CHAPTER IV

Inelastic interactions caused by carbon and silicon nuclei
4.1 Introduction

Study of nucleus-nucleus interactions at relativistic energies have attracted the attention of high energy physicists during the recent years due to the realization[1-5] that such studies might provide an opportunity for the search of possible manifestation of the collective properties of high density nuclear matter and the scale invariance in the collisions of composite systems[6].

With the availability of beams of relativistic heavy-ions at the Lawrence Berkeley Laboratory (LBL), Berkeley, USA and the Joint Institute for Nuclear Research (JINR), Dubna, USSR, it has become possible to investigate the characteristics of heavy-ion collisions at relativistic energies. Until recently, studies on nucleus-nucleus collisions were considered to be rather messy affair owing to the complicated nature of the interactions and due to the fact that theoretical models did not exist to explain the experimental data. However, with the commissioning of heavy-ion accelerators during the last two decades, many new and important developments have taken place in high energy heavy-ion physics.

The mechanism of multiparticle production of secondary particles in relativistic heavy-ion collisions is believed to be similar to that of hadron-hadron and hadron-nucleus interactions at high energies. By analyzing nucleus-nucleus interactions, one may also be able to obtain useful and interesting information about the behaviour of nucleons.
interacting collectively at relativistic energies. Furthermore, from such studies one may be able to investigate the nature of highly compressed phases of nuclear matter. It may be interesting to mention that currently the high energy nucleus-nucleus collisions are perhaps the only source to subject nuclear matter to such abnormal conditions.

In this chapter, results on the general characteristics of the interactions caused by carbon- and silicon-nuclei of primary momentum 4.5 GeV/c per nucleon with emulsion nuclei are presented. Results obtained from the study of the interactions caused by other projectiles having the same beam momentum are also presented. For this purpose, random samples consisting of 852 events of carbon and 1024 interactions produced by silicon nuclei are used to investigate the multiplicity, compound multiplicity and projectile fragment distributions, correlations among various multiplicity parameters, moments of relativistic charged particle multiplicity distributions, KNO scaling and angular distributions of secondary charged particles.

4.2 Multiplicity of secondary particles

Multiplicity, defined as the number of particles produced in an interaction, is considered to be a useful parameter for its study may yield significant information about the production mechanism. However, most of the detecting devices record only the charged particles that are produced. Multiplicity is also regarded to be a useful parameter in the study of multiparticle production process in heavy-ion
collisions. It also serves as one of the sensitive tools for checking the predictions of different phenomenological and theoretical models.

4.2.1 Multiplicity distributions

As already discussed in Chapter II, shower (minimum ionizing particles) tracks are mostly due to pions, created as relativistic particles, in carbon- and silicon-emulsion interactions at 4.5 GeV/c per nucleon. Grey tracks are produced by relatively slower particles consisting of protons, deuterons, tritons, alpha-particles and some slow mesons. Black tracks are due to slow particles and evaporated fragments. Grey and black tracks considered together are termed as heavily ionizing tracks.

Multiplicity distributions of grey, black, shower and heavily ionizing tracks produced in carbon- and silicon-emulsion interactions at 4.5 GeV per nucleon are compared with the corresponding distributions obtained for proton-emulsion[7] and α-emulsion[8] interactions at the same energy per nucleon; these distributions are shown in Figs.4.1-4.4.

It may be seen in Fig.4.1 that the \( N_g \)-distributions in \(^{12}\)C- and \(^{28}\)Si-emulsion interactions are appreciably enriched by the events of relatively higher multiplicities than those observed in the case of proton- and α-emulsion interactions at the same incident energy per nucleon. However from Fig.4.2, it is noticed that the \( N_b \)-distributions remain the same in nucleus-nucleus collisions for all projectiles considered. The \( N_g \)-distributions, plotted in Fig.4.3, exhibit
Fig. 4.1 Multiplicity distribution of grey tracks emitted in p-, α-, 12C- and 28Si-Em interactions at 4.5 A GeV/c.
Fig. 4.2 Multiplicity distribution of black tracks produced in p, α-, 12C- and 28Si-Em interactions at 4.5 A GeV/c.
Fig. 4.3 Multiplicity distribution of shower tracks produced in p-, α-, 12C- and 28Si-Em interactions at 4.5 A GeV/c.
Fig. 4.4 Multiplicity distribution of heavily ionizing tracks emitted in p-, α, \(^{12}\)C- and \(^{28}\)Si-Em interactions at 4.5 A GeV/c.
that the shape of the distribution changes appreciably with increasing projectile mass; the distribution tends to become broader with increasing projectile mass. The number of events of comparatively lower values of $N_s$ decreases with increasing projectile mass.

In Fig. 4.4, small dips in $N_h$-distributions are observed for $^{12}\text{C}$- and $^{28}\text{Si}$-emulsion events at $N_h \sim 2 - 3$. It is interesting to mention that a similar behaviour of the $N_h$-distribution has been observed by Chernov et al.[9] in their study of $^{14}\text{N}$-emulsion interactions at 2.1 GeV per nucleon.

4.2.2 Mean multiplicity

Mean multiplicities of the charged secondary particles for both $^{12}\text{C}$- and $^{28}\text{Si}$-emulsion interactions at 4.5 GeV per nucleon are listed in Table 4.1, along with the results reported by other workers[7-15] involving different projectiles at various incident energies. It is noticed from the table that the average multiplicity of the singly charged relativistic particles grows rapidly with increasing mass of the incident nucleus. It may also be seen from Table 4.1 that $\langle N_g \rangle$ increases with increasing mass of the projectile. This trend can be explained in terms of the predictions of the fire-ball model[16]; this model predicts that the number of the participant nucleons should increase due to increasing volume of the cylinder cut in the target by the projectile. This volume increases with increasing mass of the projectile and consequently the average number of the grey particles should increase. The value of $\langle N_g \rangle$, within the stated errors,
Table 4.1: Mean multiplicities of different charged secondaries emitted in nucleus-nucleus collisions for different projectiles at various energies.

<table>
<thead>
<tr>
<th>Energy</th>
<th>Projectile</th>
<th>( &lt;N_g&gt; )</th>
<th>( &lt;N_b&gt; )</th>
<th>( &lt;N_s&gt; )</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>p</td>
<td>2.81±0.06</td>
<td>3.77±0.08</td>
<td>1.63±0.02</td>
<td>7</td>
</tr>
<tr>
<td>3.6</td>
<td>d</td>
<td>2.34±0.07</td>
<td>5.78±0.15</td>
<td>3.08±0.05</td>
<td>8</td>
</tr>
<tr>
<td>3.6</td>
<td>( \alpha )</td>
<td>4.64±0.17</td>
<td>4.68±0.15</td>
<td>3.86±0.08</td>
<td>8</td>
</tr>
<tr>
<td>4.5</td>
<td>( \alpha )</td>
<td>4.70±0.20</td>
<td>4.70±0.20</td>
<td>3.90±0.10</td>
<td>9</td>
</tr>
<tr>
<td>4.5</td>
<td>C</td>
<td>5.93±0.34</td>
<td>4.49±0.24</td>
<td>7.67±0.19</td>
<td>*</td>
</tr>
<tr>
<td>2.1</td>
<td>N</td>
<td>5.29±0.31</td>
<td>4.57±0.22</td>
<td>8.85±0.28</td>
<td>11</td>
</tr>
<tr>
<td>4.5</td>
<td>O</td>
<td>7.60±0.60</td>
<td>4.88±0.29</td>
<td>10.50±0.60</td>
<td>13</td>
</tr>
<tr>
<td>4.1</td>
<td>Ne</td>
<td>7.08±0.23</td>
<td>7.63±0.17</td>
<td>8.38±0.22</td>
<td>10</td>
</tr>
<tr>
<td>4.5</td>
<td>Si</td>
<td>8.59±0.26</td>
<td>4.69±0.23</td>
<td>11.26±0.33</td>
<td>*</td>
</tr>
</tbody>
</table>

* Present work
is observed to be independent of the mass of the projectile as well as the energy of the incident nucleus.

Variations of $\langle N_g \rangle$ and $\langle N_g \rangle$ with projectile mass at 4.5 GeV per nucleon are shown in Figs. 4.5 and 4.6; the solid lines in these figures can be represented by the following equations:

$$\langle N_x \rangle = \text{const. } A^\alpha$$  \hspace{1cm} (4.1)

where $x$ represents shower or grey particles. The best fit values of $\alpha$ are $0.610 \pm 0.043$ for the variation of the average shower particles with the projectile mass and $0.332 \pm 0.022$ for the variation of mean multiplicity of grey particles with the projectile mass.

Fig. 4.7 shows the variations of $D/\langle N_g \rangle$ with the number of grey tracks for carbon- and silicon-emulsion collisions, where $D$ represents the dispersion defined as $[(\langle N_g^2 \rangle - \langle N_g \rangle^2)^{1/2}]$; $D/\langle N_g \rangle$ varies with $N_g$ in the following fashion:

$$D/\langle N_g \rangle = a (N_g)^b$$  \hspace{1cm} (4.2)

where the values of $a$ and $b$ are found to be $0.834 \pm 0.127$, $-0.352 \pm 0.051$ and $1.081 \pm 0.117$, $-0.497 \pm 0.072$ respectively for carbon- and silicon-emulsion collisions. It may be noted that $D/\langle N_g \rangle$ decreases with increasing $N_g$. This result is at odds with the predictions of the collective tube model[17] and favours the models which take into account the repeated collisions[18]. It may be of interest to mention that the collective tube model predicts that $D/\langle N_g \rangle$ remains constant with $N_g$. Similar results have also been obtained in the case
Fig. 4.5 Average multiplicity of relativistic charged particles as a function of projectile mass. The solid line discussed in the text.
Fig. 4.6 Average multiplicity of grey particles as a function of projectile mass. The solid line discussed in the text.
Fig. 4.7 Variation of $D(N_g)/N_g$ as a function of number of grey tracks.
of proton- and pion-nucleus interactions\cite{19} at high energies.

4.2.3 Multiplicity correlations

Several workers\cite{7,8,20,21} have attempted to investigate the multiplicity correlations of the type \(<N_i(N_j)>\), where \(N_i\), \(N_j\) = \(N_g\), \(N_b\), \(N_h\), \(N_s\); where \(i \neq j\), over widely different energies using different projectiles. We have also attempted to study the multiplicity correlations for the three projectiles-- proton, carbon and silicon; these correlations have been nicely fitted by using the linear relation:

\[
<N_i(N_j)> = a_{ij} + b_{ij} N_j \tag{4.3}
\]

\((i \neq j)\)

in the full range of \(N_j\) variation except the \(<N_b>-N_g\) and \(<N_g>-N_b\) correlations which have been observed to be non-linear ones. In Fig.4.8, the variations of \(<N_b>\), \(<N_g>\) and \(<N_h>\) with \(N_s\) are shown for all the three projectiles, whereas in Fig.4.9 the variations of \(<N_s>\) with \(N_b\), \(N_g\) and \(N_h\) are shown for the three projectiles at 4.5 GeV per nucleon. From these figures the following conclusions may be arrived at:

(i) in the case of p-emulsion interactions there is practically no dependence of \(<N_g>\), \(<N_b>\) and \(<N_h>\) on \(N_s\), signifying the fact that the number of shower particles produced in an event is independent of the target excitation.

But in the case of carbon-emulsion interactions these parameters, \(<N_g>\), \(<N_b>\) and \(<N_h>\), are observed to depend strongly on \(N_s\). Similar trend is visible in the case of
Fig. 4.8 Multiplicity correlations of $<N_b>$, $<N_g>$ and $<N_h>$ with $N_s$ for p-, $^{12}$C- and $^{28}$Si-Em interactions.
Fig. 4.9 Multiplicity correlations of $\langle N \rangle$ with $N_b$, $N_g$, and $N_h$ for $p^-$, $^{12}\text{C}^-$, and $28\text{Si-Em}$ interactions.
(ii) there are negative correlations between \(<N_g>\) with \(N_g\) and \(N_b\) in the case of p-emulsion interactions, whereas in the case of carbon- and silicon-emulsion reactions, the correlations are strong and positive. Similarly, the variation of \(<N_g>\) with the number of heavily ionizing particles in the case of p-emulsion interaction is weak and negative. But in the heavy-ion collisions, namely, carbon and silicon, the variations in the correlations are moderate and positive. The values of the parameter \(b_{ij}\) for the above correlations are given in Table 4.2. It may be observed from the table that the variation in the values of \(<N_c>\) with \(N_g\), \(N_b\) and \(N_h\) are faster for \(^{28}\text{Si}\)-emulsion interactions in comparison to \(^{12}\text{C}\)-emulsion interactions.

Variations of \(<N_b>\) with \(N_g\) and \(<N_g>\) with \(N_b\) are plotted for proton-, carbon- and silicon-emulsion interactions at 4.5 GeV per nucleon in Fig. 4.10. It may be seen in the figure that \(<N_g>-N_b\) correlation gets saturated beyond \(N_b \sim 12\) for proton and \(N_b \sim 16\) for the heavy-ion reactions. Furthermore, the \(<N_b>-N_g\) correlation saturates beyond \(N_g \sim 16\) for carbon- and silicon-emulsion interactions. However, in the case of proton-nucleus collisions \(<N_b>-N_g\) correlation does not saturate. Similar type of saturations have been reported by Stenlund and Otterlund\[22\] for proton- and pion-nucleus collisions over a wide range of incident energies. It has been pointed out by these workers that the saturation in these correlations should occur because the energy of the struck nucleus becomes so large that there is no way to
Table 4.2: Values of the slope coefficients ($b_{ij}$) in multiplicity correlations in proton-, carbon- and silicon-emulsion interactions.

<table>
<thead>
<tr>
<th>Projectiles</th>
<th>$&lt;N_s&gt;$</th>
<th>$&lt;N_c&gt;$</th>
<th>$&lt;N_g&gt;$</th>
<th>$&lt;N_b&gt;$</th>
<th>$&lt;N_h&gt;$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>-0.01±0.04</td>
<td>0.02±0.02</td>
<td>0.03±0.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_s$ C</td>
<td>0.45±0.01</td>
<td>0.32±0.01</td>
<td>0.75±0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Si</td>
<td>0.55±0.03</td>
<td>0.53±0.05</td>
<td>1.13±0.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>-0.10±0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_g$ C</td>
<td>0.55±0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Si</td>
<td>1.17±0.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>-0.02±0.01</td>
<td>0.44±0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_b$ C</td>
<td>0.49±0.03</td>
<td>0.87±0.14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Si</td>
<td>0.84±0.09</td>
<td>1.23±0.14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>-0.03±0.01</td>
<td>0.36±0.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_h$ C</td>
<td>0.32±0.01</td>
<td>0.76±0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Si</td>
<td>0.81±0.02</td>
<td>1.21±0.06</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 4.10 Multiplicity correlations of $\langle N_{\text{b}} \rangle$ with $N_g$ and $\langle N_{\text{g}} \rangle$ with $N_{\text{b}}$ for $p^-$, $^{12}\text{C}$- and $^{28}\text{Si}$-EM interactions.
dispose it off except by undergoing complete destruction. In such a situation, it is expected that:

(i) emitted products may have higher energies than the particles envisaged to be produced through the evaporation process, which in turn results in the increase of the grey particle multiplicity and

(ii) relatively heavier fragments might be produced during the evaporation process, thereby, causing a decrease in the multiplicity of the black tracks. It means that the $<N_g>-N_b$ and $<N_b>-N_g$ correlations are insensitive to the projectile mass in the case of nucleus-nucleus collisions.

4.3 Compound multiplicity

Most of the experiments on hadron-nucleus and nucleus-nucleus collisions have been carried out to investigate mainly the characteristics of shower particles and very little attention has been paid so far to the study of the features of grey particles envisaged to arise from the target. It is interesting to mention that the study of the emission characteristics of grey particles is of special significance because these particles are visualized to be produced during or shortly after the passage of the leading particles and hence are expected to remember a part of the history of these reactions[23]. Recently, Jurak et al[24] have examined the features of shower and grey particles taking together per event, without making any distinction between them, for the case of hadron-nucleus interactions;
the number of grey and shower particles taken together per event has been termed as the compound multiplicity, \( N_C = N_g + N_s \). Similar work has been done by Ghosh et al. [20, 25] for hadron-nucleus and nucleus-nucleus interactions.

### 4.3.1 Compound multiplicity distribution

Compound multiplicity distributions for \( p^- \), \(^{12}\text{C}^-\) and \(^{28}\text{Si}\)-emulsion interactions are shown in Fig. 4.11. It is seen that the compound multiplicity distribution rapidly becomes broader with increasing projectile mass. The values of the mean compound multiplicity and its dispersion obtained in the present study for the three projectiles are presented in Table 4.3.

### 4.3.2 Compound multiplicity correlation

Correlations among the average compound multiplicity and \( N_b \) and \( N_h \) are shown in Figs. 4.12 and 4.13. The variations of \( \langle N_C \rangle \) on \( N_b \) and \( N_h \) are reproduced reasonably well by a linear function (Eq. (4.3)); these variations are exhibited in Figs. 4.12 and 4.13. From these figures it is quite clear that the \( \langle N_C \rangle - N_b \) correlation in the case of \( p^-\)-emulsion interactions is comparatively weak in comparison to the \( \langle N_C \rangle - N_b \) correlation in the case of heavy-ion collisions. It may be noted that the \( \langle N_C \rangle - N_h \) correlation exhibits a similar behaviour. The value of the parameter \( b_{ij} \) occurring in Eq. (4.3) are listed in Table 4.2.

### 4.4 Multiplicity scaling

During the recent years, many attempts [26-29] have been
Fig. 4.11 Compound multiplicity distribution for p-, $^{12}$C- and $^{28}$Si-Em interactions.
Table 4.3: Mean values of compound multiplicity and dispersion for proton-, carbon- and silicon-emulsion interactions.

<table>
<thead>
<tr>
<th>Projectiles</th>
<th>$\langle N_c \rangle$</th>
<th>$D(N_c)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>4.45±0.09</td>
<td>2.89±0.04</td>
</tr>
<tr>
<td>C</td>
<td>13.60±0.33</td>
<td>9.53±0.23</td>
</tr>
<tr>
<td>Si</td>
<td>19.85±0.45</td>
<td>14.20±0.32</td>
</tr>
</tbody>
</table>
Fig. 4.12 Variation of $\langle N_c \rangle$ with $N_b$. 
Fig. 4.13 Variation of $\langle N_G \rangle$ with $N_h$. 
made to study the multiplicity distribution of charged shower particles produced in proton-proton, proton-nucleus and nucleus-nucleus collisions\[20,30\]. It has been observed that the probability distribution for the production of \( n \) relativistic charged particles exhibits a universal behaviour for all the interactions — \( p-p \), \( p-A \), and \( A-A \). Hence one may express \( \langle n \rangle P_n = \Psi(n/\langle n \rangle) \), where \( P_n \) is the probability of production of \( n \) charged particles, \( \langle n \rangle \) represents the average number of charged particles and \( \Psi \) is some function of the variable, \( Z (= n/\langle n \rangle) \). This behaviour of multiplicity distribution as a function of variable \( Z \) is referred to as KNO scaling after Koba-Nielsen-Olesen[31]. The probability of observing \( n \) charged particles in \( p-p \) interactions is related to the scaling function \( \Psi \) as \[31\]

\[
\frac{\sigma_n}{\sigma_{\text{inel}}} = \frac{\langle n \rangle}{\langle n \rangle^2} \Psi(n/\langle n \rangle) = \langle n \rangle^{-1} \Psi(Z) \quad (4.4)
\]

where \( \sigma_n \) is the partial cross-section for producing \( n \) charged particles at a given centre of mass energy \( \sqrt{s} \) and \( \sigma_{\text{inel}} \) is the total inelastic cross-section.

Slattery[32] has proposed the following form for the scaling function:

\[
\Psi(Z) = (AZ + BZ^3 + CZ^5 + DZ^7) \exp(-EZ) \quad (4.5)
\]

where \( A, B, C, D \) and \( E \) are constants. The values of these constants have been evaluated by Slattery[32] for \( p-p \), interactions in the energy range \( \sim (50-303) \) GeV; the values of
A, B, C, D and E are 3.79, 33.7, -6.64, 0.33, -3.04 respectively.

Martin et al[33] have, however, observed that the charged shower particle multiplicity for proton-emulsion interactions obeys a KNO type of scaling instead of an exact KNO scaling. Martin et al[33] have expressed the scaling function \( \psi(Z) \) as:

\[
\psi_m(Z) = \frac{6.84Z + 26.6Z^3 - 2.12Z^5 + 0.16Z^7}{\text{Exp}(-3.28Z)}
\]  

(4.6)

where \( \psi_m(Z) \) is the modified scaling function.

The validity of the KNO scaling at different projectile energies have been debated by many workers having diverse opinions[34]. Recently, some workers[18,29] have attempted to examine the existence of the KNO-type scaling in nucleus-nucleus collisions.

A plot of \( \psi(Z) \) versus \( N_s/\langle N_s \rangle \), where \( \langle N_s \rangle \) is the average number of charged shower particles produced in \(^{12}\text{C}\)-and\(^{28}\text{Si}\)-Em interactions, is shown in Fig.4.14. The solid curve in the figure is fitted with the following KNO-type scaling function:

\[
\psi(Z) = \frac{4.38Z + 1.50Z^3 + 0.09Z^5 + 0.03Z^7}{\text{Exp}(-2.70Z)}
\]  

(4.7)

with \( \chi^2/\text{D.F.} = 0.10 \). The values of the parameters appearing in Eq.(4.7) have been computed with the help of the CERN standard programme MINUIT.

KNO scaling leads to a linear relation between the mean
Fig. 4.14 $\psi(Z)$ versus the scaling function $Z$ for carbon and silicon projectiles. The solid curve represents the KNO type scaling function.
number of relativistic charged particles and the dispersion. Variation of \( D \) as a function of average charged particle multiplicity with the corresponding values of \( \langle n \rangle \) for different projectiles is shown in Fig. 4.15. From the figure it is clear that \( D \) increases linearly with \( \langle n \rangle \) which is in agreement with the KNO scaling law.

### 4.5 Moments of the Multiplicity Distribution

The central moments of the multiplicity distribution are defined as

\[
q!\mu_q = q!\langle (N_\text{s} - \langle N_\text{s} \rangle)^q \rangle
\]

where \( q = 1, 2, 3, 4, \ldots \). The values of the central moments of the relativistic charged particles for \( p^-\), \( ^{12}\text{C}^- \) and \( ^{28}\text{Si}^-\) emulsion interactions are given in Table 4.4. It is noticed from the table that the values of the central moments (\( 2!\mu_2 \), \( 3!\mu_3 \) and \( 4!\mu_4 \)) depend strongly on the projectile mass.

The normalized moments of the relativistic charged particles defined as

\[
C_q = \frac{q!\mu_q}{\langle N_\text{s} \rangle^{q}}
\]

where \( \langle N_\text{s} \rangle = N_\text{s} \sigma_n / \sigma\text{inel} \) is the qth moment of the shower particle multiplicity distribution, \( \sigma_n \) represents the partial cross-section for producing \( n \) charged particle and \( \sigma\text{inel} \) is the total inelastic cross-section.

The values of \( C_2, C_3 \) and \( C_4 \) for the three types of interactions are tabulated in Table 4.4. The values of these moments are observed to be constant within their statistical limits.
Fig. 4.15 Variation of dispersion with $<n>$. 

- Shower multiplicity
- Compound multiplicity
Table 4.4: Moments of the multiplicity distributions of relativistic particles produced in proton-, carbon- and silicon-emulsion interactions.

<table>
<thead>
<tr>
<th></th>
<th>proton</th>
<th>carbon</th>
<th>silicon</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle N_s \rangle$</td>
<td>$1.63\pm0.02$</td>
<td>$7.67\pm0.19$</td>
<td>$11.26\pm0.33$</td>
</tr>
<tr>
<td>$\bar{N}_2$</td>
<td>$1.08\pm0.02$</td>
<td>$5.53\pm0.13$</td>
<td>$10.70\pm0.24$</td>
</tr>
<tr>
<td>$\bar{N}_3$</td>
<td>$0.98\pm0.01$</td>
<td>$5.28\pm0.13$</td>
<td>$11.68\pm0.26$</td>
</tr>
<tr>
<td>$\bar{N}_4$</td>
<td>$1.50\pm0.02$</td>
<td>$7.47\pm0.18$</td>
<td>$15.20\pm0.34$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$1.44\pm0.14$</td>
<td>$1.52\pm0.16$</td>
<td>$1.60\pm0.12$</td>
</tr>
<tr>
<td>$C_3$</td>
<td>$2.55\pm0.17$</td>
<td>$2.89\pm0.08$</td>
<td>$2.84\pm0.06$</td>
</tr>
<tr>
<td>$C_4$</td>
<td>$5.97\pm0.46$</td>
<td>$6.23\pm0.07$</td>
<td>$6.21\pm0.06$</td>
</tr>
</tbody>
</table>
4.6 Angular distributions of secondary particles

Angular distributions of shower, grey and black tracks produced in the interactions of carbon and silicon ions with emulsion nuclei at 4.5 GeV per nucleon are displayed in Figs. 4.16-4.18. These distributions are compared with the corresponding distribution for α-emulsion interactions at the same incident energy per nucleon. The analysis of these distributions leads to the following conclusions:

(i) The angular distribution of shower particles produced in $^{28}\text{Si}$-emulsion interactions compares fairly well with those obtained for carbon-emulsion interactions except in the small angular interval around the forward direction of the projectile. This, in turn, would indicate that the singly charged particles are copiously produced in the case of heavier projectiles.

(ii) The angular distributions of the target fragments, grey and black particles, do not exhibit any peculiarity which can suggest the existence of shock wave phenomenon.

(iii) A comparison of the angular distributions of shower, grey and black particles produced in $^{12}\text{C}$- and $^{28}\text{Si}$-emulsion collisions at 4.5 GeV per nucleon with the corresponding distributions obtained for α-emulsion interactions at the same energy per nucleon[8], reveals that these distributions are essentially independent of the projectile mass.

4.7 Multiplicity distributions of projectile fragments

As already discussed in Chapter II, the fragments having momentum nearly equal to the beam momentum per nucleon and
Fig. 4.16 Angular distribution of shower tracks produced in $\alpha$, $^{12}\text{C}$ and $^{28}\text{Si}$-Em interactions at 4.5 A GeV/c.
Fig. 4.17 Angular distribution of grey tracks produced in $\alpha$, $^{12}$C, $^{28}$Si-Em collisions at 4.5 A GeV/c.
Fig. 4.18 Angular distribution of black tracks produced in α-, 12C- and 28Si-Km collisions at 4.5 A GeV/c.
flying in a narrow cone in the forward direction are projectile fragments.

Multiplicity distributions of the fragments with \(Z = 1, 2\) and \(\geq 3\) for carbon- and silicon-emulsion interactions at 4.5 A GeV are shown in Fig.4.19. It is seen from the figure that the distributions for \(Z = 1\) and \(\geq 3\) fragments strongly depend on the projectile mass. However, the distributions for \(Z = 2\) fragments exhibit a weak dependence on the projectile mass.

It may be of interest to mention that the investigation of the dependence of the average multiplicity of the fragments on the projectile mass is of much physical significance. An attempt is, therefore, made to examine the dependence of this parameter on the projectile mass. Fig.4.20 exhibits the variation of the average multiplicities for \(Z=1, 2\) and \(\geq 3\) with mass number; data are taken from references[9, 11, 35-36]. The solid lines in Fig.4.20 correspond to the following relation:

\[
<N_Z> = \text{const.} \ A^\alpha
\]  
(4.10)

The best fit values of \(\alpha\) are 0.85±0.08, 0.53±0.05 and 1.36±0.18 for the fragments having \(Z = 1, 2\) and \(\geq 3\) respectively.

4.8 Conclusions

Based on the findings of the present study, the following conclusions may be arrived at:

(1) Multiplicity distributions of grey tracks are appreciably
Fig. 4.19 Multiplicity distributions of projectile fragments produced in $^{12}$C- and $^{28}$Si-Ea interactions.
Fig. 4.20 Dependence of the average multiplicity of projectile fragments of charge $Z = 1$, 2 and $\geq 3$ on projectile mass. The solid lines discussed in the text.
enriched by the high multiplicity events with the increase in the projectile mass. This observation can be explained in terms of the fire-ball model.

(2) Multiplicity distributions of the relativistic charged particles rapidly changes with increasing projectile mass.

(3) The mean multiplicities of shower and grey tracks vary linearly with the projectile mass.

(4) The mean multiplicity of black tracks is independent of the projectile mass as well as the incident energy.

(5) There is no significant difference between the multiplicity correlations of the particles produced in $^{12}$C- and $^{28}$Si-emulsion collisions. However, one can say that the variations of the mean shower particle multiplicity, with the heavy, grey and black track multiplicities are faster in the case of $^{28}$Si-emulsion interactions in comparison to $^{12}$C-emulsion collisions. But the correlations in p-emulsion interactions are often different from those of nucleus-nucleus collisions.

(6) The $<N_g>-N_b$ and $<N_b>-N_g$ correlations are observed to saturate beyond $N_b(N_g) \sim 16$.

(7) Compound multiplicity distribution changes rapidly with increasing projectile mass.

(8) Shower particle multiplicity distribution in carbon- and silicon-emulsion collisions, are observed to obey a KNO-type scaling.

(9) The central moments of the shower particle multiplicity distribution increase linearly with increasing mean
multiplicity. However, the normalized moments are found to be essentially independent of the mean multiplicity and the projectile mass.

(10) The angular distributions of grey, black and shower tracks are practically independent of the projectile mass.

(11) The multiplicity distributions for $Z = 1$ and $> 3$ are found to be strongly dependent on the projectile mass. However, the multiplicity distributions for $Z = 2$ fragments do not manifest any appreciable change with the change in the projectile mass.
References


2. A.M. Baldin: Inv. talk at VIth Int. Conf. on High Energy Physics and Nuclear structure, Los Alamos and santa Fe, (1975).


