CHAPTER -III

METHODOLOGY

3.1. INTRODUCTION

This chapter aims to give a detailed explanation of the methodology adopted for the study with an intention to clarify the basic research purpose. This study primarily aimed at the informational efficiency of Indian futures market. In many foreign countries, it is an established fact that derivatives helped the investors but no such established facts have emerged in India. Therefore it was thought to make an attempt to see where the Indian futures market has informational efficiency to provide helps to investors. Once the informational efficiency is found then it will serve as an indication that investors in Indian futures market would get greater benefits.

Different sets of variables and methodologies have been employed by researchers in the best of futures market studies. Various researches has sued indices of share prices, volatility, open interest and other variables to establish a lead lag relationship of causality between futures and spot and price discovery. It is believed that derivatives would provide greater benefits to the investors through arbitrage and hedge. In this chapter, Research Gap, Objectives of the study, Null Hypothesis of the Study, Significance of the study, Data and Methodology, Limitations of the study and Reader’s Guide are discussed. Econometrics models used for the analysis are also provided in this chapter.

3.2. OBJECTIVES OF THE STUDY

The general objective of the study is to see the informational efficiency of futures market in India. But it is specifically intended:

1. To examine the growth and development of futures market in India in respect of values and quantities.
2. To analyze the pricing efficiency of futures market by comparing the linkage between futures and spot market.

3. To find the significance of different determinants and their impact of futures return in futures market in India.

4. To estimate the optimal hedge ratio and find the contribution of futures market in mitigating investment risk in selected securities.

3.3. NULL HYPOTHESIS OF THE STUDY

Framing hypothesis gives the research a direction to make clear empirical analysis and to specifically reveal the research results with rejection or acceptance of the null hypothesis. In order to analyze the objectives, the following null hypotheses are framed.

**H₀₁**: There is no significant relationship between spot and futures in long term and short term period.

**H₀₂**: The futures price has no information to pass on to spot market.

**H₀₃**: There is no significant lead-lag effect between spot and futures markets.

**H₀₄**: Spot return plays very negligible role in determining the futures prices.

**H₀₅**: Open interest, Trading volume, volatility of futures return and number of contracts are not the determinants of futures return.

**H₀₆**: No significant protection to investors risk through futures markets.

3.4. SIGNIFICANCE OF THE STUDY

The importance of this research and its contribution to the existing literature are discussed under this section.

On the basis of review of literature, it is found that lot of studies have been done on the futures market. Each study clearly analyzed and examined different aspects of futures market in depth. While comparing the studies from abroad and India lot of differences are seen in the movement of market, link between futures market and its underlying market and in the basic structure of variables used. Many
studies analyzed the price discovery process of NSE futures market by considering either with one variable or for different periods. Very few studies have been made on the determinants of the futures return in Indian context. This research made an attempt to fill the research gap by considering more number of variables such as futures price series, spot price series, futures return, spot return, open interest, trading volume, number of contracts and also volatility series of futures return. The whole study period (12.06.2000-30.06.2011) has been divided into four sub periods on the basis of the structural break in the data set, the four periods are introduction and development of derivatives (12.06.2000-28.02.2006), pre financial crisis period (01.03.2006-14.01.2008), financial crisis period (15.01.2008-31.10.2008) and post crisis period (01.11.2008-30.06.2011). Analyses were done for these sub periods separately and collectively. This division is to check the robustness. This study establishes the relationship between spot and futures market on the basis of its depthness, inclusion of more variables and different analysis during structural break periods. This research work provides basic knowledge on the movement of Indian futures market and its underlying market. The basic movement of both markets gives the investors and traders to take decision on their dealings in the market. Actual relationship between spot and futures market shows the picture on the opportunities of the hedging and arbitrage in the Indian market. Hence the overall result provides the policy makers in depth knowledge on the regulations which helps the traders in the futures market.

3.5. SCOPE OF THE STUDY

The scope means the boundary of the operations or the area for the study. This study includes only the variables from futures market like futures return, open interest, number of contract, turn over and volatility of futures market. Spot market return is taken as the representative of underlying market. Instead of taking only return series of spot and futures market, here index series are also taken in to consideration. By using appropriate models and checking the output of different sub periods makes the study more reliable. This study specifically excludes the effect of mispricing, volatility of spot market, the role of base market and the presence of sensitivity of spot market. By including these variables, the dimension of the study
can be expanded and it may give wide coverage on the area of the study. Even though in the absence of these variables like mispricing, volatility of spot market and base market concept this study may provide more relevance in the field of research.

3.6. DATA AND METHODOLOGY

National stock exchange of India (NSE) is the leading and very popular exchange for derivatives in India. Therefore research is carried out by considering NSE as representative of Indian derivatives market and particularly its Indian Nifty spot and Nifty futures are taken. In order to access the relationship between spot and futures market in India, daily closing indices of Nifty spot and Nifty futures from 12\textsuperscript{th} June 2000 to 30\textsuperscript{th} June 2011 (11 years, 132 near month strikes) are included. Determinants of futures return (FUTR) are identified with the help of variables like open interest (OI), turnover (TURN), volatility (VOL), number of contract (CONT) and spot return (SPOTR). Daily closing values of these variables were collected from NSE for the period 12\textsuperscript{th} June 2000 to 30\textsuperscript{th} June 2011. Risk reduction to investors of derivatives market is measured through estimation of optimal hedge ratio. For this purpose 19 companies which satisfy the conditions as it should be from the introduction of derivatives being continuously trading till 30\textsuperscript{th} June 2011 and which is part of Nifty are selected. The analysis of this study is done through the following steps.

- Structural break in time series was determined through Bai Perron Model and the real market trend in the Indian futures market.
- Individual stock prices are adjusted for Bonus issue and stock splits during the study period.
- Return is determined after converting the variable like Price series, index series, open interest, turn over and number of contracts into log form.
- Preliminary analysis is done through line graphs and descriptive statistics.
- Long term relationship between spot and futures market is determined by using Johansen Cointegration Model, after checking the stationarity properties through
Augmented Dickey Fuller and Philip Perron test and selection of lag length through LR, FPE and AIC criteria.

- Short term relationship and lead lag between spot and futures market is identified through Vector Error Correction Model.

- Causality of spot and futures price series is established by using Bivariate Error Correction model and Wald Coefficient test.

- In order to find the determinants of the futures market, Granger Causality Block/Exogeneity test is employed on variables included in the study.

- Influence of shocks, its sign and length in each variable and between other variables is examined through Impulse Response and percentage changes between variables are analyzed through Variance Decomposition Model.

- Optimal hedge ratio was determined by dividing the covariance of spot and futures return by variance of futures. Average is obtained for a particular period is the optimal hedge ratio to indicate the level of protection to investors. Covariance between spot and futures return and variance of futures return is determined through Bivariate GARCH model.

The time series data are having some special features and these all characteristics are to be analyzed to apply the apt model for analyzing the data to satisfy the objective. Stationarity characteristics, Cointegration relationship, Error Correction Model, Wald Coefficient for the causality relationships, GARCH (1,1) model for making volatility series of futures return, VAR Granger Causality /Block Exogeneity test for the causality between many variables, impulse Response function, Variance Decomposition and Bivariate GARCH model for the estimating optimum hedge ratio. These econometrics models are briefly explaining in the chapter. It will definitely give the basic ideal on the each model and its application of the different context.

3.7. PERIOD OF THE STUDY

Basically the study pertains to the period between 12th June 2000 and 30th June 2011. The period is divided in to four sub periods, 12th June 2000 to 28.02.2006 representing initial development of derivatives market in India, 1st March 2006 to 14th
January 2008 representing pre financial crisis period, from 15th January 2008 to 31st October 2008 is a financial crisis period and 1st November 2008-30th June 2011 as a post financial crisis period. But periods are divided based on the structural break identified in the data set. It was done through Bai- Perron test and market movement.

3.8. LIMITATIONS OF THE STUDY

The study is based on secondary data and errors in collection, compilation of data are due to the process and perfection desired by others not by researcher. Daily closing prices and index are considered instead of intraday or tick by tick data. Analysis on volatility spill over is not done in the study. Seasonality effect, Monday effect, expiration effects and cyclical effects and celebration effect like Divali effect are not taken into consideration in the study. Macroeconomic factors like GDP, interest rate, inflation rate are not included here. The important aspects like mispricing, vitality spill over of spot market and base market information are not considered. Although some of the areas which are not touched by the researcher still the study is able to come up with good results and research implications.

3.9. ECONOMETRICS MODELS USED IN THE STUDY

3.9.1. Stationary (Unit Root Test)

The test of stationarity which became very popular over the years is the unit root test. The unit root stochastic process will start with;

\[ y_t = p y_{t-1} + u_t \quad -l \leq p \leq l \]

Where \( u_t \) is a white noise error term. It is know that if \( p=1 \), in the case of unit root becomes a random walk model without drift, which is a non stationary stochastic process. Hence, why not simple regression \( y_t \) on its lagged value \( y_{t-1} \) and find if the estimated \( p \) is statistically equal to 1, if it is then \( y_t \) is non stationary. This is the general idea behind the unit root of stationarity.

If we plot the two series, it is seen that the data were generated by a stationary process. In econometric time series analysis, a stationary series has time independent mean, variance and auto correlation that are constant through time. The existence of
unit root is firstly tested using the ADF test in 1981 through the following relationship.

\[ \Delta S_t = \alpha + \beta T + p S_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta S_{t-i} + u_t \]  

(1)

Where \( \Delta S_t = S_t - S_{t-1} \), \( S_t \) is the index of the spot market, and \( k \) is chosen so that the deviations \( u_t \) to be white noise. The same relationship is used to determine the order of the futures price index \( F_t \). The null and the alternative hypothesis for the existence of unit root in \( S_t \) and \( F_t \) is \( H_0: P=0, H_1: P<0 \). If the null hypothesis of only a unit root cannot be rejected, then the stock prices follow a random walk.

Phillip and Perron (1988) have modified the ADF test, as the ADF tests are only valid under the crucial assumption i.i.d. process. In practice, it may be more realistic to allow for some dependence among the \( u_t \)'s. In that case, the asymptotic distribution is changed. Philips and Perron (1988) have weakened the i.i.d. assumption by using a non-parametric correction to allow for some serial correlation and heteroskedasticity.

\[ y_t = \alpha_0 + a y_{t-1} + u_t \]  

(2)

The PP test tends to be more robust to a wide range of serial correlation and time-dependent heteroskedasticity. The asymptotic distribution of the PP test-statistic is the same as the ADF test-statistics.

3.9.2. Cointegration

If there are two non-stationary time series that becomes stationary while differencing such linear combination are said to be cointegrated. Cointegration relationship provides particular types of long run equilibrium relationship. In technically two or more first order I(1) integrated variables are cointegrated, then it can give long run information on one variable, it helps to predict the movement of another.

The Johansen’s Maximum likelihood procedure (Johansen, 1988) is implemented to estimate cointegration relationships. This is the preferred method of testing for cointegration as it allows restrictions on the cointegrating vectors to be tested directly, with the test statistic being \( \chi^2 \) distributed. This specific procedure provides a unified frame work of estimating and testing the cointegration relationship.
in VAR error correction mechanism, which incorporate different short term and long run dynamic relationship in a variable system.

The Johansen’s procedure firstly specifies the following unrestricted N-variable VAR’

\[ X_t = \mu + \sum_{i=1}^{k} \Pi_i x_{t-i} + \epsilon_t \]  \tag{3}

Where \( x_t = (f_t, s_t) \), \( \mu \) is a vector of intercept terms and \( \epsilon_t \) is a vector of error term. Johansen (1988) and Johansen and Juselius (1990) reparameterizied the above equation in the following way,

\[ \Delta x_t = \mu + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \Pi x_{t-k} + \epsilon_t \]  \tag{4}

This is now a VAR reparameterised in the error correction form, where \( \Pi = - (\Pi - \Pi_1 - \ldots - \Pi_k) \) represent the long response matrix. Writing this matrix as \( \Pi = \alpha \beta \), then the linear combinations \( \beta x_{t-k} \) will be I(0) in, with \( \alpha \) being adjustment coefficients, and the matrix \( \Pi \) will be of reduced ranks. The Johansen approach can be used for cointegration by assessing the rank \( r \) of the matrix \( \Pi \). If \( r=0 \) then all the variables are no cointegrating vectors. If \( r=N \) then all the variables are I(0) and, given that any linear combinations of stationary variables will also be stationary, there \( N \) cointegrating vectors. Last if \( 0<r<N \) there will be \( r \) cointegrating vectors.

Evidence of price changes in one market generating price changes in the other market so as to bring about a long-run equilibrium relationship from the equation

\[ F_t - \delta_0 - \delta_1 S_t = \epsilon_t \]  \tag{5}

Where \( F_t \) and \( S_t \) are contemporaneous futures and cash prices at time \( t \), \( \delta_1 \) and \( \delta_0 \) are parameter, and \( \epsilon_t \) is the deviation from parity. If \( F_t \) and \( S_t \) are nonstationary then the OLS method is inappropriate because the standard errors are not consistent. This inconsistency does not allow hypothesis testing of the cointegrating parameters \( \delta_1 \). If \( F_t \) and \( S_t \) are nonstationary but the deviation \( \epsilon_t \) are stationary, \( F_t \) and \( S_t \) are cointegrated and an equilibrium relationship exists between them (Engle and Granger, 1987). For \( F_t \) and \( S_t \) to be cointegrated, they must be integrated of the same order. Performing unit root tests on each price series determine the order of integration. If each series is nonstationary in the levels, but the first difference and the deviation \( \epsilon_t \)
are stationary, then the prices are cointegrated of order (1,1) denoted C1 (1,1) with the cointegrating coefficient $\delta_1$.

### 3.9.3. Error Correction Model and Causality

Eagle Granger (1987) revealed the fact that the estimates of a VAR are misspecified in the case of cointegrated variables, because the error correction terms that are attached to error correction models are not accounted. The cointegration between two series involves a continuous adjustment of innovations prices, so that these would not become larger in the long run. Eagle and Granger (1987) have shown that all the cointegrated series can include an error correction (the Granger representation theorem) and, on the contrary, the existence of cointegration is necessary condition in order to construct error correction models. The acceptance that each pair of cash and futures prices composes a cointegrating system leads to the implementation of an error correction model for each series, which is characterized by the ability to overcome problems caused by spurious results. If $\Delta S_i$ and $\Delta F_i$ denotes the first difference of the futures and cash prices, the following cointegrating regressions are possible.

$$\Delta S_i = \alpha_1 + a_{z} z_{i-1} + \sum_{i=1}^{n} \alpha_{11}(i) \Delta F_{i-1} + \sum_{i=1}^{n} \alpha_{12}(i) \Delta F_{i-1} + \varepsilon_{zi} \tag{6}$$

$$\Delta F_i = \alpha_2 + a_{F} z_{i-1} + \sum_{i=1}^{n} \alpha_{21}(i) \Delta S_{i-1} + \sum_{i=1}^{n} \alpha_{22}(i) \Delta F_{i-1} + \varepsilon_{Fi} \tag{7}$$

Where $z_i = S_i - [b + a F_i]$ is the error correction term. Equation (5) and (6) represent a vector auto regression in first difference. The Error Correction term enters in to two equations with a one period lag and is estimated from the Cointegrating Regression, with constant terms being included to make the mean of the error series zero. The coefficients $\alpha_i, ka_i, a_F$ attached to the error correction term measures the single period response of the left hand side variable to departure from equilibrium. At least one speed of adjustment coefficients must be nonzero for the model to be an error correction model. Granger (1986), the link between cointegration and causality stems from the fact that if spot and futures indices are cointegrated, then causality must exist in at least one direction and possibly in both direction. Temporal causality can be assessed by examining the statistical significance and the relative magnitudes of error
correction coefficients and coefficients on the lagged variables (Wahab and Lashgari, (1993)).

3.9.4. Wald Test (Coefficient Restrictions)

The Wald coefficient restriction test is used to find the causal relationship between spot and futures market while restricting coefficient of spot and futures separately. The Wald coefficient restrictions test (F-test) tests whether multiple coefficients are simultaneously equal to 0 (or some other value). For the purpose estimate both restricted and unrestricted equation and take the RSS of both models denoted as $RSS_k$ and $RSS_U$, respectively. Then the following formula can be furnished to estimate F-stat:

$$F = \frac{(RSS_k - RSS_U) / (k_u - k_r)}{SSR_U / (n-k_u)}$$

This follows an F-type distribution with $(k_u - k_r, n-k_u)$ degrees of freedom.

3.9.5. VAR Model

$$Y_t = C_0 + \sum_{k=1}^{p} A_k Y_{t-k} + \varepsilon_t, \text{ E} (\varepsilon_t, \varepsilon_t^\prime) = \Omega$$

Where $Y_{t,k}$ is a $n \times 1$ column vector of $n$ stationary variables at time $t-k$, $C_0$ is a $n \times 1$ column vector of constants, $A_k$ is an $n \times n$ matrix of coefficients, $p$ is the number of lags, and $\varepsilon_t$ is a $n \times 1$ column vector of white noise innovation terms with symmetric and positive definite variance-covariance matrix $\Omega$.

VAR models were popularized in econometrics by Sims (1980) as a natural generalization of univariate autoregressive models. VAR is a systems regression model that can be considered a kind of hybrid between the univariate time series models and the simultaneous equation models. VARs have often been advocated as an alternative to large scale simultaneous equations structural models.

The simplest case that can be entertained is a bivariate VAR where there are only two variables $y_{1t}$ and $y_{2t}$, each of whose current values depend on different combinations of the previous $k$ values of both variables and error terms. An important feature of the VAR model is its flexibility and the case of generalization. Instead of having only two variables, $y_{1t}$, $y_{2t}$ and $y_{3t},...,y_{gt}$, each of which has an equation.
Another useful facet of VAR models is the compactness with which the notation can be expressed. This could be written as

\[ y_{1t} = \beta_{10} + \beta_{11}y_{1t-1} + \alpha_{11}y_{2t-1} + u_{1t} \]  
(8)

\[ y_{2t} = \beta_{20} + \beta_{21}y_{2t-1} + \alpha_{21}y_{1t-1} + u_{2t} \]  
(9)

There are g=2 variables in the system. Extending the model to the case where there are k lags of each variable in each equation is also easily accomplished.

3.9.6. Granger Causality

One test of causality is whether the lags of one variable enter in to the equation for another variable. In a two equation model with p lags, \{y_t\} does not granger cause \{z_t\} if and only if all of the coefficient of \(A_{21}(L)\) are equal to zero. Thus if \{y_t\} does not improve the forecasting performance of \{z_t\} then \{y_t\} granger causes \{Z_t\}. If all variables in the VAR are stationary, the direct way to test Granger causality is to use a standard F-test of the restrictions:

\[ a_{21}(1)=a_{21}(2)=a_{21}(3) = \ldots a_{21}(p)=0 \] 

it is straight forward to generalize this notion to the n-variable case of \((T-c)(\log|\Sigma_r| - \log|\Sigma_u|)\). Since \(A_{ij}(L)\) represent the coefficients of lagged values of variable j on variables i, can be set equal to zero. Granger causality actually measures whether current and past values of \{y_t\} helps to forecast futures values of \{z_t\}. The following equation is considered for \(z_t\)

\[ z_t^+ = \phi_{21}(0)e_{y_{t1}} + \sum_{i=0}^{n} \phi_{22}(i)e_{z_{t-i}} \]  

If we forecast \(z_{t+1}\) conditional on the value of \(z_t\) we obtain the forecast error \(\phi_{21}(0)e_{y_{t+1}} + \phi_{22}(0)e_{z_{t+1}}\) given the value of information concerning \(y_t\) does not aid in reducing the forecast error for \(z_{t+1}\). The only additional information contained in \(y_t\) is the current past values of \{\epsilon_{yt}\}. However such values do not affect \(z_t\) and so cannot improve of the \(z_t\) sequences. A block exogeneity test is useful for detecting whether to incorporate an additional variable in to a VAR. This multivariate generalization of the Granger causality test should actually be called a block-causality test. Estimate the \(y_t\) and \(z_t\) equations using lagged values of \{\(y_t\),\{\(z_t\}\} and \{\(w_t\}\) and calculate \(\Sigma_u\). re estimate excluding the lagged values of \{\(w_t\}\} and \(\Sigma_r\). find the likelihood ratio statistic:
When a VAR includes many lags of variables, it will be difficult to see which set of variables have significant effects on each dependent variable and which do not. In order to address this issue tests are usually conducted that restrict all of the lags of a particular variable to zero. The VAR (3) could be written out to express the individual equation as

\[ y_{1t} = \alpha_{t0} + \beta_{11}y_{1t-1} + \beta_{12}y_{2t-1} + \gamma_{11}y_{1t-2} + \gamma_{12}y_{2t-2} + \delta_{11}y_{1t-3} + \delta_{12}y_{2t-3} + u_{1t} \]

\[ y_{2t} = \alpha_{20} + \beta_{21}y_{1t-1} + \beta_{22}y_{2t-1} + \gamma_{21}y_{1t-2} + \gamma_{22}y_{2t-2} + \delta_{21}y_{1t-3} + \gamma_{22}y_{2t-3} + u_{2t} \]

Assuming that all of the variables in the VAR are stationary, the joint hypothesis can easily be tested with in the F-test framework, since each individual set of restrictions parameters drawn from only one equation. The evaluation of the significance of variables in the context of a VAR almost invariably occurs on the basis of joint test on all of the lags of a particular variable in an equation. Granger causality really means only a correlation between the current value of one variable and the past values of others.

### 3.9.7. Impulse response function

An autoregression has a moving average representation, a vector autoregression can be written as a vector moving average. In fact

\[ x_t = \mu + \sum_{i=0}^{\infty} A_t^i e_{t-1} \]  

(10)

This equation is the VMA representation of \( x_t = A_0 + A_t x_{t-1} + e_t \) in that the variables (\( y_t \) and \( Z_t \)) are expressed in terms of the current and past values of the two types of shocks. The VMA representation is an essential feature of Sims’s (1980) methodology in that it allows us to trace out the time path of the various shocks on the variables contained in the VAR system. Writing the two-variable VAR in matrix form,

\[ \begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \]

(11)

Or
This equation expresses \( y_t \) and \( z_t \) in terms of the \( \{e_{1t}\} \) and \( \{e_{2t}\} \) sequences. However, it is insightful to rewrite (12) in terms of the \( \{\varepsilon_{1t}\} \) and \( \{\varepsilon_{2t}\} \) sequences. The vector of errors can be written as

\[
\begin{bmatrix}
e_{1t} \\
e_{2t}
\end{bmatrix} = \frac{1}{1-b_{12}b_{21}} \begin{bmatrix} 1 & -b_{12} \\
-b_{21} & 1
\end{bmatrix} \begin{bmatrix}
\varepsilon_{1t-1} \\
\varepsilon_{2t-1}
\end{bmatrix}
\] (13)

So that (12) and (13) can be combined to form

\[
\begin{bmatrix}
y_t \\
z_t
\end{bmatrix} = \begin{bmatrix} y_t \\
z_t
\end{bmatrix} + \frac{1}{1-b_{12}b_{21}} \sum_{i=0}^{\infty} \begin{bmatrix} a_{11} & a_{12} \\
-a_{21} & a_{22}
\end{bmatrix}^t \begin{bmatrix} 1 & -b_{12} \\
-b_{21} & 1
\end{bmatrix} \begin{bmatrix}
\varepsilon_{1t-1} \\
\varepsilon_{2t-1}
\end{bmatrix}
\] (12)

Hence, the moving average representation of (12) and (13) can be written in the error term of the \( \{\varepsilon_{1t}\} \) and \( \{\varepsilon_{2t}\} \) sequences:

\[
\begin{bmatrix}
y_t \\
z_t
\end{bmatrix} = \begin{bmatrix} y_t \\
z_t
\end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} \phi_{11}(i) & \phi_{12}(i) \\
\phi_{21}(i) & \phi_{22}(i)
\end{bmatrix} \begin{bmatrix}
\varepsilon_{1t-i} \\
\varepsilon_{2t-i}
\end{bmatrix}
\]

Or, more compactly,

\[
x_t = \mu + \sum_{i=0}^{\infty} \phi_i \varepsilon_{t-i}
\] (14)

The moving average representation is an especially useful tool to examine the interaction between the \( \{y_t\} \) and \( \{z_t\} \) sequences. The coefficient of \( \phi_i \) can be used to generate the effect of \( \varepsilon_{1t} \) and \( \varepsilon_{2t} \) shocks on the entire time paths of the \( \{y_t\} \) and \( \{z_t\} \) sequences. The accumulated effects of unit impulses in \( \varepsilon_{1t} \) and \( \varepsilon_{2t} \) can be obtained by the appropriate summation of the coefficients of the impulse response functions. The four sets of coefficients \( \phi_{11}(i), \phi_{12}(i), \phi_{21}(i) \) and \( \phi_{22}(i) \) are called the impulse response function. Plotting the impulse response function is the practical way to visually represent the behavior of the \( \{y_t\} \) and \( \{z_t\} \) series in response to the various shocks.

Block F- test and an examination of causality on a VAR will suggest which of the variable in the model have statistically significant impacts on the futures values of each of the variables in the system. But F-test results will not. By construction, be able to explain the sign of the relationship or how long these effects required to take
place. F-test results will not reveal whether changes in the value of a given variable have a positive or negative effect on other variables in the system, or how long it would take for the effects of that variable to work through the system. Such information will be given by an examination of the VAR’s impulse responses. Impulse Responses trace out the responsiveness of the dependent variables in the VAR to shocks to each of the variables. It is probably fairly easy to see what the effects of shocks to the variables will be in such a simple VAR, the same principle can be applied in the context of VAR containing more equations or more lags, where it is much more difficult to see by eye are the interactions between the equations.

3.9.8. Variance Decomposition

Variance decompositions model a slightly different method for examining VAR system dynamics. They give the proportion of the movements in the dependent variables that are due to their own shocks, versus shocks to the other variables. A shock to their $i^{th}$ variable will directly affect that variable of course, but it will also be transmitted to all the other variables in the system through the dynamic structure of the VAR. variance decomposition determine how much of the $s$-step head forecast error variance of a given variable is explained by innovations to each explanatory variables for $s=1, 2, 3,...$. In practice, it is usually observed that own series shocks explain most of the errors variances of the series in a VAR. To some extent, impulse response and variance decompositions offer very similar information. Runkle (1987) argues that confidence bands around the impulses response and variance decomposition should always be constructed.

Since unrestricted VARs are overparameterized, they are not particularly useful for short term forecast. However, understanding the properties of the forecast errors is exceeding helpful uncovering interrelationships among the variables in the system. The coefficients of $A_0$ and $A_1$ and wanted to forecast the various values of $x_{t+1}$ conditional on the observed value of $x_t$, taking the conditional expectations of $x_{t+1}$, we can obtain

$$E_x x_{t+1} = A_0 + A_1 x_t$$
It is noted that one step ahead forecast error is $x_{t+1} - E_t x_{t+1} = e_{t+1}$. If we take conditional expectations, the two steps ahead forecast error is $e_{t+2} + A_1 e_{t+1}$ more generally, it is easily verified that the $n$-step-ahead forecast is

$$E_t x_{t+n} = (I + A_1 + A_1^2 + ... + A_1^{n-1}) A_n + A_n^k x_t$$

and that the associated forecast error is

$$e_{t+n} A_n A_{n-1} e_{t+n-1} + A_1^2 e_{t+n-2} + ... + A_n^{n-1} e_{t+1}$$

(15)

The VMA and the VAR models contain exactly the same information but it is convenient to describe the properties of the forecast errors in term of the $\{\varepsilon_t\}$ sequences. If the conditional forecast $x_{t+1}$, the one-step-ahead the forecast error is $\phi_0 e_{t+1}$. In general,

$$x_{t+n} = \mu + \sum_{i=0}^{\infty} \phi_i e_{t+n-i}$$

So that the $n$-period forecast error $x_{t+n} - E_t x_{t+n}$ is

$$x_{t+n} - E_t x_{t+n} = \sum_{i=0}^{n-1} \phi_i e_{t+n-i}$$

Denoting the $n$-step-ahead forecast error variance of $y_{t+n}$ as $\sigma_y(n)^2$.

$$\sigma_y(n)^2 = \sigma_y^2 [\phi_1(0)^2 + \phi_1(1)^2 + ... + \phi_1(n-1)^2] + \sigma_z^2 [\phi_2(0)^2 + \phi_2(1)^2 + ... + \phi_2(n-1)^2]$$

Because all values of $\phi_k(i)^2$ are necessarily non negative, the variance of the forecast error increase as the forecast horizon $n$ increases. Note that it is possible to decompose the $n$-step–ahead forecast error variance into the proportions due to each shock. The proportions of $\sigma_y(n)^2$ due to shocks in the $\{\varepsilon_{yt}\}$ and $\{\varepsilon_{zt}\}$ sequences are

$$\frac{\sigma_y^2 [\phi_1(0)^2 + \phi_1(1)^2 + ... + \phi_1(n-1)^2]}{\sigma_y(n)^2} \quad \text{and} \quad \frac{\sigma_z^2 [\phi_2(0)^2 + \phi_2(1)^2 + ... + \phi_2(n-1)^2]}{\sigma_z(n)^2}$$

The forecast error variance decomposition gives the proportion of the movements in a sequence due to its own shocks versus shocks to the other variables. Variance decompositions can be useful tools to examine the relationship among
economic variables. If the correlations among the various innovations are small, the identification problem is not likely to be especially important.

3.9.9. GARCH Model

GARCH (1,1) specification proposed by Bollerslev (1986) for the variances leads to an MA(1) given by

\[ r_t = \theta_0 + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \theta_3 \varepsilon_{t-3} + \varepsilon_t \]

\[ \varepsilon_t / \psi_t = \tilde{t} (0, h_t) \]

\[ h_t = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \beta \varepsilon_{t-1} \]

Where \( \psi_{t-1} \) denotes all available information at time \( t-1 \) and \( \alpha_1, \alpha_0, \alpha_1 \) and \( \beta \) are constant and non-negative parameters with \( \alpha_1 + \beta < 1 \).

A more formal way to introduce time varying volatility is through the GARCH model of Engle (1982) and Bollerslev (1986). Assume that \( H_t \) can be specified

\[ \text{Vech} (H) = C + \sum_{i=1}^{m} A_i \text{vech}(\varepsilon_{t-i} \varepsilon_{t-i}^\top) + \sum_{i=1}^{q} B_i \text{vech}(H_{t-i}) \] (16)

Where \( c \) is a \((2 \times 1)\) vector of parameters, the \( A_i \) are \((3 \times 3)\) matrices of parameters for \( i = 1,2,\ldots,m \), the \( B_i \) are \((3 \times 3)\) matrices of parameters for \( i = 1,2,\ldots,q \), and \( \text{Vech} \) is the column stacking operator that stacks the lower triangular portion of a symmetric matrix. This is a bivariate GARCH \((m,q)\) model and it allows autocorrelation in the squared prediction errors to be modeled flexibly, in much the same way that an ARIMA specification provides a flexible means of modeling the autocorrelation in the level of time series. Assuming that the conditional distributions of the prediction errors are normal, the log-likelihood function for a sample of \( T \) observations on cash and futures prices is

\[ L(\Theta) = -T \log 2\pi - 0.5 \sum_{t=1}^{T} \log |H_t(\Theta)| - 0.5 \sum_{t=1}^{T} \varepsilon_t(\Theta) H_t^{-1}(\Theta) \varepsilon_t(\Theta) \] (17)

Where \( \Theta = \{\alpha, C, A_1, A_2, \ldots, A_m, B_1, B_2, \ldots, B_q\} \) is the set of all conditional mean and variance parameters. Notice that \( H_t \) and \( \varepsilon_t \) are functions of the sample data on cash and futures prices and the parameter vector, \( \Theta \). Thus, given a sample, estimation proceeds by maximizing the log-likelihood with respect to the unknown parameters.
Estimating $\Theta$ by maximum likelihood involves a complex nonlinear optimization problem. Once this has been accomplished, however, the time path of the optimal hedge ratios can be computed easily by taking the ratio of $h_t^{21}$ to $h_t^{22}$ at each time period in the sample.

The GARCH model represents a flexible specification for modeling time varying volatility in assets prices, and maximum likelihood is an optimal approach to inference. Thus the GARCH model has significant theoretical advantages over moving sample variances and covariance. On the other hand, the GARCH model is much more difficult and demanding to estimate. A natural question is whether the additional efforts required to estimating the GARCH model provides a significantly improved hedging performance, compared to simpler approaches. This question is investigated with an example because results will invariably depend on the particular application under study.

3.9.10. The Bivariate GARCH Method

As most of the financial time series data possess ARCH effects, the hedge ratio from the VAR models has turned out to be extraneous. To take care of ARCH effects in the residual of the error correction model, a VEC multivariate GARCH model of Bollerslev, Engle and Wooldridge (1988) can be deployed. The main advantages of this model are that it simultaneously models the conditional variance and covariance of two interacted series. So it is possible to retrieve the time varying hedge ratios based on the conditional variances and covariance of the spot and futures prices. A standard MGARCH (1, 1) model is expressed as follows.

$$
\begin{align*}
\begin{bmatrix}
    h_{ss} \\
    h_{sf} \\
    \vdots \\
    h_{ff}
\end{bmatrix}
&=
\begin{bmatrix}
    C_{ss} \\
    C_{sf} \\
    \vdots \\
    C_{ff}
\end{bmatrix}
+
\begin{bmatrix}
    \alpha_{11} & \alpha_{12} & \alpha_{13} \\
    \alpha_{21} & \alpha_{22} & \alpha_{23} \\
    \vdots & \vdots & \vdots \\
    \alpha_{n1} & \alpha_{n2} & \alpha_{n3}
\end{bmatrix}
\begin{bmatrix}
    \varepsilon_{st}^2 \\
    \varepsilon_{sf}^2 \\
    \vdots \\
    \varepsilon_{ff}^2
\end{bmatrix}
+
\begin{bmatrix}
    \beta_{11} & \beta_{12} & \beta_{13} \\
    \beta_{21} & \beta_{22} & \beta_{23} \\
    \vdots & \vdots & \vdots \\
    \beta_{n1} & \beta_{n2} & \beta_{n3}
\end{bmatrix}
\begin{bmatrix}
    \varepsilon_{st} \varepsilon_{sf} \\
    \varepsilon_{sf} \varepsilon_{ff} \\
    \vdots \\
    \varepsilon_{ss} \varepsilon_{ff}
\end{bmatrix}_{t-1}
\begin{bmatrix}
    h_{ss} \\
    h_{sf} \\
    \vdots \\
    h_{ff}
\end{bmatrix}_{t-1}
\end{align*}
$$

Where $h_{ss}, h_{ff}$ are the conditional variance of the errors ($\varepsilon_{ss}, \varepsilon_{ff}$) form the mean equations. In this study mean equation is the bivariate vector error correction (VEC) model. As the model has 21 parameters to be estimated, Bollerslev, Engle and Wooldridge (1988) proposed a restricted version of the above model with $\alpha$ and $\beta$
matrixes having only diagonal elements. This Diagonal VEC (DVEC) model is expressed as

\[ h_{sst} = c_{ss} + \alpha_{11} \varepsilon_{sst-1}^2 + \beta_{11} h_{sst-1} \]  \hspace{2cm} (19)

\[ h_{sft} = c_{sf} + \alpha_{22} \varepsilon_{sft-1} \varepsilon_{ft-1} + \beta_{22} h_{sft-1} \]  \hspace{2cm} (20)

\[ h_{fft} = c_{ff} + \alpha_{33} \varepsilon_{ft-1}^2 + \beta_{33} h_{fft-1} \]  \hspace{2cm} (21)

The time varying hedge ratio has been calculated as the ratio between covariance of spot and futures price with variance of futures price. So \( h_{sft}/h_{fft} \) will be the time varying hedge ratio. The return series used in this study is calculated as the first difference of logarithmic individual stock spot and future index and the index spot and future. Alternative specification of return might have a significant impact on a hedge ratio (N.Bhadurai, 2010).

### 3.9.11. Diagonal VEC-GARCH Model (DVEC- GARCH)

However the multivariate GARCH model is facing the problem of large number of parameters, (21 parameters) which may make the calculation very difficult. To overcome this problem Bollerslev, Engle and Kraft have introduced a Diagonal Vector GARCH (DVEC-GARCH) Model in 1988. The dynamic hedge ratio is estimated in the study through Diagonal Vec-GARCH model. To model the conditional variance, it is needed a model which has the capability of dealing with the volatility. Such models are generally called as ARCH family models. Since the dynamic hedge ratio under minimum variance criterion is the ratio of the covariance of the conditional spot and futures over the conditional futures variance at time t, a natural approach would be to estimate a bivariate GARCH model of spot and futures prices. Then the bivariate GARCH model is considered as the model which is able to capture the time varying nature of return series, volatility spill over between markets or assets and conditional covariance between spot and futures market.

By considering the VECM model as mean equation Brooks et.al (2002) have employed a VECM \((k)\) GARCH model to estimate time varying nature of the second moment. By assuming \( \varepsilon_t / \Omega_t \sim N(O,H_t) \) and by defining \( h_t \) as Vech \((H_t)\), which
denotes the vector half operator that arrange the lower triangular elements of \( N \times N \) matrix in to \( [N (N+1)/2] \) vector, the bivariate VECM GARCH can be written as

\[
\text{Vech}(H_t) = C_0 + A \text{vec}(\epsilon_{s,t-1}\epsilon_{f,t-1}) + B_h h_{t-1}
\]  

(22)

This can be explained as,

\[
H_t = \begin{bmatrix}
h_{ss,t} \\
h_{sf,t} \\
h_{ff,t}
\end{bmatrix} = \begin{bmatrix}
e_{ss,t} \\
e_{sf,t} \\
e_{ff,t}
\end{bmatrix} + \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix} \times \begin{bmatrix}
\epsilon_{s,t-1}^2 \\
\epsilon_{s,t-1} \epsilon_{f,t-1} \\
\epsilon_{f,t-1}^2
\end{bmatrix} + \begin{bmatrix}
b_{11} & b_{12} & b_{13} \\
b_{21} & b_{22} & b_{23} \\
b_{31} & b_{32} & b_{33}
\end{bmatrix} \times \begin{bmatrix}
h_{ss,t-1} \\
h_{sf,t-1} \\
h_{ff,t-1}
\end{bmatrix}
\]

Where \( h_{ss,t} \) and \( h_{ff,t} \) represent the conditional variance of the errors \( \epsilon_{st} \), \( \epsilon_{ft} \) form the mean equations, while \( h_{sf,t} \) represents the conditional covariance between spot and futures prices.

The simplified Diagonal VECH GARCH (1,1) (DVEC GARCH) model, introduced by Bollerslev et al(1988).

\[
r_{s,t} = \alpha_s + \epsilon_{st}
\]

\[
r_{ft} = \alpha_f + \epsilon_{ft}
\]  

(23)

\[
\begin{bmatrix}
\epsilon_{st} \\
\epsilon_{ft}
\end{bmatrix} \sim_{t-1} \mathcal{N}(0,H)
\]  

(24)

\[
H_t = U + A \otimes \epsilon_{t-1}\epsilon_{t-1} + B \otimes H_{t-1}
\]  

(25)

\[
H_t = \begin{bmatrix}
h_{ss,t} & 0 \\
h_{sf,t} & h_{ff,t}
\end{bmatrix} = \begin{bmatrix}
u_{ss} & 0 \\
u_{fs} & u_{ff}
\end{bmatrix} + \begin{bmatrix}
a_{ss} & 0 \\
a_{sf} & a_{ff}
\end{bmatrix} \times \begin{bmatrix}
\epsilon_{s,t-1} \epsilon_{s,t-1} & 0 \\
\epsilon_{f,t-1} \epsilon_{s,t-1} & \epsilon_{f,t-1} \epsilon_{f,t-1}
\end{bmatrix} \\
+ \begin{bmatrix}
b_{ss} & 0 \\
b_{fs} & b_{ff}
\end{bmatrix} \times \begin{bmatrix}
h_{ss,t-1} & 0 \\
h_{fs,t-1} & h_{ff,t-1}
\end{bmatrix}
\]  

(26)
Where equation (1) is the mean equation of the model, $e_t$ is the innovation term, which follows a normal distribution with mean zero and conditional variance $H_t$, $\psi_t$ is the information set at time $t-1$ and $\otimes$ is the Hadamard product. Equation (24) and (25) show that conditional variance follow an ARMA (1,1) process, which depends on its last period variance and last period squared residual. As shown in equation (26) only consider the triangular part of the symmetric metrics of $U, A$ and $B$. The covariance matrix must be positive semi-definite (PSD) but $H_t$ in the DVEC model cannot be guaranteed to be PSD. Therefore, it is considered the fourth model-matrix Diagonal GARCH (1, 1) model, modified form Bollerslev et al. (1994).

$$H_t = UU + AA \otimes e_{t-1}e_{t-1} + b \otimes H_{t-1}$$  \hspace{1cm} (27)

Where $b$ is just a scalar. Equation (27) is a simple PSD version of the DVEC model.

### 3.9.12. Optimal Hedge Ratio

There are many techniques available for reducing and managing risk, the simplest and the most widely used, is hedging with futures contracts. A hedge is achieved by taking opposite positions in spot and futures market simultaneously, so that any loss sustained from an adverse price movement in one market should to some degree be offset by a favorable price movement in the other. The ratio of number of units of the futures units that are purchased relative to the number of units of the spot assets is known as hedge ratio. Since risk in this context is usually measured as the volatility of portfolio returns, an intuitively plausible strategy might be to choose that hedge ratio which minimizes the variance of the return of a portfolio containing the spot and futures position. It is known as the optimal hedge ratio. The variance of the change in the value of the hedged position is given by

$$\nu = \sigma_s^2 + h^2 \sigma_f^2 - 2h\rho \sigma_s \sigma_f$$

Minimizing this expression, $h$ would be

$$h = p \frac{\sigma_s}{\sigma_f}$$

According to this formula, the optimal hedge ratio is time invariant, and would be calculated using historical data. The standard deviation and the correlation between

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movements in the spot and futures series could be forecast from a multivariate GARCH model, so that the expression above is replaced by

\[ h_t = p_t \frac{\sigma_{r,t}}{\sigma_{F,t}} \]

Multivariate GARCH models are in spirit very similar to their univariate counterparts, except that the former also specify equations for how the covariance moves over time. Several different Multivariate GARCH model for estimation have been proposed in the literature, including the VECH, the Diagonal VECH and the BEKK models. In each case, it is assumed that there are two assets, whose return variances and covariance are to be modeled. In the case of two assets, the conditional variances equations for the unrestricted VECH model contain 21 parameters. As the number of assets employed in the model increases, the estimation of the VECH model can quickly become infeasible. Hence, the VECH model’s conditional variance covariance matrix has been restricted to the form developed by Bollerslev, Engle and Wooldridge (1988), in which \( A \) and \( B \) are assumed to 9 and the model, known as a diagonal VECH, is then characterized by

\[ h_{ij,t} = \alpha_{ij} + \alpha_{ij} u_{i,t-1} u_{j,t-1} + \beta_{ij} h_{ij,t-1} \] for \( i,j = 1,2 \)

Where \( \alpha_{ij}, \beta_{ij} \) are parameters. The diagonal VECH multivariate GARCH model could also be expressed as an infinite order multivariate ARCH model, where the covariance is expressed as a geometrically declining weight average of past cross products of unexpected returns, with recent observations carrying higher weights. An alternative solution to the dimensionality problem would be to use orthogonal GARCH or factor GARCH models. It is said that the VECH model is having one disadvantage that there is no guarantee of a positive semi-definite covariance matrix. A variance-covariance or correlation matrix must always be positive semi-definite, and in the case where all the returns in a particular series are all the same so that their variance is zero is disregarded, then the matrix will be positive definite. Among other things, this means that the variance covariance have matrix will have all positive numbers on the leading diagonal, and will be symmetrical about this leading diagonal. These properties are intuitively appealing as well as important from a mathematical point of view, for variances can never be
negative, and the covariance between two series is the same irrespective of which of the two series is taken first, and positive definiteness ensures that this is the case. The modified correlation matrix may or may not positive definite, depending on the values of the correlation that are put in, and the values of the remaining correlations. If by chance, the matrix is not positive definite, the upshot is that for some weighting of the individual assets in the portfolio, the estimated portfolio variance could be negative.

3.10 .READER’S GUIDE

This study is divided in to 7 chapters in which First chapter deals with introduction of study, back ground of the derivatives product, importance of derivatives, futures market in India and the growth of derivatives market in India. Second chapter covers the review of literature in the different area of the study. Through the thorough review of existing literature on the inter relationship between futures market and spot market, causal relationship, lead lag relationship between spot and futures market, determinants of futures markets and its role to predict the futures market return and the risk reduction efficiency of futures market. On the basis of the review, it is being found the gap to make empirical analysis. Third chapter is associated with the methodology of the study-objectives of the study -significance of the study- null hypothesis of the study-detailed explanation on the econometrics models which are applied in the study-limitations of the study. Fourth chapter is examining the relationship of spot market and futures market in India by using different methodology to find the long run relationship between spot and futures market, short term relationship and causality relations among them. Fifth chapter explains the determinants of futures market through causality of spot and futures market- causality of open interest and turnover, trading volume and volatility of futures return by using different models of VAR system. Sixth Chapter deals with the analysis of risk protection level and estimation of optimal hedge ratio for the different sub study period and. Seventh chapter provides the findings, conclusion, suggestions and scope for further research.