CHAPTER 7

COMBINED RANGE AND RANGE RATE RESOLUTION
7.1 INTRODUCTION

The principal aim throughout this work has been to design waveforms for a radar environment where the relative Doppler spread of the target is negligible. However, when there is significant Doppler shift, the reflections from a target are no longer replicas of the transmitted waveform and the matched filter (MF) response in time and frequency has to be considered.

The following discussions of combined range and range rate (Doppler) resolution of a signal is included for completeness. Detailed analysis of resolution in range and range rate of general types of waveforms can be found in many contemporary books, [22], [23]. Moreover, consideration will only be given to properties pertinent to the types of waveforms described in previous chapters.

Although, in principle, the various methods developed to design pulse trains having good range resolution can be extended to the more general case of range and Doppler resolution, the computational efforts involved for longer sequences is quite formidable even for modern computers. For this reason waveform synthesis for range and velocity resolution is commonly done by trial and judicious use of available information (e.g. ambiguity function).

It has been shown (Chapter 2) that the range resolution property of a signal depends on the shape of its spectrum envelope. Based on the time frequency duality it can be argued, therefore, that resolution in range rate depends only on the envelope of the signal in the time domain. Consequently, combined range and velocity resolution depends on the complete waveform structure in time and frequency. Hence signals with good resolution in one parameter may perform very poorly when combined resolution in both parameters is required.

For combined resolution in range and velocity the waveform must be investigated in terms of the complete MF response in delay and Doppler. This generalized response is given by Woodward's Ambiguity Function (ABF) which has already been introduced in chapter 2 and is repeated here for convenience.
It is noted that in the literature, the terms \( x(\tau,\nu) \), \( |x(\tau,\nu)| \) and \( |x(\tau,\nu)|^2 \) are often used synonymously for the ABF.

The ABF plays a central part in the analysis of combined resolution. This is so because the width of the main response peak of the ABF serves as a measure for close-target visibility in range-Doppler, while the low-level response and subsidiary spikes give an indication of the self clutter and target masking problem by mutual interference. Since the volume of the ABF over the entire \((\tau,\nu)\)-plane is constant, the signal design problem for combined range and velocity resolution may therefore be regarded as shifting the unavoidable ambiguity (volume) to those parts of the \((\tau,\nu)\)-plane where it causes least interference for a given environment and application. Some of the general resolution properties of the various types of pulse trains considered previously are discussed subsequently using the ABF description.

### 7.2 Ambiguity Function of Pulse Trains

One derivative of the ambiguity function involves computing the signal output of the receiver filter. The ambiguity function is the complex modulation of this output signal. The transmitted signal is

\[
S_i(t) = a \cdot u(t) e^{j2\pi f_0 t}
\]

where \( a \) = amplitude factor

\( u(t) \) = complex transmitted modulation

\( f_0 \) = carrier frequency
Depending on the radar cross section, range, and Doppler velocity of the target, the return signal differs from the transmitted signal by

(i) having a different amplitude $a_r$, determined by the radar range equation.

(ii) being delayed by $\tau$ and

(iii) being Doppler shifted by $\nu$

$$S_r(t) = a_r \nu(t - \tau)e^{j2\pi(\nu(t) - \nu)\tau} \quad \text{...(7.3)}$$

In the general case the receiver filter is matched to some signal $S_m(t)$ with modulation $\nu(t)$, and thus has the impulse response

$$h_m(t) = KS^*(T_0 - t)$$

$$= Ka^\nu_m(T_0 - t - \tau_m)e^{-j2\pi(f_0 - v_m)\tau_0 - \tau_m} \quad \text{...(7.4)}$$

where, $\tau_m, v_m$ are the delay and the Doppler shifts to which the filter is matched. If $u(t) = \nu(t)$, then the filter is matched filter. If $u(t) \neq \nu(t)$, the filter is called mismatched filter. The signal at the filter output in case of a matched filter is given by

$$Z(t) = \frac{Ka^\nu_m}{2}e^{-j2\pi(f_0 - v_m)t} \left[ \int_{-\infty}^{\infty} v(t)v^\ast(t + \tau)e^{-j2\pi\nu t} \, dt \right] \quad \text{...(7.5)}$$

The quantity within brackets is known as ambiguity function

$$x(\tau, \nu) = \int_{-\infty}^{\infty} v(t)v^\ast(t + \tau)e^{-j2\pi\nu t} \, dt \quad \text{...(7.6)}$$

In discrete function, the same is given by equation 7.1

Pulse trains whose ACF decreases at a faster rate for small Doppler shifts are the non-linear FM type approximation. It can be seen from figures 7.1 to 7.3 and the ABF's of these waveforms basically exhibits the ridge lime structure which suggests a relatively strong range Doppler coupling. However, the linear FM property is more and more eliminated as the order of the spectrum tapering increases.
For certain applications the inability to resolve targets in range and velocity along the ridge might be unacceptable. In these circumstances a signal whose ABF approaches that of a single strong spike (thumbtack) as shown in Figs. 7.4 and 7.8 might be adequate. However, the expense of implementing many Doppler channels may be prohibitive.

The choice of a thumbtack ABF may be justified for high close target resolution in the absence of any prior information of the target environment. The close-target resolvability is, however, achieved at the expense of introducing self clutter. Therefore, if the target space is confined to a narrow region, there is no reason to spread the volume of the ABF uniformly over the \((x, v)\) plane. Moreover, if visibility of small targets is of overriding importance, it is preferable to choose an ABF whose volume is concentrated in strong spikes or a narrow ridge. Such an ABF trades uniform poor visibility for weak targets (thumbtack ABF) against good visibility for most targets and extremely poor visibility for some targets. Fig. 7.7 illustrates the relatively large increase in sidelobe levels off the delay axes for an optimum binary sequence of length 101. It can clearly be seen that noise-like waveforms are inherently suited to approximate thumbtack ABF's. In general, however, binary sequences are usually better suited to improving range resolution rather than velocity resolution.

In summary, all of the waveforms discussed in previous chapters are optimum for some particular clutter environment. The different pulse trains yield a wide variety of ABF shapes. The contours may be of the diagonal ridge structure as for linear FM by signals, or may consists of a single strong spike surrounded by a low level pedestal for noise like waveforms, in various combination, of these basic structures. The waveforms have different tolerances to Doppler shifts. This may be exploited for hardware savings if ambiguity in the range-Doppler coverage can be tolerated.

Another advantage of discrete coding which has not been mentioned is the flexibility of eliminating the range Doppler ambiguity of codes, for example, by simply scrambling the order of the sub-pulses. Moreover, the variations possible with discrete coded pulse trains are virtually unlimited in that the phase, amplitude, frequency and time of transmission of each sub-pulse can be varied. The resulting multi-function capability and adaptability to a particular target environment is clearly one of the most attractive features discrete coding.
Fig. 7. (a) ABF of 128-element non-linear FM type pulse train for $n = 1$.
(b) Peak response as a function of doppler shift $v$. 
Fig. 72  (a) ABF of non-linear FM type pulse train for $n = 2$
(b) Peak response as function of doppler shift $v$. 
Fig. 7.3  (a) ABF of 128-element non-linear FM type pulse train for $n = 3$
(b) Peak response as a function of doppler shift $v$. 

- $n = 3$
- $c = 0$
- $\omega T = 1$
Fig. 7.4  (a) ACF of optimum 128-element binary sequence when the initial sequence is chosen randomly.
(b) Peak response as a function of doppler shift \( v \).
Fig. 7.5 (a) ABF of 100-element Huffman code derived from a random zero pattern.

(b) Peak response as a function of doppler shift $v$. 
Fig. 7. (a) ADF of optimum 128-element binary sequence when the initial sequence is chosen randomly.
(b) Peak response as a function of doppler shift \( v \).
Figure 7.7: Plot of autoambiguity diagram for 31 element binary code.
Figure 7.8: Plot of autoambiguity diagram for 53 element binary code.
Figure 7.9: Plot of autoambiguity diagram for 91 element binary code.
Figure 7.10: Plot of autoambiguity diagram for 101 element binary code.