Preface

The present thesis is based on my study and research since 1986 as a research scholar in the department of Mathematics of Aligarh Muslim University (AMU), Aligarh, subsequently as a regular student of Industrial Mathematics course at the university of Kaiserslautern, Germany, for a period of two years (1991-1993) and finally as a regular research candidate under the "Sandwich Programme" of German Academic Exchange Services (DAAD) and Government of India during August 1996 - September 1998. In the first place I would like to thank the professors, Prof. M.Z. Khan, department of Mathematics, AMU, Prof. H. Neunzert, Director, Institute of Industrial and Business Mathematics (ITWM), university of Kaiserslautern, Germany and Prof. A.H. Siddiqi, department of Mathematics, AMU, without whose able guidance I could not have succeeded in completing this thesis. The present thesis entitled “Wavelet Packet and its applications”, falls under the general heading of Industrial Mathematics which is an emerging area of Mathematical Sciences. As Prof. Neunzert and Prof. Siddiqi have put in the preface of their Book “Industrial Mathematics” is nothing but modelling, discretization and visualization of solutions of organizational, technical, financial and Industrial problems [62]. In the last two decades, two important concepts, namely “Wavelets” and “Fractal” have revolutionized various areas of science and technology. These concepts have attracted the attention of persons working in different areas of science, technology, finance and management. The study of these two topics require deep knowledge of modern Algebra and Analysis. My study and research during the period of 1986-91 in the field of Algebra proved a boon for my investigations in the present thesis.

The present thesis contains six chapters. Section 1.1 of Chapter 1 deals with introduction of Image compression including classical technique JPEG; Walsh function, background on group representations, frame and Shannon sampling theorem. Section 1.2 and 1.3 of Chapter 1 is devoted to the two new techniques of image compression namely wavelets and fractal. An introduction of wavelet packets is given in Section 1.5. Readers interested in acquiring thorough knowledge of these areas for their application to image compression may look into the current publications like [5], [11], [26], [51], [62] and [72]. The second chapter is devoted to our study of the diamond wavelet packets, more precisely, wavelet packets related to the affine diamond
group are described and the corresponding frames and the point-wise convergence of their expansion are considered. In Chapter 3, we have studied the concept of sharp operator and its applications to image classification, more precisely, the measure of oscillation behaviour of an image alongwith application to the classification of images is investigated. In Chapter 4, distortion measures, important for quality control of images have been investigated. Distortion measures related to total variation and Sobolev error etc. have been studied. The Chapter 5 is based on the comparative study of compression techniques, like wavelets, fractal and JPEG. However, the study of compression ratios for hybrid wavelet-fractal method is yet to be investigated for different types of images. In Chapter 6, we have studied the error estimates for wavelet packet expansion generalizing some well known results concerning wavelets. We have examined the recent results of Chambolle et al [13], concerning the study of wavelet based image processing algorithms and variational problems, for wavelet packets. In the end a fairly comprehensive bibliography is presented.

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