COMPRESSIONS

IN

ABELIAN AND NON-ABELIAN GROUPS
This dissertation entitled 'Compressions In Abelian And Non-Abelian Groups' is the research work done by me since 23rd of August, 1963 in the Department of Mathematics and Statistics, A.M.U.Aligarh.(India) under the encouraging, inspiring and close supervision of Prof. M.A.Kazim, Reader in the Department. The work was all along financially supported by the council of scientific and industrial research, Govt. of India.

The concepts of normality and conjugacy which play a fundamental role in the structural study of non-abelian groups are incidently completely trivial in the case of abelian groups. This, in fact, led me to think seriously to fill up the vast gap in that direction. In this attempt, the concept of 'Compression' is introduced and studied in several of its various aspects (though not completely for lack of space) in this humble presentation. The power of the concept both in abelian and non-abelian groups will be to a larger extent evident through the pages of this work. The technique used is not always very sophisticated but mostly very simple and easily readable which was sometimes found essential for the growth of the concept. A number of other concepts based on this study are introduced like self compression, complete self compression (§5, chapter II), c.s.c-subgroups, smallest c.s.c-subgroups (chapter III), essential equality (Def.3.3), C-simple groups (Def.3.4), compressor (Def.4.1), c-power of an element (Def.4.3)
compression classes, locally c.s.c-groups (Def.5.1), power C-transform (Def.5.2), generalised compressor (Def.6.1); compression series (Def.7.3), compression chains (Def.7.5), C-measure and invert C-measure (Def.9.4, 9.3) and some others in the sequel.

At the outset I must acknowledge my deep indebtedness and respect to my supervisor for his continued influence, and my due homage to Prof. R. Baer and Prof. B.H. Neumann who have thrilled me on occasions by their works which I have used, and the deep thanks for the encouragement and suggestions I received from them in my personal contacts through correspondence. I should also thank Prof. G. Higman, with whom I came in close contact when preparing the work during three months seminar at Chandigarh (Punjab, India.), for his constructive criticism which enlightened me a lot.

The thesis consists of two parts: Part I - Abelian groups Part II - Non-abelian groups and comprises ten chapters with one appendix out of which first seven chapters are devoted to the study of compression in abelian groups on the non-abelian group pattern which has facilitated the direct use of arguments from abelian groups to non-abelian groups. All those theorems proved in these chapters while found holding true even in non-abelian groups have been classified at the end of each chapter. The last three chapters exclusively deal with compression in non-abelian groups. The results proved here are
in addition to those already remarked to be valid both for abelian and non-abelian groups.

In all nearly 180 Theorems accompanied by nearly 80 corollaries and several remarks have been established in the course of this dissertation. To satisfy the requirements of clause VIII of chapter XXV Academic ordinances, every chapter is equipped with a comprehensive introduction in the beginning pointing out the main theorems proved and a bibliography at the end of it indicating the sources used, with the contents articlewise classified and tabulated in the Table of Contents.

In chapter I, we introduce the concept of 'Compression' and study the basic properties and other behaviours of C-transforms of subsets and subgroups. In theorem 1.14, we characterise images of a pair of subsets under a homomorphism as a pair of compression subsets. Chapter II deals with self compression a special form of the concept of compression. We develop the concept in several directions and give a criterion for self compression of all non-trivial subgroups of a periodic group. Chapter III studies the properties of the smallest completely self compressed (c.s.c-) subgroup and of groups which have no proper c.s.c-subgroups (C-simple groups). Since the smallest c.s.c-subgroup is the basic one as far as the concept of compression goes, it has been studied in details in its several aspects. The theorems on relations between the rank of a group and of its smallest c.s.c-subgroup, the index
of the smallest c.s.c-subgroup in the group are of great interest (theorem 3.6 to 3.8) and are used in later study. The notions of essential equality and C-simplicity are quite interesting in their applications. In chapter IV a detailed study of compressor, the analogue of normaliser, obtained from the concept of compression has been made in details. A quite interesting theory of the ascending chain of compressors is developed with the help of the concept of c-power of an element (§11). Chapter V provides a glance into compression classes and some results parallel to those proved in an interesting but simple paper of B.H. Neumann (Bibliography - Chapter V, (10)). A new concept of power-C-transform (Def.5.2) is introduced to study compression classes by a finer tool. In Chapter VI a slight generalisation of the concept of compressor is studied and in Chapter VII we deal with compression series and chains, and obtain some interesting results.

The content of Chapters VIII, IX & X is the continued study of the problems of compression in non-abelian groups similar to those of Abelian groups. Some interesting study is made pertaining to equivalence of this relation of compression in non-abelian cases. Moreover, some results parallel to (Bibliography - Chapter VIII (7) and (8)) are proved in chapter VIII. Several criteria for self compression of subsets and subgroups are provided in chapter IX with various
other intrinsic properties, e.g., the compressor is the normal subset of the normaliser. An interesting study of compressor is made in chapter X, and some really interesting situations have been dealt regarding the relations of compressor and normaliser.

Lastly, the material of this dissertation has not yet been sent for publication as the author finds still a lot of scope to work in different directions of its aspects in abelian and non-abelian groups utilising the theory of compression developed here. However, some work of the author closely related to this work has been either published or is to be published in near future. These papers are on Translative mappings (1) and (2) appendix and on a generalisation to normality (3), appendix is to appear in the Mathematics Student.

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(P.K. Sharma)

A.M.U. Aligarh.

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