of strike because it was developed for one velocity of strike, therefore, it can not be used for any other data.

A model for the prediction of hole-diameter based on the principle of fluid mechanics was developed by Chant (2004) [135], given by

\[
\frac{D_h}{D_p} = \sqrt{2\pi} \left[ \left( \frac{\rho_p}{\rho_t} \right) \left( \frac{V}{c} \right) \right]^{0.5} \left( \frac{T_t}{D_p} \right) + 1.0
\]  
(2.13)

Hill (2004) [136] proposed a model based on the data taken from different sources including the data pertaining to oblique strike but the angle of strike has not been used in his analysis thus making their model independent of it.

\[
\frac{D_h}{D_p} = 3.309 \left[ \frac{\rho_p}{\rho_t} \right]^{0.022} \left( \frac{V}{c} \right)^{0.298} \left( \frac{V}{c_p} \right)^{0.033} \left( \frac{T_t}{D_p} \right)^{0.359}
\]  
(2.14)

Hill [136] proposed another model with additive constant:
but the difference between the two models is very small.

**Concluding Remarks:**

It is observed that most of the models presented above were mainly developed by different researchers mainly for their own data, except the model proposed by Hill, which has used a wide variety of data. Some other observations about these models are given in below and some more are discussed in chapter 4 where the analysis of data is presented.

- Apparently, the hole diameter equations given by Maiden and Nysmith & Denardo are not dimensionally homogeneous but these may be made so because of the consideration of single material i.e. Aluminum.
- The model of Maiden and Nysmith & Denardo are independent of the density of materials of projectile as well as the target. This is just because the models were developed for single material, i.e. Aluminum.
- Models developed for more than one material, such as Sawle, Chant and Hill models, incorporate the velocity of sound in the target and/or projectile material.
- Most of the models were developed for normal strike of projectile with the exception of Schonberg's model, which incorporates the angle of strike.
- As the angle of strike approaches to zero (i.e. normal strike), the hole approaches to circular, therefore, major and minor diameter, given by Schonberg's model should approach each other which is not happening. Therefore, these equations cannot be used for low angle of strike especially when the strike is normal.
2.5. AXIAL CRUSHING OF SHELLS

Thin-walled shells, such as tubes, frusta, hemispherical shells, and domes are potential candidates for their use as energy absorbing elements in crashworthiness applications in aircraft and other transport vehicles due to their high specific absorbing capacity and the stroke efficiency. The main advantage is that designers have greater flexibility in tailoring the material to meet the specific requirements of loading and changing environment.

2.5.1. Axisymmetric Crushing of Cylindrical Tubes

The plastic-mechanics of structural elements like tubes of circular and non-circular sections, spherical shells, and conical frusta, have received considerable attention during the last four decades. Their application in the design for absorbing kinetic energy in situations of a crash or an accident is common. Various factors that determine the efficiency of performance of the energy absorbers, and their selection criteria have been discussed in detail in [151]. The axial collapse mechanisms of thin-walled tubes of circular, square or rectangular sections under static or dynamic loading in particular, have been studied by various investigators in the past [152-154].

There are practical situations when thin-walled members are subjected to axial impacts. Indeed, the impetus for some of the early impact studies on circular tubes was an attempt to reveal the characteristics of structural crashworthiness for railway coaches [155]. A more recent example, from the aerospace industry, is the dynamic buckling which might arise from the sudden thrust of a rocket engine ignition [57].

Metallic and non-metallic tube structures are used as energy absorbers in road and rail vehicles or aircraft. They achieve a controlled reduction of the kinetic crush energy for the purpose of limitation of critical passenger acceleration and injuries. An ideal absorber is defined as one which maintains the maximum allowable retarding force throughout the stroke, apart from elastic loading and unloading effects [156, 157]. However, a designer
must often trade off considerations of cost, volume, stroke, weight, deceleration, etc. against efficiency. Above all, an energy absorber must be reliable and in many cases versatile enough to absorb a dynamic load which may strike at a random location.

The energy absorption process of metallic tube structures is based on the formation of plastic folds, which is dependent on the tube geometry and the time dependent loading conditions. The critical buckling load value is also an important design criterion for aerospace structures that are subjected to axial impulsive loading. In the static case, the buckling can theoretically be treated as a bifurcation problem. Dynamic buckling is not a bifurcation problem but a definition of acceptable displacement. Generally, it is impossible to obtain an exact expression for the dynamic buckling load.

Round tubes which collapse progressively in axi-symmetric concertina, diamond or mixed mode are very good energy absorbers and consequently several experimental and analytical studies have been devoted to the understanding of the mechanics of deformation involved in their collapse.

Alexander (1960) [154] gave an analysis of the axi-symmetric concertina mode of deformation, which forms the basis of many later studies. He derived a simple expression for the mean crushing force, \( P_m \), and the size of fold, \( h \), for cylindrical tubes of diameter, \( D_c \), and thickness, \( t \), by considering two separate plastic dissipation mechanisms: axial bending in concentrated hinge lines and circumferential stretching of the shell material between the hinge lines.

\[
\frac{P_m}{M_p} = 20.73 \left( \frac{D_c}{t} \right)^{0.5} + 6.283 \\
(2.16)
\]

\[
h = 0.952 \sqrt{D_c t} \\
(2.17)
\]
where, \( P_m \) and \( M_p = \sigma_y r^2 / 4 \) are the mean crushing force and full plastic bending moment per unit length respectively; and \( \sigma_y \) denotes the yield stress of the tube. This simple model did provide very good results of the size of fold, however, the mean collapse load is underestimated [Grzebeita (1990) [158]]. The main kinematic assumption made by Alexander was that one fold was formed at a time. It was further assumed that a given element goes through the entire crushing process before its neighbor begins to deform. After the element is crushed, the tube returns to its ultimate compressive capacity at the end of each cycle; a characteristic which is not observed experimentally. Furthermore, in the model with stationary plastic hinge, a cylindrical shell is deformed into a system of flat annular discs meaning that the entire length of the tube is available for crushing. Despite its various limitations, the assumption that one folding wave forms at a time has remained unchallenged until the present time.

Abramowicz and Jones (1984) [152] proposed an improved model with the tube wall bending in the meridional direction into two oppositely curved arcs instead of a straight line:

\[
\frac{P_m}{M_p} = 20.79 \left( \frac{D}{t} \right)^{0.5} + 11.90 \tag{2.18}
\]

They used ultimate stress instead of the yield stress to account for the strain hardening, while the material considered was again rigid plastic. Effective crushing distance was introduced in their analysis and the mean collapse load thus obtained showed improved results. The bending moment capacity used in the analysis was \( M_p' = \sigma_u r^2 / 2\sqrt{3} \) (\( \sigma_u \) being ultimate stress) and the size of fold was found as \( h = 0.877\sqrt{D_t} \) (about 92% of the size of fold found by Alexander).

Both of the above analyses were directed towards determining the mean collapse load. Grzebeita (1990) [158], however, gave a method to determine the load history between a peak and a minimum in a load oscillation of the load-compression curve. The bending
moment capacity that he assessed was $M'_p = 2\left[1 - (3/4)P^2\right]M_p/\sqrt{3}$. If the second term in the bracket is neglected, one obtains $M'_p = 2M_p/\sqrt{3}$. Grzebeita proposed a model wherein the curvature was confined around hinges, and the central span of one-third length was flat. In his analysis the fold length was taken from the analysis given by Alexander [154], arguing that it agreed with experiments quite well. In all the above analyses [Alexander [154], Abramowicz and Jones [152], Grzebeita [158]], the Von-Mises yield criterion has been employed. Wierzbicki et al. (1992) [159] studied the axisymmetric collapse mode of round tubes by considering the internal folding. The half fold length determined in their analysis is $h = 1.244\sqrt{D_t}$. The plastic hinge moment capacity taken by them is equal to the hinge moment, i.e. $M_p = \sigma_m t^2/4$, where $\sigma_m$ is the average flow stress which is equal to 92% of the ultimate stress of the material.

Wierzbicki and Bath (1986) [160] modified Alexander's solution and replaced stationary plastic hinges with moving hinges. This led to realistic deformed shape and improved prediction of the mean crushing force. However, because the main simplifying assumption that one folding wave forms at a time remained unchanged, the modified model retained the unrealistic features of Alexander's solution:

$$\frac{P_m}{M_p} = 22.27\left(\frac{D_t}{t}\right)^{0.5}$$ (for the stationary hinge model) (2.19)

$$\frac{P_m}{M_p} = 31.74\left(\frac{D_t}{t}\right)^{0.5}$$ (for the moving hinge model) (2.20)

Wierzbicki and Bath [160] addressed the fact that experimentally the tube wall is observed to fold both inwards and outwards. They introduced a parameter known as the eccentricity factor, $e$, which was defined as the ratio of the inside fold length to the total fold length. This then enabled a complete force-displacement curve to be developed, not just an average force. This work has been further refined by Singace et al. (1995) [161]:

42
\[
\frac{P_m}{M_p} = 22.27 \left( \frac{D_c}{t} \right)^{0.5} + 5.632
\]  

(2.21)

All of these studies predict that the normalized mean crushing load, \( \frac{P_m}{M_p} \), is roughly proportional to \( (D_c/t)^{0.5} \) for the conventional range of \( D_c/t \) values. However, it may be observed that a curve fit to a large number of previous experimental data taken from reference [162] demonstrates that \( \frac{P_m}{M_p} \) is closely proportional to \( (D_c/t)^{0.33} \). It seems that some important characteristics have been overlooked in previous theoretical studies as pointed out by Guillow et al. [162].

The original analysis developed by Alexander was motivated by a specific need for predicting energy absorption of tubes subjected to dropped objects in nuclear reactor applications. Interestingly, much of the current industrial research effort is driven by similar practical needs [Magee and Thornton, 1978 [163]; Mahmood et al., 1986 [164]]. An enormous amount of empirical knowledge has been accumulated to date on the strength and energy absorption of tubes in terms of geometry, material properties and loading conditions [Jones, 1989 [44]]. Parallel to experimental efforts, several specialized FE codes for crashworthiness applications such as PAM-CRASH [Haug et al., 1986 [165]] and DYNA-3D [Hallquist and Benson, 1986 [166]] can now reproduce with some realism the actual crushing process.

Despite all these advances, a full understanding of the physics of the problem is lacking or lags behind practical applications. For the calculation of mean crushing load, Alexander [154] had taken mean circumferential strain. But the calculation of variation of crushing load requires the consideration of variation of circumferential strain with change in the rotation of fold [152].
Abbas et al. [167, 168] considered total inside folding in their formulation. The early models discussed above, in general, do not satisfy the experimental observations [153, 169] where it has been observed that the folds formed are partly inside and partly outside the initial mean diameter of the tube. In some recent studies [153], models were developed with partly inside/outside folding but the folding parameter, \( m \), which is the ratio of inside to total fold length was either taken from experiments or it was assumed. Some useful models were later developed for determining the value of folding parameter mathematically and incorporated the change in the thickness of the tube during fold formation [170-172]. The straight fold and curved fold partly inside and partly outside models of Gupta and Abbas [171-174] for axial crushing tube take into account the variation in the thickness of tube over the fold length. The straight fold model incorporates a parameter \( r' \), which is the ratio of the yield stress values of the tube material in compression and tension. The influence of its variation is considered and is found to help in explaining the process of fold formation. The value of the fold parameter has been determined by minimizing the average crushing load. Incorporating changes in thickness in the models, the fold parameter comes closer to experiments. The total outside and the total inside fold models can be easily derived from these models.

2.5.2. Axisymmetric Crushing of Frusta

The performance of frusta as an energy absorbing device has been compared in the past [175] with tubes through experimental investigations. Several papers dealing with the theory of the collapse of frusta [167-168,175] have appeared in literature. In frusta with low semi-apical angle (< 10 degree), first few folds are observed in experiments to be of concertina type [151]. It is due to this reason that the mathematical analysis for axisymmetric crushing is required.

Seide [176] investigated the axisymmetric buckling of circular cones under axial compression and obtained a simple expression to relate the cone-buckling load to the buckling load of a cylinder of same thickness. Postlethwaite and Mills [177] performed the axial crushing of conical shells of semi-apical angles ranging from 5° to 20° and
studied their energy absorption capacity. Analytical and experimental results were presented for the axisymmetric buckling of stainless steel and aluminium truncated conical shells of semi-apical angles of $5^\circ$ by Ramsey [178, 179]. He considered time dependence of external load in the equilibrium equations.

As a part of studies on crushing analysis of rotationally symmetric plastic shells, analysis of conical shells subjected to a centrally applied load has been reported by De Oliveira and Wierzbicki [180]. It was assumed that the conical shell deformed as an inverted shell with toroidal surface at the top due to a point load at the centre. Expressions were proposed for the prediction of load-deformation behaviour and the radius of the toroidal surface at the top.

Mamalis and Johnson [175] have studied the axial compression of aluminium conical frusta of semi-apical angle from $5^\circ$ to $10^\circ$ under quasi-static loading. The load-deformation behaviour, initial peak load, mean collapse load and various modes of collapse were reported from the experimental observations. The initial peak load and mean collapse load was found to increase with the increase in slenderness ratios (i.e. diameter/thickness ratio) of the conical frusta. It was also observed that collapse began with the formation of an axisymmetric diamond pattern. Mamalis et al. [181] have also performed the axial compression on steel thin-walled frusta of semi-apical angle from $5^\circ$ to $10^\circ$ at elevated strain rates. The load-deformation behaviour and the collapse in this case were similar to similar to those in [175].

Mamalis et al. [182] later reported in-extensional axial collapse of thin PVC conical shells of semi-apical angles $5-35^\circ$. The mode of collapse was found to be the same as that reported in [175, 181]. They have also proposed an analytical model for the predication of the mean collapse load for conical shells collapsing in diamond mode, by using the concepts of stationary and included rolling plastic hinges of constant radius observed in experiments.
To understand the influence of end constraints on the behaviour of thin shell, experimental studies were carried out by El-Sobky and Singace [183]. In that it was postulated that pre-buckling elastic stress pattern in axially compressed right frustum would determine its initial and progressive buckling mode. They also studied and compared the dynamic behaviour of conical frusta with the static response [184]. It was pointed out that due to inertia effects, the absolute value of the energy absorbed by frusta were higher in dynamic loads than that under quasi-static loads.

Gupta and Prasad [185, 186] studied experimentally the behaviour of conical frusta of large semi-apical angles. Experimental results of frusta tested at various strain rates were presented.

Mamalis et al. [187] proposed an analytical model for the frusta collapsing in concertina mode, in which the effect of slenderness ratio (i.e. diameter/thickness ratio) and semi-apical angles on the modes of collapse, initial peak load and mean collapse load have been studied. The crumpling in the concertina mode of PVC thin-walled conical frusta of semi-apical cone angles 14-35° under axial compression has also been reported. It was observed that the mode of collapse was concertina in case of high slenderness ratio otherwise it was progressive collapse in diamond mode. The results, from the proposed analytical model have been compared with the experimental results reported in [175, 181].

Mamalis et al. [182, 187] have also proposed an analytical model for the progressive extensible plastic collapse of thin-walled conical shells collapsing in diamond mode after the formation of an initial ring. For the experimental validation of the theoretical model, PVC conical frusta, cylindrical and conical shells were tested in axial compression. The load deformation behaviour obtained from experiments was found to match well with the predictions of the analytical model. A mathematical model for axisymmetric axial crushing of thin frusta with total outside straight folds has been developed by Gupta and Abbas [188]. The results were compared with experiments and good agreement was found.
2.5.3. Diamond Mode of Collapse for Cylindrical Tubes and Frusta

The consideration of circumferential buckling leads to non-symmetric (i.e. diamond) mode of collapse which has been observed in a large number of experiments carried out using cylinders and frusta [152, 175] with increasing rate of loading. So far, no definite demarcation between concertina and diamond mode of collapse has been established on the basis of the strain rate. Pugsley and Macaulay [175] proposed the following empirical relations for the cylinders.

\[
\frac{P_b}{P_y} = 3.2 \frac{t}{D_c} + 0.12 \tag{2.22}
\]

where, \(P_b\) is the buckling load and \(P_y\) is the yield end load which given by \(P_y = \sigma_y \pi D_c\).

Equation (2.22) does not fit to the test results presented by Mamalis and Johnson [175], therefore, they proposed a different relation for the cylinders:

\[
\frac{P_b}{P_y} = 7.0 \frac{t}{D_c} + 0.07 \tag{2.23}
\]

The relations proposed for the frustas [175] having 5° and 10° semiapical angle and collapsing in axisymmetric as well as non-axisymmetric modes are

\[
\frac{P_b}{P_y} = 7.0 \frac{t}{D} + 0.11 \quad \text{for } 5^\circ \text{ semiapical angle} \tag{2.24}
\]

\[
= 14.0 \frac{t}{D} + 0.17 \quad \text{for } 10^\circ \text{ semiapical angle} \tag{2.25}
\]
where, $P_\pi = \sigma_n \pi (D_1 - \ell)$, $\bar{P}_n =$ mean buckling load, $\bar{D} =$ mean diameter of frusta

$\bar{P}_n = \frac{1}{\sqrt{2}} (D_2^2 + D_1^2)$, $D_1 = 2R_1$ and $D_2 = 2R_2$ are the small and large end diameters of frustum, respectively.

2.6. NEURAL NETWORKS

Neural networks are composed of simple elements operating in parallel. These elements are inspired by biological nervous systems. As in nature, the network function is determined largely by the connections between elements. A neural network may be trained to perform a particular function by adjusting the values of the connections (weights) between elements. Commonly neural networks are adjusted, or trained, so that a particular input leads to a specific target output. Such a situation is shown below. The network is adjusted, based on a comparison of the output and the target, until the network output matches the target (Fig. 2.1).

![Fig. 2.1 Neural Network Algorithm](image)

Typically many such input/target pairs are used, in this supervised learning, to train a network. Batch training of a network proceeds by making weight and bias changes based on an entire set (batch) of input vectors. Incremental training changes the weights and biases of a network as needed after presentation of each individual input vector.
Incremental training is sometimes referred to as "online" or "adaptive" training. Neural networks have been trained to perform complex functions in various fields of application including pattern recognition, identification, classification, speech, vision and control systems.

Today neural networks can be trained to solve problems that are difficult for conventional computers or human beings. The supervised training methods are commonly used, but other networks can be obtained from unsupervised training techniques or from direct design methods. Unsupervised networks can be used, for instance, to identify groups of data. Certain kinds of linear networks and Hopfield networks are designed directly. In summary, there are a variety of kinds of design and learning techniques that enrich the choices that a user can make. The field of neural networks has a history of some five decades but has found solid application only in the past fifteen years, and the field is still developing rapidly. Thus, it is distinctly different from the fields of control systems or optimization where the terminology, basic mathematics, and design procedures have been firmly established and applied for many years. In the present work, neural network has been employed the estimation of the diameter of hole in the plates by spherical projectile.

2.6.1. Principle of Artificial Neural Network (ANN)

Artificial Neural Network, are computation networks that attempt to simulate the networks of nerve cells of the human or animal central nervous system. They are collections of simple, highly connected processing elements that respond (or "learn") according to sets of inputs. As such they are capable of realizing a greater variety of non-linear relationships of considerable complexity between input and output data sets.

The brain is composed of over 100 different kinds of special cells called neurons. The number of neurons in the brain is estimated to range from 50 billion to over 100 billion. These neurons are divided into interconnected groups called networks and provide specialized functions. Each group contains several thousand neurons that are highly interconnected with each other. Thus the brain can be viewed as a collection of networks.
ANN is a model that emulates the neural network of the biological brain. It is composed of basic units called neurons that are the processing elements in a network. Each neuron receives input data, processes it, and delivers a single output (Fig. 2.2).

The input can be raw data or output of other processing elements. The output can be the final product or it can be an input to another neuron. An ANN is composed of a collection of interconnected neurons that are often grouped in layers. The two basic layered architectures are (a) two layers: input and output and (b) three layers: input, intermediate (called hidden) and output. The input layer receives data from the outside world and sends signals to the subsequent layers. The outside layer interprets signals from the previous layer to produce a result that is transmitted to the outside world as the network’s understanding of the input data.

Each input corresponds to a single attribute of a pattern of other data in the external world. The network can be designed to accept sets of input values that are either binary-valued or continuously valued. The output of the network is the solution to the particular
problem. The initial output is usually incorrect thus the network has to be trained until it gives the proper output. The output data is usually rescaled by the so-called connection weights, one for each wire coming to a neuron from another one. Thus if the \textit{i}-th neuron receives a single input from the \textit{j}-th, and the connection weight for this wire from neuron \textit{j} to neuron has the value \(\omega_{ij}\) then the activity received by the \textit{i}-th neuron will the amount \(\omega_{ij}\). The total activity received by the \textit{i}-th neuron will be

\[
A_i = \sum_j \omega_{ij} u_j,
\]  

\[(2.26)\]

where \(u_j\) is the activity of the \textit{j}-th neuron, being unity if the \textit{j}-th neuron is active and zero if inactive. The \textit{i}-th neuron responds with a single output which depends on the value of its activity at that time. The weights represent the relative strengths of the various connections that transfer data from layer to layer. The objective in training a neural network is to find a set of weights that will correctly interpret all the sets of input values that are of interest for a particular problem.

In normal regression methods, the analysis begins with the prior choice of a relationship (usually linear) between the output and input variables. A neural network is capable of realizing a greater variety of non-linear relationship, of considerable complexity. The data are presented to the network in the form of input and output parameters, and the optimum non-linear relationship is found by minimizing a penalized likelihood. In fact, the network tests many kinds of relationship in its search for an optimum fit. As in regression analysis, the results then consist of a specification of the function, which in combination with a series of coefficients (called weights), relates the inputs to the outputs. The search for the optimum representation can be computer intensive, but once the process is completed (that is, the network has been trained), the estimation of outputs is very rapid.

Bayesian framework for networks allows quantitative assessment of the relative probabilities of models of different complexity, and quantitative errors can be applied to
the predictions of the models. This work has been applied to the complex problem of predicting growth of hole diameter.

Fig. 2.2 shows the structure of the neural network used in our model. A data set \((x_1, x_2,\ldots)\) is first fed directly to the network through the input layer and, subsequently, the Bayesian neural network produces an expected result \((y)\) in the output layer. The output \((y)\) is determined by the architecture of the network.

Three neuron models namely, tansig, logsig and purelin, have been used in the architecture of the network with the back-propagation algorithm. In the back-propagation algorithm, the feed-forward (FFBP) and cascade-forward (CFBP) type network was considered. Each input is weighted with an appropriate weight and the sum of the weighted inputs and the bias forms the input to the transfer function. The neurons employed use the following differentiable transfer function to generate their output:

Log-Sigmoid Transfer Function:
\[
y_j = f\left(\sum_i \omega_{ij} x_i + \phi_j\right) = \frac{1}{1 + e^{-\left(\sum_i \omega_{ij} x_i + \phi_j\right)}}
\] (2.27)

Linear Transfer Function:
\[
y_j = f\left(\sum_i \omega_{ij} x_i + \phi_j\right) = \sum_i \omega_{ij} x_i + \phi_j
\] (2.28)

Tan-Sigmoid Transfer Function:
\[
y_j = f\left(\sum_i \omega_{ij} x_i + \phi_j\right) = \frac{2}{1 + e^{-2\left(\sum_i \omega_{ij} x_i + \phi_j\right)}} - 1
\] (2.29)

The weight, \(\omega\), and biases, \(\phi\), of these equations are determined in such a way as to minimize the energy function. The Sigmoid transfer functions generate outputs between 0 and 1 or -1 and +1 as the neuron's net input goes from negative to positive infinity depending upon the use of log or tan sigmoid. When the last layer of a multilayer network has sigmoid neurons (log or tan), then the outputs of the network are limited to a small range, whereas, the output of linear output neurons can take on any value.
Further, in order to see if advanced training schemes provide better learning than the basic back propagation, a radial basis function (RBF) network was also used which though requires more neurons but it is sometimes more efficient. The Radial basis transfer function is given by:

$$y_j = f\left(\sum \omega \phi \right) = e^{-\left(\sum \omega_i x_i \phi_j \right)}$$  \hspace{1cm} (2.30)

### 2.6.2. Training of Neural Network

Both the input and output variable, are sometimes first normalized within the range 0 to 1 as follows:

$$x^N = \frac{x - x_{min}}{x_{max} - x_{min}}$$ \hspace{1cm} (2.31)

where $x^N$ is the normalized value of $x$; $x_{max}$ and $x_{min}$ are the maximum and minimum values of variable, $x$. This normalization is not essential to the neural network approach, but allows the network to be trained better.

Using the normalized input and output, the coefficients (weights) $\omega$ and bias $\phi$ are determined in such a way as to minimize the following energy function

$$F(\omega) = \delta E_D + \sum E_{\omega(e)}$$ \hspace{1cm} (2.32)

The minimization was implemented using a variable metric optimizer. The gradient of $F(\omega)$ was computed using a back-propagation algorithm. The energy function consists of the error function, $E_D$ and regularization, $E_{\omega}$. The error function is the sum-squared error as follows
\[ E_D(\omega) = \frac{1}{2} \sum_m \left( y(x^m; \omega) - t^m \right)^2 \] (2.33)

Where \( \{x^m, t^m\} \) is the data set, \( x^m \) represents the inputs, \( t^m \) represents the targets, and \( m \) is a label of the pair. The error function \( E_D \) is smallest when the model fits the data well, that is when \( y(x^m; \omega) \) is close to \( t^m \). The coefficients \( \omega \) and \( \phi \), shown in Eqs. (2.27) to (2.30), make up the parameter vector \( \omega \). A number of regularizers \( E_{\omega(c)} \) are added to the data error. These regularizers favour functions \( y(x; \omega) \), which are smooth functions of \( x \).

The simplest regularizers method uses a single regularizer, \( E_{\omega} = \frac{1}{2} \sum \omega_i^2 \). A slightly more complicated regularization method, known as the automatic relevance determination model, is used in this study. Each weight is assigned to a class \( c \), depending on which neurons it connects. For each input, all the weights connecting that input to the hidden nodes are in a single class. The biases of the nodes are in another class, and all the weights from the hidden nodes to the outputs are in a final class. \( E_{\omega(c)} \) is defined as the sum of the squares of the weights in class \( c \) as follows:

\[ E_{\omega(c)}(\omega) = \frac{1}{2} \sum_{\omega_i} \omega_i^2 \] (2.34)

This additional term favours small values of \( \omega \) and decreases the tendency of a model to over-fit noise in the data set. The control parameters \( \chi_c \) and \( \delta \), together with the number of hidden nodes, determine the complexity of the model. These hyper-parameters define the assumed Gaussian noise level \( \mu_c^2 = 1/\delta \) and the assumed weight variances, \( \mu_{\omega(c)}^2 = 1/\chi(c) \). The noise level inferred by the model is \( \mu_c \). The parameter \( \chi \) has the effect of encouraging the weights to decay. Therefore, a high value of \( \mu_{\omega} \) implies that the particular input parameter explains a relatively large amount of the variation in the output. Thus, \( \mu_{\omega} \) is regarded as a good expression of the significance of each input.
through not of the sensitivity of the output to that input. The values of the hyper-
parameters are inferred from the data using the Bayesian methods given in Mackay
(1992a). In this method, the hyper-parameters are initialized to values chosen by the
operator and the weights were set to small initial values. The objective function \( F(\omega) \)
was minimized to a chosen tolerance and the values of the hyper-parameters are then
updated using a Bayesian approximation given in Mackay (1992a). The \( F(\omega) \) function
was minimized again, starting from the final state of the previous optimization, and the
hyper-parameters are updated again.

Two different styles of training, namely incremental training and batch training are used.
In the incremental training, the weights and biases of the network are updated each time
an input is presented to the network. In batch training the weights and biases are only
updated after all of the inputs are presented.
**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{mp}$</td>
<td>half-width of the plate</td>
</tr>
<tr>
<td>$c, c_i$</td>
<td>Speed of sound in projectile and target materials respectively</td>
</tr>
<tr>
<td>$D_h$</td>
<td>Diameter of hole in target plate</td>
</tr>
<tr>
<td>$D_p$</td>
<td>Diameter of the projectile</td>
</tr>
<tr>
<td>$\bar{D}$</td>
<td>Mean diameter of frusta</td>
</tr>
<tr>
<td>$D_c$</td>
<td>Diameter of the cylindrical tube</td>
</tr>
<tr>
<td>$D_q$</td>
<td>Parameters</td>
</tr>
<tr>
<td>$E_D$</td>
<td>Error function</td>
</tr>
<tr>
<td>$h$</td>
<td>Size of fold</td>
</tr>
<tr>
<td>$L_{mp}$</td>
<td>half-length of the plate</td>
</tr>
<tr>
<td>$M_p = \frac{\sigma_s t^2}{4}$</td>
<td>Full plastic bending moment per unit length</td>
</tr>
<tr>
<td>$F(\omega)$</td>
<td>Objective function</td>
</tr>
<tr>
<td>$P_b$</td>
<td>Buckling load</td>
</tr>
<tr>
<td>$P_m$</td>
<td>Mean crushing force</td>
</tr>
<tr>
<td>$\bar{P}_b$</td>
<td>Mean buckling load</td>
</tr>
<tr>
<td>$P_y$</td>
<td>Yield end load</td>
</tr>
<tr>
<td>$R_{cp}$</td>
<td>Radius of circular plate</td>
</tr>
<tr>
<td>$S_w$</td>
<td>Half-length of the wedge strike</td>
</tr>
<tr>
<td>$T_i$</td>
<td>Thickness of the target plate</td>
</tr>
<tr>
<td>$t$</td>
<td>Thickness of frusta or cylinder</td>
</tr>
<tr>
<td>$V$</td>
<td>Striking velocity of projectile</td>
</tr>
<tr>
<td>$w_{pm}$</td>
<td>Maximum permanent deflection of plate</td>
</tr>
<tr>
<td>$Y$</td>
<td>Yield strength of the plate</td>
</tr>
</tbody>
</table>
Greek symbols

\( \dot{\varepsilon}_m \)  Mean circumferential strain
\( \rho_p \)  Density of projectile
\( \rho_t \)  Density of target
\( \sigma_y \)  Yield stress
\( \sigma_d \)  Dynamic flow stress
\( \sigma_o \)  Static flow stress
\( \sigma_u \)  Ultimate stress
\( \sigma_{av} \)  Average flow stress
\( \psi \)  Calibre-radius-head for ogival nose projectile
\( \sigma_{US} \)  Shear strength of the target material
\( \theta \)  Angle of strike or spray angle of projectile
\( \phi \)  Bias
\( \omega \)  Weight

Subscripts

bl  Ballistic limit
cp  Circular plate
h  Hole
m  Maximum
pl  Plug
P  Plastic
p  Projectile
rp  Rectangular plate
t  Target
US  Ultimate strength