CHAPTER II

Models of Multiparticle Production

2.1 Introduction

In order to explain various experimental results on multiparticle production a large number of models have been proposed from time to time and now this number has become too large to go thorough all of them one by one. Here we confine our attention only to those which in addition to explaining some of main experimental results of hadron-nucleus collisions are simple and at the same time sufficiently general. A broad classification has generally been made according to the particle production being assumed to be instantaneous or delayed through the formation of intermediate state/states. In the former class of models, known as the 'single step models' - e.g. the earlier multiperipheral models and the Bremsstrahlung model, cascading is obvious in collision with a nuclear target. In the latter class of models known as the 'double step models' cascading is suppressed due to the formation of intermediate state/states of well defined life time/times that in general decay outside the nucleus due to the time dilation effect. The double step models may be further
subdivided into two groups. One group is of the models where the incident particle is assumed to interact successively with a number of nucleons inside the nucleus, along its trajectory, and thereby generates intermediate states. The nuclear multiparticle final state is thus a superposition of the independent contributions from all these states. The other group is of those models where the incident particle is assumed to interact collectively with all the nucleons along its trajectory and forms a single system which finally contributes towards multiparticle final state. The two fireball models\textsuperscript{9-11}, the diffractive excitation model\textsuperscript{1,12-14}, the energy flux cascade model\textsuperscript{15,16} etc. fall under the former group and the statistical model\textsuperscript{17}, the hydrodynamical model\textsuperscript{18} and a number of recently developed tube models\textsuperscript{19-24} come under the latter group. We shall, however, discuss here only the diffractive excitation, the energy flux cascade, the hydrodynamical and the coherent tube models.

Although, we know that the inter-nuclear cascade model is no longer acceptable regarding the results of hadron-nucleus collisions (see Chapter I), we shall consider it, in a bit detail, just to see how badly it fails and how
it leads to double step models.

Some of the recently developed multiperipheral models and their parton treatment, which at first sight look quite fascinating involve a long chain of assumptions (that are hard to test experimentally) and still have no definite quantitative predictions regarding the main experimental results of hadron-nucleus collisions, have not been considered.

2.2 The interannuclear cascade model.

This model is based on the assumption that the multiparticle final state in a hadron-nucleon collision is reached immediately after the collision, i.e., in a distance $\ll$ nucleon size. Subject to a hadron-nucleus collision the incident particle will interact successively with a number of nucleons inside the nucleus producing secondaries at each collision. Each of these secondaries further interact with the downstream nucleons in the nucleus producing secondaries at each collision. Thus an interannuclear cascade will develop. As the size of the target nucleus increases the probability of secondary interactions increases. Thus a strong dependence of the final state particles on the target size is expected as the energy of the incident particle
increases. As the size of the target nucleus increases the average transverse momentum, $\langle P_T \rangle$, the mean inelasticity (the fraction of the primary energy going with secondaries), $\langle K \rangle$, and the dispersion in the distribution of secondaries increases. The model predicts a large and energy dependent value of the mean number of heavy prongs, $\langle N_h \rangle$, where $N_h$ represents the number of particles with relative velocity $\beta < 0.7$. As we shall see in the next chapter, neither of these predictions agree with the experimental results. To improve the predictions of the model certain adhoc assumptions are made. Thus it is assumed that:

(i) the interaction cross section for secondary hadrons inside the nucleus is sufficiently smaller than the corresponding interaction cross section for free nucleons.

(ii) only a small fraction of secondary particles contribute to cascading and the remaining particles interact collectively.

However, the first assumption is equivalent to assuming either local reduction in the density of nucleons in the nucleus or partial loss of normal field of secondary
hadrons. Agreements with experiments up to energies \( \leq 70 \text{ GeV} \) have been achieved through the former assumption.\(^{29-31}\) Quantitative estimations through the latter one are difficult due to the absence of any rigorous field theory.

The second assumption is equivalent to a model involving a leading particle and a cluster showing a weak interaction with the nuclear matter. In the first approximation this assumption leads to a dependence of the mean multiplicity of created particles on target size \( A \) as \( \sim A^{1/3} \).\(^5\)

It may be interesting to note that the above two assumptions that are necessary to improve the predictions of the model are essentially equivalent to the introduction of an extended virtual phase and therefore, it is better to treat the virtual phase directly in space-time, i.e., particle production through intermediate states.

2.3.1 The diffractive excitation model

The intermediate states are assumed to be generated by the diffractive excitation of the beam and target particles.\(^{1,12-14}\). In the first collision two excited objects
are produced— a slow one and a fast one. Because of the time dilation at high enough energies the life time of the fast object exceeds the time of transit through the nucleus and thus the fast object contributes to the final state particles only after it has left the nucleus. The fast object continues through the nuclear matter with a mean free path identical to that of the incident particle and in the subsequent collision produces another slow excited object. Thus in $\nu$ successive collisions inside the nucleus one fast and $\nu$ slow excited objects are formed. Interactions of slow excited objects, if any, may be ignored as far as generation of new states is concerned. They all decay to give rise to $\frac{1}{2} \langle n \rangle$ secondaries, where $\langle n \rangle$ is the average number of particles produced in hadron-nucleon collisions at the same energy of the incident particle. Thus the 'mean normalized multiplicity', $R_A$, which is the ratio of the average number of particles produced in hadron-nucleus collisions to that produced in hadron-nucleon collisions at the same energy, may be given as

$$R_A = \frac{1}{2} (1 + \frac{\nu}{\nu})$$

where $\frac{\nu}{\nu}$ is the average number of collisions inside the nucleus.
The model predicts essentially an energy independent value of \( R_A \) on target size \( A \). This should, however, be deceived as the limiting value of \( R_A \) which any such two centre model can predict. At truly high energies the secondary interactions of the so called slow excited objects cannot be overlooked and thus a cascading is always suspected.

The model predicts essentially target size independent value of \( \langle K \rangle \) and \( \langle p_T \rangle \). The predictions of the model regarding the angular distribution of particles produced have been discussed below along with that of the energy flux cascade model.

2.3.2 The energy flux cascade model.

In this model it is assumed\(^{15,16}\) that the energy flux of the hadronic matter is the essential variable that determines the early evolution of the system. Subsequent to a hadronic collision an energy flux is formed which has rapidity distribution within it exactly similar to that of the observed asymptotic distribution of the produced particles. (This is achieved by projecting the observed asymptotic flux backward in time via the classical trajectories of the free particles, i.e., the model treats the observed hadron-nucleon final state spectrum as an input parameter.) As the time proceeds, the energy flux expands, with the faster component
forming the head or so-called 'hard hadron' and slower component as tail or 'soft hadron'. Any portion of the energy flux is assumed to act like a real hadron in the subsequent collisions if its spatial thickness is equal to the characteristic thickness of a single hadron in its rest frame. Thus in a hadron-nucleus collision the energy flux produced in the first collision of the incident particle with a nucleon of the nucleus expands and after one mean free path it corresponds to two hadrons - a hard one with approximately full energy and a soft one with energy $\sim E^{1/3}$. In the next collision only the hard hadron generates an energy flux which again gives one soft hadron and essentially the original hard hadron. In this way after $V$ successive collisions one hard and $V$ soft hadrons are produced. The soft hadrons do not produce energy flux except perhaps at very high energies and cascading, if any, is, therefore, inhibited.

Rapidity ($Y$) distribution of secondaries stemming from soft and hard hadrons is shown in figure 1 in arbitrary units. Considered in the rapidity space each of the soft hadron is equivalent to a rapidity slice whose thickness is equal to $1/3 \gamma$, where $\gamma$ represents the total rapidity width. The rest of the $\gamma$ space is associated with
FIG. 1: Rapidity distribution in arbitrary units for class one double step models. Separation at $\frac{3}{2}$ corresponds to energy flux cascade and at $\frac{1}{2}$ to diffractive excitation.
the hard hadron. Taking the energy dependence of the mean multiplicity in hadronic collisions as $\propto \ln S$, where $S$ is the square of c.m. energy, an integration of $\frac{dN}{dy}$ over $y$ gives the mean normalized multiplicity, $R_A$, as

$$R_A = \frac{2}{3} + \frac{1}{3}$$

Thus the model predicts essentially energy independent value of $R_A$ in agreement with the experimental results. The forward part of the rapidity spectrum for all $y > y/3$ in hadron-nucleus collision is identical to that of hadron-nucleon collisions. The excess of the particles appears only in the lower rapidity region $-y < y/3$, or at larger angles. Since all the soft hadrons are identical, i.e., acquire equal rapidity space, the shape of the rapidity distribution of the excess particles should remain unchanged as the size of the target nucleus increases. In other words, the centroid of the excess particles is fixed irrespective of the target mass. The dispersion of the rapidity distribution of particles produced in hadron-nucleus collision remains unchanged with increasing target size and is equal to that observed in hadron-nucleon collisions.
The separation between the soft and hard components of the energy flux at $\frac{y}{3}$ is arbitrary. A more general description may be given by taking the separation point at $\eta y^{33}$. Then the value of $R_A$ is given by

$$R_A = (1-\eta) + n \sqrt{\eta}$$

For $\eta = 1/2$ the predictions of the model are same as that of the diffractive excitation model. The discussion given above is valid also for the diffractive excitation model by taking the separation point at $\frac{y}{2}$ instead of $\frac{y}{3}$ between the soft and hard components. The model gives very weak dependence of $\langle K \rangle$ on $A$ and can explain energy independence of $\langle N_h \rangle$ and $\langle p_{1\perp} \rangle$.

2.4.1 The hydrodynamical model.

The hydrodynamical model was proposed by Blenkij and Landau in 1956. According to this model a hadron-hadron collision is considered as a perfectly (inelastic) central collision in which two Lorentz contracted discs hit each other so hard that they arrest each other and coalesce to form a single system of continuous hadronic matter like a fluid. After the collision the matter expands hydrodynamically.
and the outflow continues until its density falls to the characteristic density of hadronic matter. At this stage the system starts decaying giving rise to the real physical particles in accordance with the laws of statistical thermodynamics. The average multiplicity is given as $\langle n \rangle \sim S^{1/4}$, where $S$ is the square of C.M. energy in hadron-hadron collisions.

In case of hadron-nucleus collision, at sufficiently high energies, the single hadronic system formed during first interaction of the incident hadron with a nucleon of the nucleus interacts further with next nucleon along its trajectory before the expansion phase is over (due to the time dilation) and a Lorentz contracted bigger system of hadronic matter is formed. In the next subsequent collision the phenomenon repeats itself and in this way the incident particle, in effect, cuts out a tube of hadronic matter of radius equal to that of interaction radius. If there are $N$ nucleons in the tube a coalesced hadronic body of $(N+1)$ hadrons is formed. This hadronic body decays outside the nucleus into real particles.

The mass number dependence of the mean normalized multiplicity obtained by Blenklj and Landau on the basis of
the above model for proton-nucleus collisions is expressed as \( R_A \propto A^{0.19} \). More refined estimates lead to a value of \( R_A \propto A^{0.15} \). The model predicts the rapidity distribution of particles to be Gaussian in shape, its width increasing with increasing target size. Further details about the rapidity distribution are given along with the coherent tube models.

Some of the results predicted on the basis of the above picture agree with the experimental results. However, the extreme realization of always perfectly central collisions where two hadrons hit each other so hard that they arrest each other and intermingle together to form a single state of continuous hadronic matter is an over approximation of the real situation. For, from the deep inelastic scattering studies it is known that collisions are dominantly soft in the sense that only small energy-momentum transfer takes place. The extreme situation visualized in the model leads to large transverse momentum events. In fact, this idea has been recently used by Fredriksson\textsuperscript{21} and Meng-ta-Chung\textsuperscript{24} to explain some of the large transverse momentum events (the so called 'violent collisions') in hadron-nucleus collisions.

The model cannot explain the leading particle effect
and inelasticity. The model predicts $K \approx 1$ close to unity in all the collisions. The validity of the model (regarding the statistical treatment) is questionable below 1000 GeV.

2.4.2 Coherent tube models.

The central idea of this class of hadron-tube collision models is that at sufficiently high energies a Lorentz contracted tube of $V$ nucleons (of longitudinal thickness characteristic hadronic thickness in the rest frame of the incident hadron) is considered as structureless medium acting effectively like a single target (the so-called 'big hadron'). A hadron-effective target collision is considered exactly identical to that of hadron-nucleon collision occurring at higher energy ($\sqrt{s_{\text{HN}}}$, i.e., the whole of the hadron-tube multiparticle final state spectrum is simply a hadron-nucleon multiparticle final state spectrum at some higher energy (the so-called 'universality hypothesis'). Some of the immediate consequences of the approach are as follows.

The mean multiplicity of particles in hadron-nucleon collisions may be expressed to rise as $S^x$, with $x \approx 0.2-0.3$. 
or $\ln S$, it follows that mean normalized multiplicity
may be given as $R_A \propto \nu \text{ or } 1+ \frac{\ln \nu}{\ln S}$. Since the
average number of collisions is $\propto A^{1/3}$ for proton-nucleus
collisions, it is expected that $R_A$ should remain constant
$\propto A^{0.7} - A^{0.1}$ if power dependence is considered. The dependence
is too weak to be compatible with the experimental results.

The expected rapidity distribution of particles is
shown in figure 2 in arbitrary units. Subject to a hadron-
ucleus collision the range of the rapidity distribution
increases by an amount $\ln \nu$ at the left end of the distribution,
since $S_{hN} = \sqrt{S_{NN}}$. Thus the centre of the distribution
shifts by an amount $1/2 \ln \nu$ compared to the position of
hadron-nucleon C.M. (due to the motion of the C.M.). The shift
in the position of the centre of the distribution is in close
agreement with the experimental results. The dispersion,
$D(y) = (\langle y^2 \rangle - \langle y \rangle^2)^{1/2}$, of the $y$ distribution should
increase with increasing target size. Since $D^2(y)$ varies as
$\nu$, it is clear from the figure that

$$\frac{D^2(y)_{hA}}{D^2(y)_{hN}} \propto \frac{\ln \sqrt{S}}{\ln S} \propto 1 + \text{constant} \frac{\ln \nu}{\ln S}$$

at a given energy of the incident particle\(^{39}\). This result
FIG 2  Rapidity distribution in arbitrary units for coherent tube models.
FIG. 2  Rapidity distribution in arbitrary units for coherent tube models.
is also a consequence of the universality hypothesis according to which \( D(y) \) in hadron-nucleus collisions at energy \( E \) should be same as that observed in hadron-nucleon collisions at higher energy \( \gg \sqrt{s} \). Since experimentally it is found that \( D(y) \) in hadron-nucleon collisions increases with increasing energy it follows that \( D(y) \) in hadron-nucleus collisions should increase with increasing target size if the universality hypothesis is true. The above discussion regarding the rapidity distribution also holds for the hydrodynamical model.

The universality (or hadron-big hadron collision) hypothesis, if confirmed, will be of tremendous consequences bringing us above the present kinematical limits by turning ISR into heavy ion collision beams. Careful analysis of the experimental data is, therefore, necessary in order to test the hypothesis.

**References**

where $E$ is the total energy of the particle in that frame and $p$ its longitudinal momentum. Under the Lorentz transformation along the beam axis rapidity transforms as

$$y_{\text{Lab.}} = y_{\text{C.M.}} + \gamma$$

where $\gamma = \ln \frac{E}{E_{\text{target}}}$ with $E$ being C.M. energy. The width of the rapidity distribution therefore grows as $\ln E$.

32. A Convenient parameter for describing the longitudinal motion of the relativistic particles is the rapidity \( \gamma \), defined in any frame by:

\[
\gamma = \frac{1}{2} \ln \left( \frac{E + P}{E - P} \right)
\]