Chapter III

Generalised Radiation Theory of the Mössbauer Radiation

3.1 Multipole Expansion of Radiation Field:

The absorption and induced emission of electromagnetic radiation by nuclei can be understood on the basis of perturbation theory. An electromagnetic wave described by the vector potential

\[ \vec{A} = A_0 \left[ e^{i(kz-wt)} + e^{-i(kz-wt)} \right] \] .. 3.1

interacts with the charge and magnetic moment of the nucleons. Thus the interaction Hamiltonian can be written as

\[ H' = \sum_{\alpha} \left[ \frac{e_\alpha}{M_{\alpha} c} \vec{A} \cdot \vec{P}_\alpha + \frac{e_\alpha \hbar}{2M_{\alpha} c} \vec{\mu} \cdot \text{curl}\vec{A} \right] \] .. 3.2

in which \( M_{\alpha}, e_\alpha, \vec{\mu}_\alpha \) are the mass, charge and magnetic moment of the nucleons. This interaction induces transitions between the unperturbed nuclear energy levels.

The transition between the two nuclear states accompanied by the emission of a quantum of radiation involves a definite change in quantum numbers \( J, M_J \) and \( P_r \) (parity) in conformity with the conservation principles. Thus it may be useful to consider the waves emitted or absorbed by nucleus as eigenwaves of the angular momentum and parity. Such eigen waves are called the electric or magnetic multipole fields. Therefore, the plane wave representation (Eq. 3.1) for the vector field \( \vec{A} \) is not at all convenient in considering the angular distribution and polarization of the Mössbauer radiation. Hence it is convenient to adopt a technique in which an arbitrary electromagnetic field can be expressed in terms of an infinite series of vector spherical harmonics multiplied by appropriate radial functions. These then can be related to radia-
tion quanta having definite angular momentum and parity.

The multipole fields are solutions of the Maxwell's wave equation in vacuum for a given total angular momentum and parity. According to established conventions, the quantum numbers of the successive electric and magnetic multipoles are as follows. Electric \(2^J\)-pole: \(EJ\); total angular momentum \(J\) and total parity \((-1)^{J}\).

Magnetic \(2^J\)-pole: \(MJ\); total angular momentum \(J\) and total parity \((-(-1)^{J})\). The value of \(J\) for dipole, quadrupole etc., is 1, 2 and so on, respectively. Such multipole fields can be described either by giving the vector potential \(\vec{A}\) or one of the fields \(\vec{E}\) and \(\vec{H}\).

The vector solutions of the wave equation

\[
(v^2 + k^2) \vec{A} = 0,
\]

for the electromagnetic radiation are\(^{45}\)

\[
\vec{A}_E^M = \sqrt{2} \text{ curl } (j_j(kr)Y_j^M(\theta, \phi)) \text{ and } \vec{A}_M^M = \sqrt{2} k j_j(kr)Y_j^M(\theta, \phi)
\]

where \(\vec{A}_E^M\) and \(\vec{A}_M^M\) define the electric and magnetic multipole fields, respectively. The functions \(j_j(kr)\) are the well known spherical Bessel functions and \(Y_j^M(\theta, \phi)\) are the spherical harmonics of definite total angular momentum and parity and are written in terms of \(C\)-coefficients as

\[
Y_j^M(\theta, \phi) = Y_j^M, J, 1 = -\left[\frac{(J+M)(J-M+1)}{2J(J+1)}\right]^{1/2} Y_j^{M-1} \hat{e}_+ + \sqrt{J(J+1)} Y_j^M \hat{e}_0 + \left[\frac{(J-M)(J+M+1)}{2J(J+1)}\right]^{1/2} Y_j^{M+1} \hat{e}_- .
\]

The unit vectors \(\hat{e}_+\) and \(\hat{e}_0\) are the eigen vectors of spin 1 and correspond to a system of complex rotational axes as
FIG. 3.1. UNIT VECTOR IN THE CARTESIAN AND SPHERICAL POLAR COORDINATE SYSTEMS.
Using the wave equation \( \nabla \times \nabla \times \vec{A} - k^2 \vec{A} = 0 \), we can easily obtain the magnetic field of an electric multipole from Eq. (3.4\( ^e \)), on the other hand, the electric field of a magnetic multipole as the time derivative of \( \vec{A} \) in Eq. (3.4\( ^M \)). Thus (apart from uninteresting phases)

\[
\vec{H}^M_{\xi j} = \vec{E}^M_{\mu j} = \sqrt{2} k^2 j_j(kr) \times \mathbf{\chi}^M_{\xi j}(\theta, \phi). \tag{3.7}
\]

These fields in the far zone are tangential to the wave front. The remaining fields can be obtained by using the fact that \( \vec{E} \) and \( \vec{H} \) are perpendicular to \( \vec{r} \), the direction of propagation, and to each other. Thus (apart from phases)

\[
\vec{E}^M_{\xi j} \approx \vec{H}^M_{\xi j} \approx \sqrt{2} k^2 j_j(kr) \vec{r} \times \mathbf{\chi}^M_{\xi j}(\theta, \phi); \quad kr \gg 1. \tag{3.8}
\]

Equations (3.7) and (3.8) together with Eq. (3.5), completely specify the fields in the far zone, where they can be experimentally investigated. This aspect will be useful in the study of the Mössbauer resonance associated with single crystal measurements. From these considerations it is obvious that the spherical coordinate system will be most appropriate to study the angular distributions and polarization of the Mössbauer radiations. To achieve this end the unit vectors \( \hat{x} \), \( \hat{y} \) and \( \hat{z} \) in rectangular coordinate system can be related to \( \hat{r} \), \( \hat{\theta} \) and \( \hat{\phi} \), the unit vectors in spherical coordinate system, as (referring to Fig. 3.1)

\[
\begin{align*}
\hat{x} &= \hat{r} \sin \theta \cos \phi + \hat{\theta} \cos \theta \cos \phi - \hat{\phi} \sin \phi, \\
\hat{y} &= \hat{r} \sin \theta \sin \phi + \hat{\theta} \cos \theta \sin \phi + \hat{\phi} \cos \phi, \\
\hat{z} &= \hat{r} \cos \theta - \hat{\theta} \sin \theta.
\end{align*}
\]
Table 3.1: General expressions for the components of the electric and magnetic field strengths.

<table>
<thead>
<tr>
<th>Electric</th>
<th>Magnetic</th>
<th>2J-pole radiation; kr&gt;&gt;1</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Br</td>
<td>Er</td>
</tr>
<tr>
<td>-E_φ</td>
<td>B_θ</td>
<td>E_θ</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[1/2 {(J+M)(J-M+1)/J(J+1)}^{1/2}Y_{j}^{M-1}\cos \theta e^{i \phi} ]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-M/ {(J+1)}^{1/2}Y_{j}^{M}\sin \theta/2 {(J-M)(J+M+1)/J(J+1)}^{1/2}Y_{j}^{M+1}\cos \theta e^{-i \phi} ] \mathbf{c}</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E_θ</td>
<td>B_φ</td>
<td>E_φ</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[1/2 {(J+M)(J-M+1)/J(J+1)}^{1/2}Y_{j}^{M-1}\cos \theta e^{i \phi} ]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-i/2 {(J-M)(J+M+1)/J(J+1)}^{1/2}Y_{j}^{M+1}\cos \theta e^{-i \phi} ] \mathbf{c}</td>
</tr>
</tbody>
</table>
Table 3.2: Field components for M1 and E2 radiations.

<table>
<thead>
<tr>
<th>m</th>
<th>$E_\theta$</th>
<th>$E_\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>m= -1</td>
<td>$\sqrt{3/16\pi} e^{-i\phi} c$</td>
<td>$-i\sqrt{3/16\pi} \cos \theta e^{-i\phi} c$</td>
</tr>
<tr>
<td>m= 0</td>
<td>0</td>
<td>$+i\sqrt{3/8\pi} \sin \theta c$</td>
</tr>
<tr>
<td>m= +1</td>
<td>$\sqrt{3/16\pi} e^{+i\phi} c$</td>
<td>$+i\sqrt{3/16\pi} \cos \theta e^{+i\phi} c$</td>
</tr>
<tr>
<td>m= -2</td>
<td>$-i\sqrt{5/16\pi} \sin \theta \cos \phi e^{-2i\phi} c$</td>
<td>$-\sqrt{5/16\pi} \sin \theta e^{-2i\phi} c$</td>
</tr>
<tr>
<td>m= -1</td>
<td>$+i\sqrt{5/16\pi} (1-\cos^2 \theta) e^{-i\phi} c$</td>
<td>$-\sqrt{5/16\pi} \cos \theta e^{-i\phi} c$</td>
</tr>
<tr>
<td>m= 0</td>
<td>$+i\sqrt{5/8\pi} \sin \theta \cos \phi c$</td>
<td>0</td>
</tr>
<tr>
<td>m= +1</td>
<td>$-i\sqrt{5/16\pi} (1-\cos^2 \theta) e^{+i\phi} c$</td>
<td>$-\sqrt{5/16\pi} \cos \theta e^{+i\phi} c$</td>
</tr>
<tr>
<td>m= +2</td>
<td>$-i\sqrt{5/16\pi} \sin \theta \cos \phi e^{2i\phi} c$</td>
<td>$+\sqrt{5/16\pi} \sin \theta e^{2i\phi} c$</td>
</tr>
</tbody>
</table>
and with the help of equations (3.6) and (3.5), we can obtain the components of the electric and magnetic fields in spherical coordinate representation as the coefficients of unit vectors $\hat{e}_r$, $\hat{e}_\theta$ and $\hat{e}_\phi$. Thus the multipole fields along the $\hat{e}_\theta$ and $\hat{e}_\phi$ directions are calculated in terms of $X^M_J$ as

\[ (X^M_J)_\theta = \left[ \frac{1}{2} \left\{ \frac{(J+M)(J-M+1)}{J(J+1)} \right\} \right]^{1/2} Y^M_J \cos \theta e^{i\phi} - \frac{M}{\sqrt{J(J+1)}} Y^M_J \sin \theta \]

\[ + \left[ \frac{1}{2} \left\{ \frac{(J-M)(J+M+1)}{J(J+1)} \right\} \right]^{1/2} Y^{M+1}_J \cos \theta e^{-i\phi} \] C and \hspace{1cm} 3.10a

\[ (X^M_J)_\phi = \left[ \frac{1}{2} \left\{ \frac{(J+M)(J-M+1)}{J(J+1)} \right\} \right]^{1/2} Y^{M-1}_J e^{i\phi} - 0 \]

\[ - \frac{1}{2} \left\{ \frac{(J-M)(J+M+1)}{J(J+1)} \right\} Y^{M+1}_J e^{-i\phi} \] C \hspace{1cm} 3.10b

where $C$ is a constant given by

\[ C = \frac{1}{r} (2/\pi)^{1/2} (-1)^{J+1} e^{ik(r-ct)} \]

The general expressions for the $\theta$ and $\phi$ components of the electric and magnetic field strengths for the $2^J$-pole radiation are given in table (3.1). The table (3.2) gives these results in a more explicit form for the magnetic dipole ($M_1$) and electric quadrupole ($E_2$) cases. These results are in agreement with those of Tolhoek and Cox (with few sign differences?) obtained by a different method.

3.2 Angular Distribution in the Presence of Nuclear Quadrupole Hyperfine Interaction:

In the Mössbauer resonance when the decaying nucleus is located in a non-cubic crystalline environment, the nuclear hyperfine field interactions will admix the projection quantum number states of the specified nuclear angular momentum state. Therefore, in calculating the angular distribution and polarization of the Mössbau-
er radiation, we need to know the interaction operator which can be expressed as

$$H' = \sum_{\beta} b_{\beta} \delta_{M}^{J} (-1)^{M} A_{J}^{M}(\hat{r}) T_{J}^{M}(\hat{r}_{N})$$

in which $A_{J}^{M}(\hat{r})$ and $T_{J}^{M}(\hat{r}_{N})$ refer to the radiation field and the source nuclei, respectively. The $b_{\beta}$ are certain constants which are actually irrelevant for our purpose and can be lumped into the $A_{J}^{M}$ or $T_{J}^{M}$. The matrix elements of $H'$ between states of specified angular momentum $J_{f}$ and $J_{1}$ associated with radiation of angular momentum $J$ are then

$$\langle J_{f} M_{f} | H' | J_{1} M_{1} \rangle = \sum_{\beta} \sum_{J} (-1)^{M} \left[ \begin{array}{ccc} J_{1} & J_f & J_{f} \\ M_{1} & M & M_{f} \end{array} \right] * A_{J}^{M} * (J_{f} | \delta_{J} | J_{1})$$

where $\left[ \begin{array}{ccc} J_{1} & J_f & J_{f} \\ M_{1} & M & M_{f} \end{array} \right]$ are the C-coefficients and $(J_{f} | \delta_{J} | J_{1})$ is the reduced matrix element independent of the magnetic quantum numbers and dependent on the details of the nuclear structure which is unperturbed by the hyperfine interactions.

In the following useful results are derived for some specific transitions.

3.2.1 Magnetic Dipole Radiation (M1) in Fe$^{57}$:

In this case the M1 Mössbauer radiation occurs in the transition from the excited state $|3/2^-\rangle$ to the ground state $|1/2^-\rangle$. Due to the nuclear quadrupole hyperfine interaction the excited state gets split into two two-fold degenerate states. The electric field gradient asymmetry parameter $\gamma$ admixes the nuclear states of $|I, m\rangle$ with $|I, m+2\rangle$ and $|I, m-2\rangle$. By diagonalization of the interaction matrix we get the doubly degenerate eigenvalues as

$$E_+ = \frac{e^2 q Q}{4 I(2I-1)^{1/2}} (1 + \gamma^2/3)$$

and

$$E_- = -\frac{e^2 q Q}{4 I(2I-1)^{1/2}} (1 + \gamma^2/3)$$
Table 3.3: Radiation amplitudes for M1 when $\eta \neq 0$.

| $|\Psi_n\rangle$ | $|\frac{1}{2}\rangle$ | $|-\frac{1}{2}\rangle$ |
|-----------------|------------------|------------------|
| $|\Psi_1\rangle$ | $-\frac{1}{\sqrt{2}} a_1 A_1^1 - \frac{1}{\sqrt{6}} a_2 A_1^2$ | $-\frac{1}{\sqrt{3}} a_2 A_1^0$ |
| $|\Psi_2\rangle$ | $-\frac{1}{\sqrt{3}} a_2 A_1^0$ | $-\frac{1}{\sqrt{2}} a_1 A_1^{-1} - \frac{1}{\sqrt{6}} a_2 A_1^1$ |
| $|\Psi_3\rangle$ | $+\frac{1}{\sqrt{3}} a_1 A_1^0$ | $+\frac{1}{\sqrt{6}} a_1 A_1^{-1} - \frac{1}{\sqrt{2}} a_2 A_1^{-1}$ |
| $|\Psi_4\rangle$ | $+\frac{1}{\sqrt{6}} a_1 A_1^{-1} - \frac{1}{\sqrt{2}} a_2 A_1^1$ | $+\frac{1}{\sqrt{3}} a_1 A_1^0$ |
Table 3.4: $E_\theta$ and $E_\phi$ for M1 when $\eta \neq 0$.

| $|i\rangle$ | $E_\theta$ | $|+\frac{1}{2}\rangle$ | $|\frac{1}{2}\rangle$ | $|\frac{3}{2}\rangle$ | $|\frac{5}{2}\rangle$ |
|-----------|-----------|----------------|----------------|----------------|----------------|
| $|1\rangle$ | $\frac{1}{\sqrt{32\pi}}(-\sqrt{3}a_1e^{i\phi}-a_2e^{-i\phi})$ | $i\frac{1}{\sqrt{32\pi}}\cos(-\sqrt{3}a_1e^{i\phi}+a_2e^{-i\phi})$ | $-i\frac{1}{\sqrt{8\pi}}(a_2\sin\theta)$ | $i\frac{1}{\sqrt{8\pi}}\cos(\sqrt{3}a_1e^{-i\phi}-a_2e^{i\phi})$ | $i\frac{1}{\sqrt{8\pi}}(a_2\sin\theta)$ |
| $|2\rangle$ | $0$ | $\frac{1}{\sqrt{32\pi}}(-\sqrt{3}a_1e^{-i\phi}-a_2e^{i\phi})$ | $-i\frac{1}{\sqrt{8\pi}}\sin\theta$ | $i\frac{1}{\sqrt{32\pi}}\cos(\sqrt{3}a_1e^{i\phi}-a_2e^{-i\phi})$ | $\frac{1}{\sqrt{8\pi}}(a_1\sin\theta)$ |
| $|3\rangle$ | $0$ | $\frac{1}{\sqrt{32\pi}}(a_1e^{i\phi}-\sqrt{3}a_2e^{-i\phi})$ | $i\frac{1}{\sqrt{8\pi}}\sin\theta$ | $i\frac{1}{\sqrt{32\pi}}\cos(a_1e^{i\phi}+\sqrt{3}a_2e^{-i\phi})$ | $\frac{1}{\sqrt{8\pi}}(a_1\sin\theta)$ |
| $|4\rangle$ | $\frac{1}{\sqrt{32\pi}}(a_1e^{i\phi}+\sqrt{3}a_2e^{i\phi})$ | $0$ | $i\frac{1}{\sqrt{8\pi}}\cos(-a_1e^{i\phi}-\sqrt{3}a_2e^{i\phi})$ | $\frac{1}{\sqrt{8\pi}}(a_1\sin\theta)$ | $i\frac{1}{\sqrt{8\pi}}(a_2\sin\theta)$ |
The eigen functions for the energy level $E_+$ are

$$|\Psi_1\rangle = a_1 |3/2, 3/2\rangle + a_2 |3/2, -1/2\rangle \quad \text{and} \quad |\Psi_2\rangle = a_1 |3/2, -3/2\rangle + a_2 |3/2, 1/2\rangle$$

and for the energy level $E_-$ are

$$|\Psi_3\rangle = -a_1 |3/2, 1/2\rangle + a_2 |3/2, -3/2\rangle \quad \text{and} \quad |\Psi_4\rangle = -a_1 |3/2, -1/2\rangle + a_2 |3/2, 3/2\rangle$$

where $a_1 = (1 + \sqrt{1 + \eta^2/3})/N [(1 + \sqrt{1 + \eta^2/3})^2 + \eta^2/3]$ and $a_2 = (\eta/\sqrt{3})/N [(1 + \sqrt{1 + \eta^2/3})^2 + \eta^2/3]$.

Thus each of the states in Eq. (3.14) will decay to $|1/2, \pm 1/2\rangle$ states satisfying conservation laws. The transitions from $|3/2, 3/2\rangle$ to $|1/2, \pm 1/2\rangle$ and $|3/2, 1/2\rangle$ to $|1/2, \pm 1/2\rangle$ will lead to a quadrupole doublet with two distinct energies. Using Eqs. (3.11) and (3.12) the radiation amplitudes for $\gamma \neq 0$ can be calculated as e.g.,

For $|\Psi_1\rangle = |1/2, 1/2\rangle$:

$$\langle 1/2, 1/2 | \frac{1}{x} (-1)^M A_1^{-1} A_1^M | \Psi_1\rangle = -a_1 \left[ \begin{array}{c} 3/2 \ 1/2 \\ 3/2-1 \ 1/2 \end{array} \right] A_1^2 - a_2 \left[ \begin{array}{c} 3/2 \ 1/2 \\ 3/2 \ 1/2 \end{array} \right] A_1^{-1}$$

$$= - \sqrt{2} a_1 A_1^1 - \frac{1}{\sqrt{6}} a_2 A_1^{-1}.$$

Similarly these are calculated for all other transitions too and results are given in table 3.3. The field components $E_\theta$ and $B_\phi$ for all these transitions are tabulated in table 3.4.

The intensity of the Mössbauer transition corresponding to the $E_+$ energy is

$$I_+ = \left( \frac{a_2^2}{3} |A_1^0|^2 + \frac{a_2 a_1}{\sqrt{2}} A_1^1 + \sqrt{2} a_2 a_1^{-1} | + \frac{a_2}{\sqrt{6}} A_1^1 + \frac{A_1^{-1}}{\sqrt{2}} \right)^2.$$
\[ a \left(2a_1^2 + 1\right)(1 + \cos^2 \theta) + 4a_2^2 \sin^2 \theta + 2\sqrt{3}a_1 a_2 \sin^2 \theta \cos 2\phi. \] 3.15

Similarly the intensity for the transition related to the \( E^- \) energy is

\[ I_\alpha \left(2a_2^2 + 1\right)(1 + \cos^2 \theta) + 4a_1^2 \sin^2 \theta - 2\sqrt{3}a_1 a_2 \sin^2 \theta \cos 2\phi \] 3.16

3.2.2 Electric Quadrupole Radiation (\( E_2; 2^+ \rightarrow 0^+ \) Transition):

There are a large number of Mössbauer nuclei decaying with electric quadrupole radiation. We have considered the case of the nucleus having excited state spin \( 2^+ \) and ground state spin \( 0^+ \). Due to nuclear quadrupole hyperfine interaction the excited state is split into several energy levels and no splitting in the ground state. The eigen functions with their eigen values for the \( 2^+ \) state are.\(^5\)

I: \[ |\Psi_1\rangle = b_1 |2, 2\rangle + b_1 |2, -2\rangle + b_2 |2, 0\rangle; E_1 = 6A\sqrt{1 + \eta^2/3} \]

II: \[ |\Psi_2\rangle = -\frac{1}{\sqrt{2}} |2, 2\rangle + \frac{1}{\sqrt{2}} |2, -2\rangle; E_2 = 6A \]

III: \[ |\Psi_3\rangle = \frac{1}{\sqrt{2}} |2, 1\rangle + \frac{1}{\sqrt{2}} |2, -1\rangle; E_3 = -3A(1 - \eta) \]

IV: \[ |\Psi_4\rangle = -\frac{1}{\sqrt{2}} |2, -1\rangle + \frac{1}{\sqrt{2}} |2, 1\rangle; E_4 = -3A(1 + \eta) \]

V: \[ |\Psi_5\rangle = b_3 |2, 2\rangle + b_3 |2, -2\rangle + b_4 |2, 0\rangle; E_5 = -6A\sqrt{1 + \eta^2/3} \] 3.17

where \( b_1 = \frac{\eta}{\sqrt{4 \eta^2 + 12 - 12\sqrt{(1 + \eta^2/3)}}} \), \( b_2 = \frac{\sqrt{2\eta^2 + 12 - 12\sqrt{(1 + \eta^2/3)}}}{\sqrt{4 \eta^2 + 12 - 12\sqrt{(1 + \eta^2/3)}}} \), \( b_3 = -\frac{\eta}{\sqrt{4 \eta^2 + 12 + 12\sqrt{(1 + \eta^2/3)}}} \), \( b_4 = \frac{\sqrt{2\eta^2 + 12 + 12\sqrt{(1 + \eta^2/3)}}}{\sqrt{4 \eta^2 + 12 + 12\sqrt{(1 + \eta^2/3)}}} \),

and the constant \( A = e^2 qQ/4I(2I - 1) = e^2 qQ/24 \).
The transition amplitudes have been calculated by using the interaction operator Eq. (3.11) with expression (3.12) and the results are as under:

I: \( \langle 0,0|H|\Psi_1 \rangle = b_1 \begin{bmatrix} 2 & 2 & 0 \\ 2 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} (A_2^{-2} + A_2^2) + b_2 \begin{bmatrix} 2 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} A_2^0 \)

\[
= \frac{1}{\sqrt{5}} b_1 (A_2^{-2} + A_2^2) + \frac{1}{\sqrt{5}} b_2 A_2^0
\]

II: \( \langle 0,0|H|\Psi_2 \rangle = \frac{1}{12} \begin{bmatrix} 2 & 2 & 0 \\ 2 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} (A_2^{-2} - A_2^2) = \frac{1}{12} \sqrt{10} (A_2^{-2} - A_2^2) \)

III: \( \langle 0,0|H|\Psi_3 \rangle = \frac{1}{12} \begin{bmatrix} 2 & 2 & 0 \\ -2 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} (A_2^{-2} + A_2^2) = -\frac{1}{12} \sqrt{10} (A_2^{-2} - A_2^2) \)

IV: \( \langle 0,0|H|\Psi_4 \rangle = \frac{1}{12} \begin{bmatrix} 1 & 2 & 0 \\ 2 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} (A_2^{-1} - A_2^1) = \frac{1}{12} \sqrt{10} (A_2^{-1} - A_2^1) \)

V: \( \langle 0,0|H|\Psi_5 \rangle = b_3 \begin{bmatrix} 2 & 2 & 0 \\ 2 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} (A_2^{-2} + A_2^2) + b_4 \begin{bmatrix} 2 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} A_2^0 \)

\[
= \frac{1}{\sqrt{2}} b_3 (A_2^{-2} + A_2^2) + \frac{1}{\sqrt{2}} b_4 A_2^0 \]

where \( A_j^M \) have their usual meaning as in the preceding section. The intensities of the resulting Mössbauer radiations are obtained, as for M1, with the help of table 3.2 and the results are given below corresponding to the respective eigen energies of the expression in Eq. (3.17) as

I: \( b_1^2 \left( \frac{5}{4\pi} \right) (\sin^2 \theta \cos^2 \phi + \sin^4 \theta \sin^2 2\phi) + b_2^2 \left( \frac{15}{8\pi} \right) \sin^2 \theta \cos^2 \phi - 2b_1 b_2 \left( \frac{5}{4\pi} \right)^* \)

\[
\sqrt{2} \sin^2 \theta \cos^2 \phi.
\]

II: \( \left( \frac{5}{8\pi} \right) (\sin^2 \theta \cos^2 \phi + \sin^4 \theta \cos^2 2\phi). \)

III: \( \left( \frac{5}{16\pi} \right) [(1 - 3\cos^2 \theta + 4\cos^4 \phi) + (4\cos^2 \phi - 1) \sin^2 \theta \cos 2\phi]. \)

IV: \( \left( \frac{5}{16\pi} \right) [(1 - 3\cos^2 \theta + 4\cos^4 \phi) - (4\cos^2 \phi - 1) \sin^2 \theta \cos 2\phi]. \)

V: \( b_3^2 \left( \frac{5}{4\pi} \right) (\sin^2 \theta \cos^2 \phi + \sin^4 \theta \sin^2 2\phi) + b_4^2 \left( \frac{15}{8\pi} \right) \sin^2 \theta \cos^2 \phi - 2b_3 b_4 \left( \frac{5}{4\pi} \right)^* \)

\[
\sqrt{2} \sin^2 \theta \cos^2 \phi. \]

... 3.19
3.3 Transformation of Radiation Field Amplitudes:

All the known calculations, done hitherto, are in the Principal Axis System (PAS) which for practical purposes need to be transformed into the Crystal Fixed Axis System (CFAS) so that the results measured in one system may be transformed into the other. For this purpose the rotation D-matrix method has been used as following:

\[
R_{M}^{J}(\Theta, \Phi)R^{-1} = \sum_{M}^{J} D_{MM}^{J}(\alpha, \beta, \gamma)A_{M}^{J}(\Theta, \Phi) \quad \ldots \quad 3.20
\]

where \( R = e^{-iaJz} e^{-i\beta Jy} e^{-i\gamma Jz} \) is the unitary transformation operator defining the 3D-rotation. The elements of the rotation matrix \( D_{MM}^{J}(\alpha, \beta, \gamma) \) are well known. The angles \( \alpha, \beta \) and \( \gamma \) are the usual Euler angles. The radiation amplitudes calculated for M1 and E2 Mössbauer radiations in the previous sections have been transformed with the help of results for D-matrices given in Appendix-I. The results for the radiation amplitudes are obtained as below.

3.3.1 Field Amplitudes for the M1 Radiation:

For \( |\Psi^{'}_{1} >^{M1} |1/2, 1/2> \):

\[
R(- \frac{1}{\sqrt{2}}a_{1}A_{1}^{1} - \frac{1}{\sqrt{6}}a_{2}A_{1}^{-1})R^{-1} = \frac{1}{\sqrt{2}}a_{1}(D_{1}^{1},1A_{1}^{1} + D_{-1}^{-1},1A_{1}^{-1} + D_{0}^{1},1A_{1}^{0})
\]
\[
- \frac{1}{\sqrt{6}}a_{2}(D_{1}^{1},-1A_{1}^{1} + D_{-1}^{-1},-1A_{1}^{-1} + D_{0}^{1},-1A_{1}^{0})
\]
\[
= -[(\frac{1}{\sqrt{2}}a_{1}D_{1}^{1},1 + \frac{1}{\sqrt{6}}a_{2}D_{1}^{-1},-1)]A_{1}^{1} + \frac{1}{\sqrt{2}}a_{1}D_{1}^{1},-1 + \frac{1}{\sqrt{6}}a_{2}D_{1}^{-1},-1)A_{1}^{-1}
\]
\[
+ (\frac{1}{\sqrt{2}}a_{1}D_{0}^{1},1 + \frac{1}{\sqrt{6}}a_{2}D_{0}^{-1},-1)A_{1}^{0}] . \quad \text{(A)}
\]

Similarly it can be shown that

For \( |\Psi^{'}_{1} > - |1/2, -1/2> \):

\[
= -\frac{1}{\sqrt{3}}a_{2}(D_{1}^{1},1A_{1}^{1} + D_{-1}^{-1},1A_{1}^{-1} + D_{0}^{1},A_{1}^{0}) \quad \text{(B)}
\]
For $|\Psi_2'\rangle = |1/2,1/2\rangle$:

$$= -\frac{1}{\sqrt{3}}a_2(D_{1,0}^{1},oA_{1}^{1}+D_{-1,0}^{1},oA_{1}^{-1}+D_{0,0}^{1},oA_{1}^{0}) .$$  \hspace{1cm} (C)

For $|\Psi_2'\rangle = |1/2,-1/2\rangle$:

$$= \left[ (\frac{1}{\sqrt{6}}a_2^{D_{1,1}}D_{1,1},-)A_{1}^{-1}+(\frac{1}{\sqrt{6}}a_2^{D_{1,1}}D_{-1,1},-)A_{1}^{-1} \
+ (\frac{1}{\sqrt{6}}a_2^{D_{1,1}}D_{0,1},-)A_{1}^{0} \right] .$$ \hspace{1cm} (D)

For $|\Psi_3'\rangle = |1/2,1/2\rangle$:

$$= \frac{1}{\sqrt{3}}a_1(D_{1,0}^{1},oA_{1}^{1}+D_{-1,0}^{1},oA_{1}^{-1}+D_{0,0}^{1},oA_{1}^{0}) .$$  \hspace{1cm} (E)

For $|\Psi_3'\rangle = |1/2,-1/2\rangle$:

$$= \left[ (\frac{1}{\sqrt{6}}a_1^{D_{1,1}}D_{1,1},-)A_{1}^{1}+(\frac{1}{\sqrt{6}}a_1^{D_{1,1}}D_{-1,1},-)A_{1}^{-1} \
+ (\frac{1}{\sqrt{6}}a_1^{D_{1,1}}D_{0,1},-)A_{1}^{0} \right] .$$ \hspace{1cm} (F)

For $|\Psi_4'\rangle = |1/2,1/2\rangle$:

$$= \left[ (\frac{1}{\sqrt{2}}a_2^{D_{1,1}}D_{1,1},-)A_{1}^{1}+(\frac{1}{\sqrt{2}}a_2^{D_{1,1}}D_{-1,1},-)A_{1}^{-1} \
+ (\frac{1}{\sqrt{2}}a_2^{D_{1,1}}D_{0,1},-)A_{1}^{0} \right] .$$ \hspace{1cm} (G)

For $|\Psi_4'\rangle = |1/2,-1/2\rangle$:

$$= \frac{1}{\sqrt{3}}a_1(D_{1,0}^{1},oA_{1}^{1}+D_{-1,0}^{1},oA_{1}^{-1}+D_{0,0}^{1},oA_{1}^{0}) .$$  \hspace{1cm} (H) - 3.2)

Now substituting the values for $D_{M}^{j}$; $J=1$ from Appendix-I and that of $A_{j}^{M}$ in terms of $E_{\theta}$ and $E_{\phi}$ from table 3.4 in the above obtained expressions, we get the electric field amplitudes $E_{\theta c}$ and $E_{\phi c}$ in the CFAS as following:

For the transition (A):

$$E_{\theta c} = \frac{1}{16\pi} \left[ \left( \frac{1}{\sqrt{2}}a_1 + \frac{1}{\sqrt{6}}a_2 \right) \left\{ -\cos(\alpha-\phi)\cos \gamma + \sin(\alpha-\phi)\sin \gamma \cos \beta \right\} \right.
\left. + i \left( \frac{1}{\sqrt{2}}a_1 - \frac{1}{\sqrt{6}}a_2 \right) \left\{ \cos(\alpha-\phi)\sin \gamma + \sin(\alpha-\phi)\cos \gamma \cos \beta \right\} \right]$$
\[ E_{\phi c} = \sqrt{\frac{3}{16\pi}} \left[ (\frac{1}{\sqrt{2}}a_1 + \frac{1}{\sqrt{6}}a_2) \left\{ \cos(\sin(\alpha-\phi))\cos\gamma + \cos(\alpha-\phi)\sin\gamma \cos\beta \right. \right. \\
+ \sin\beta \sin\gamma \sin\gamma \right] + i(\frac{1}{\sqrt{2}}a_1 - \frac{1}{\sqrt{6}}a_2) \left\{ \cos(-\sin(\alpha-\phi))\sin\gamma + \cos(\alpha-\phi)\right. \\
\left. \cos\gamma \cos\beta + \sin\beta \sin\gamma \cos\gamma \right\} \] .

For the transition (B):
\[ E_{\theta c} = i[\frac{a_2}{2\sqrt{2}\pi}] \sin\beta \sin(\alpha-\phi)] \]
\[ E_{\phi c} = i[+ \frac{a_2}{2\sqrt{2}\pi}(\cos(\alpha-\phi)\cos\theta \sin\beta + \cos\beta \sin\theta)]. \]

For the transition (C):
\[ E_{\theta c} = i[- \frac{a_2}{2\sqrt{2}\pi}] \sin\beta \sin(\alpha-\phi)] \]
\[ E_{\phi c} = i[+ \frac{a_2}{2\sqrt{2}\pi}(\cos(\alpha-\phi)\cos\theta \sin\beta + \cos\beta \sin\theta)] . \]

For the transition (D):
\[ E_{\theta c} = -\sqrt{\frac{3}{16\pi}} \left[ (\frac{1}{\sqrt{2}}a_1 + \frac{1}{\sqrt{6}}a_2) \left\{ \cos(\alpha-\phi)\cos\gamma - \sin(\alpha-\phi)\sin\gamma \cos\beta \right. \right. \\
+ \cos(\alpha-\phi)\sin\gamma \cos\gamma \cos\beta \right\} \\
-E_{\phi c} = -\sqrt{\frac{3}{16\pi}} \left[ (\frac{1}{\sqrt{2}}a_1 + \frac{1}{\sqrt{6}}a_2) \left\{ \cos(\sin(\alpha-\phi))\cos\gamma + \cos(\alpha-\phi)\sin\gamma \cos\beta \right. \\
+ \sin\beta \sin\gamma \sin\gamma \right] + i(\frac{1}{\sqrt{2}}a_1 - \frac{1}{\sqrt{6}}a_2) \left\{ \cos(-\sin(\alpha-\phi))\sin\gamma + \cos(\alpha-\phi)\cos\gamma \cos\beta + \sin\beta \sin\gamma \cos\gamma \right\} \] .

For the transition (E):
\[ E_{\theta c} = + i[\frac{a_1}{2\sqrt{2}\pi}] \sin(\alpha-\phi) \sin\beta \]
\[ E_{\phi c} = + i[- \frac{a_1}{2\sqrt{2}\pi}(\cos(\alpha-\phi)\cos\theta \sin\beta + \cos\beta \sin\theta)]. \]

For the transition (F):
\[ E_{ec} = \sqrt{\frac{3}{16\pi}} \left[ (\frac{1}{\sqrt{2}}a_1 - \frac{1}{\sqrt{6}}a_2) \left\{ \cos(\alpha-\phi)\cos\gamma - \sin(\alpha-\phi)\sin\gamma \cos\beta \right. \right. \\
+ \cos(\alpha-\phi)\cos\gamma \cos\beta + \sin\beta \sin\gamma \cos\gamma \right\} + \text{contd..} \]
\[
\begin{align*}
E_c &= \sqrt{\frac{3}{16\pi}} \left[ \left( \frac{1}{\sqrt{6}} a_1 - \frac{1}{\sqrt{2}} a_2 \right) \{ \cos(\alpha - \phi) \cos \gamma \sin(\alpha - \phi) \cos \beta - \sin(\alpha - \phi) \sin \gamma \cos \beta \} + 
\left( \frac{1}{\sqrt{6}} a_1 + \frac{1}{\sqrt{2}} a_2 \right) \{ \cos(\alpha - \phi) \sin \gamma + \sin(\alpha - \phi) \cos \gamma \cos \beta \} \right], \\
\phi_c &= \sqrt{\frac{3}{16\pi}} \left[ \left( \frac{1}{\sqrt{6}} a_1 - \frac{1}{\sqrt{2}} a_2 \right) \{ \cos(\alpha - \phi) \cos \gamma \sin(\alpha - \phi) \cos \beta + \sin(\alpha - \phi) \sin \gamma \cos \beta \} + 
\left( \frac{1}{\sqrt{6}} a_1 + \frac{1}{\sqrt{2}} a_2 \right) \{ \cos(\alpha - \phi) \sin \gamma - \sin(\alpha - \phi) \cos \gamma \cos \beta \} \right]
\end{align*}
\]

For the transition (G):
\[
E_c = \sqrt{\frac{3}{16\pi}} \left[ \left( \frac{1}{\sqrt{6}} a_1 - \frac{1}{\sqrt{2}} a_2 \right) \{ \cos(\alpha - \phi) \cos \gamma \sin(\alpha - \phi) \cos \beta \} + 
\left( \frac{1}{\sqrt{6}} a_1 + \frac{1}{\sqrt{2}} a_2 \right) \{ \cos(\alpha - \phi) \sin \gamma + \sin(\alpha - \phi) \cos \gamma \cos \beta \} \right], \\
\phi_c = \sqrt{\frac{3}{16\pi}} \left[ \left( \frac{1}{\sqrt{6}} a_1 - \frac{1}{\sqrt{2}} a_2 \right) \{ \cos(\alpha - \phi) \cos \gamma \sin(\alpha - \phi) \cos \beta - \sin(\alpha - \phi) \sin \gamma \cos \beta \} + 
\left( \frac{1}{\sqrt{6}} a_1 + \frac{1}{\sqrt{2}} a_2 \right) \{ \cos(\alpha - \phi) \sin \gamma - \cos(\alpha - \phi) \cos \gamma \cos \beta \} \right]
\]

For the transition (H):
\[
E_c = +1 \cdot \left[ \frac{a_1}{2\sqrt{2\pi}} \sin(\alpha - \phi) \right], \\
\phi_c = +1 \cdot \left[ -\frac{a_1}{2\sqrt{2\pi}} (\cos(\alpha - \phi) \cos \gamma \sin \phi + \sin(\alpha - \phi) \sin \beta \cos \phi) \right].
\]

3.3.2 Field Amplitudes for the E2 Transitions:

The transformed radiation amplitudes, in terms of D-matrices, for E2 (2+ → 0+) transitions given in Eq. (3.18) are as following:

For the transition | \Psi_1^+ \rangle → |0,0\rangle:
\[
I: \frac{1}{\sqrt{5}} \left[ (b^2_1 b^2_2 + b^2_1 b^2_1, -2 + b^2_2, 0) A^2_2 + (b^2_1 b^2_2, -2 + b^2_1, 0, 0) A^2_2 - (b^2_1 b^2_1, 2 + b^2_2, 0) A^2_2 + (b^2_1 b^2_1, -2 + b^2_2, 0, 0) A^2_2 \right] A^2_2 
\]

H: For the transition | \Psi_2^+ \rangle → |0,0\rangle.
For the transition $|\Psi_2\rangle \rightarrow |0,0\rangle$:

$$II: \frac{1}{\sqrt{10}}\left[(D_2^2, D_2^2, -2)A_2^2 + (D_{-2,2}^2 - D_{-2,-2}^2)A_2^{-2} + (D_{1,2}^2 - D_{1,-2}^2)A_2^1 + (D_{-1,2}^2 - D_{-1,-2}^2)A_2^{-1} + (D_{0,2}^2 - D_{0,-2}^2)A_2^0 \right].$$

For the transition $|\Psi_3\rangle \rightarrow |0,0\rangle$:

$$III: \frac{1}{\sqrt{10}}\left[(D_2^2 + D_2^2, 1)A_2^2 + (D_{-2,1}^2 + D_{-2,-1}^2)A_2^{-2} + (D_{1,1}^2 + D_{1,-1}^2)A_2^1 + (D_{-1,1}^2 + D_{-1,-1}^2)A_2^{-1} + (D_{0,1}^2 + D_{0,-1}^2)A_2^0 \right].$$

For the transition $|\Psi_4\rangle \rightarrow |0,0\rangle$:

$$IV: \frac{1}{\sqrt{10}}\left[(D_2^2, D_2^2, -1)A_2^2 + (D_{-2,1}^2 + D_{-2,-1}^2)A_2^{-2} + (D_{1,1}^2 + D_{1,-1}^2)A_2^1 + (D_{-1,1}^2 + D_{-1,-1}^2)A_2^{-1} + (D_{0,1}^2 + D_{0,-1}^2)A_2^0 \right].$$

For the transition $|\Psi_5\rangle \rightarrow |0,0\rangle$:

$$V: \frac{1}{\sqrt{5}}\left[(b_3^2 D_2^2, 2 + b_3^2 D_2^2, -2 + b_4^2 D_2^2, 2 + b_4^2 D_2^2, 0)A_2^2 + (b_3^2 D_{-2}^2, -2 + b_3^2 D_{-2}^2, -2 + b_4^2 D_{-2}^2, 0)A_2^{-2} + (b_3^2 D_{1}^2, 2 + b_3^2 D_{1}^2, -2 + b_4^2 D_{1}^2, 0)A_2^1 + (b_3^2 D_{-1}^2, -2 + b_3^2 D_{-1}^2, -2 + b_4^2 D_{-1}^2, 0)A_2^{-1} + (b_3^2 D_{0}^2, 2 + b_3^2 D_{0}^2, -2 + b_4^2 D_{0}^2, 0)A_2^0 \right].$$

Now substituting the values for $D_J^{M,M}$; $J=1$ from Appendix-I and that of $A_J^M$ in terms of $E_\Theta$ and $E_\phi$ from table 3.2 in the above obtained transformed expressions, we get the electric field amplitudes $E_\Theta$ and $E_\phi$ in the transformed coordinate system for which the results for all the above said transitions are given below. The constants occurring in expressions are defined as: $c = \frac{1}{\sqrt{2}} b_1$, $d = \frac{1}{\sqrt{5}} b_2$.

For the transition I:

$$E_\Theta = \sqrt{\frac{5}{16\pi}} \sin \theta \cos \theta \left[ c \left( \frac{1 + \cos \phi}{2} \right)^2 (\sin 2(\alpha - \phi + i \gamma) + \cos 2(\alpha - \phi + i \gamma)) \right] + \frac{\sqrt{2}}{8} \sin^2 \beta \theta \cos \phi \phi.$$
\[
\begin{align*}
\sin^2(a-(l)+\gamma) + \cos^2(a-(l)+\gamma) - \cos^2(a-(l)+\gamma)
\end{align*}
\]
\[-\left(\frac{1-\cos\theta}{2}\right)^2 \left\{ \sin(\alpha-\phi-\gamma)+\cos(\alpha-\phi-\gamma) \right\} \right] + \sqrt{\frac{1}{32\pi}} \sin\theta \cos\phi \theta + \sin^2(\frac{\theta}{2}) \left\{ \sin(\alpha-\phi-\gamma)+\cos(\alpha-\phi-\gamma) \right\} \]

\[
\left\lbrack \left(\frac{1-\cos\theta}{2}\right)^2 \left\{ -\sin(\alpha-\phi-\gamma)+\cos(\alpha-\phi-\gamma) \right\} \right\rbrack + \left\lbrack \left(\frac{1+\cos\theta}{2}\right)^2 \left\{ +\sin(\alpha-\phi+\gamma) \right\} \right\rbrack
\]

\[
\left\lbrack -\cos(\alpha+\phi+\gamma) \right\rbrack + \sqrt{\frac{1}{32\pi}} \left(1-2\cos^2\theta\right) \sin\theta \left\lbrack \cos^2(\frac{\theta}{2}) \right\rbrack \sin(\alpha-\phi+2\gamma)
\]

\[
\left\lbrack +\cos(\alpha+\phi+\gamma) \right\rbrack + \sin^2(\frac{\theta}{2}) \left\lbrack \sin(\alpha-\phi+2\gamma)+\cos(\alpha-\phi+2\gamma) \right\rbrack
\]

\[
\sqrt{\frac{1}{32\pi}} \left(1-\cos^2\theta\right) \sin\theta \left\lbrack \sin^2(\frac{\theta}{2}) \right\rbrack \cos(\alpha+\phi+2\gamma)
\]

\[
\left\lbrack +\sin^2(\frac{\theta}{2}) \left\lbrack \cos(\alpha-\phi+2\gamma)+\sin(\alpha-\phi+2\gamma) \right\rbrack \right\rbrack
\]

For the transition III:

\[
\frac{1}{\sqrt{32\pi}} \sin\theta \cos\theta \sin\theta \left\lbrack \cos^2(\frac{\theta}{2}) \right\rbrack \left\lbrack \sin(2(\alpha-\phi)+\gamma)+\cos(2(\alpha-\phi)+\gamma) \right\rbrack
\]

\[
+\left\lbrack \sin^2(\frac{\theta}{2}) \left\lbrack \sin(2(\alpha-\phi)-\gamma)+\cos(2(\alpha-\phi)-\gamma) \right\rbrack \right\rbrack + \sqrt{\frac{1}{32\pi}} \sin\theta \cos\theta \sin\theta \left\lbrack \sin^2(\frac{\theta}{2}) \right\rbrack \left\lbrack \sin(2(\alpha-\phi)+\gamma)-\cos(2(\alpha-\phi)+\gamma) \right\rbrack
\]

\[
+\left\lbrack \sin^2(\frac{\theta}{2}) \left\lbrack \sin(2(\alpha-\phi)-\gamma)-\cos(2(\alpha-\phi)-\gamma) \right\rbrack \right\rbrack + \cos^2(\frac{\theta}{2}) \left\lbrack \sin(2(\alpha-\phi)+\gamma)-\cos(2(\alpha-\phi)+\gamma) \right\rbrack
\]

\[
+\left\lbrack \sin^2(\frac{\theta}{2}) \left\lbrack \sin(2(\alpha-\phi)-\gamma)-\cos(2(\alpha-\phi)-\gamma) \right\rbrack \right\rbrack + \frac{i}{2} \sqrt{\frac{1}{32\pi}} (1-2\cos^2\theta) \left[ (1+2\cos\theta)(1-\cos\theta) \sin(\alpha-\phi-\gamma)+1+2\cos\theta)(1-\cos\theta) \sin(\alpha-\phi-\gamma)+1 \right]
\]

\[
+\sin(\alpha-\phi-\gamma) \left\lbrack (2\cos\theta-1)/(\cos\theta+1) \right\rbrack
\]

\[
\left\lbrack \sin(\alpha-\phi-\gamma)-\cos(\alpha-\phi-\gamma) \right\rbrack
\]

\[
+\sin(\alpha-\phi-\gamma) \left\lbrack (2\cos\theta-1)/(\cos\theta+1) \right\rbrack
\]

\[
\left\lbrack \sin(\alpha-\phi-\gamma)-\cos(\alpha-\phi-\gamma) \right\rbrack
\]

\[
+\sin(\alpha-\phi-\gamma) \left\lbrack (2\cos\theta-1)/(\cos\theta+1) \right\rbrack
\]
$$+ \sqrt{\frac{1}{32 \pi}} \sin \beta \cos \gamma \sin 2 \theta \sin \gamma,$$

$$E_{\phi_c} = \left[ \sqrt{\frac{1}{32 \pi}} \sin \theta \sin \beta \cos^2 \left( \frac{\beta}{2} \right) \left\{ -\cos(2(\alpha - \phi) + \gamma) + \sin(2(\alpha - \phi) + \gamma) \right\} \right]$$

$$+ \sin^2 \left( \frac{\beta}{2} \right) \left\{ -\cos(2(\alpha - \phi) - \gamma) + \sin(2(\alpha - \phi) - \gamma) \right\} \right] - \sqrt{\frac{1}{32 \pi}} \sin \theta \sin \beta \sin \left( \frac{\beta}{2} \right)$$

$$\left\{ + \cos(2(\alpha - \phi) - \gamma) + \sin(2(\alpha - \phi) - \gamma) \right\} + \cos^2 \left( \frac{\beta}{2} \right) \left\{ \cos(2(\alpha - \phi) + \gamma) \right\}$$

$$+ \sin(2(\alpha - \phi) + \gamma) \right\} \right] + \sqrt{\frac{1}{32 \pi}} \cos \theta \left[ (1 + 2 \cos \beta)(1 - \cos \beta) \left\{ \cos(\alpha - \phi - \gamma) - \sin(\alpha - \phi - \gamma) \right\} \right]$$

$$+ \left\{ 2 \cos \beta(\cos \beta + 1) \left\{ \cos(\alpha - \phi + \gamma) + \sin(\alpha - \phi + \gamma) \right\} \right\} + \text{Zero}].$$

For the transition IV:

$$E_{\theta_c} = \left[ -\sqrt{\frac{1}{32 \pi}} \sin \theta \cos \theta \sin \beta \cos^2 \left( \frac{\beta}{2} \right) \left\{ \sin(2(\alpha - \phi) + \gamma) + \cos(2(\alpha - \phi) + \gamma) \right\} \right]$$

$$- \sin^2 \left( \frac{\beta}{2} \right) \left\{ \sin(2(\alpha - \phi) - \gamma) + \cos(2(\alpha - \phi) - \gamma) \right\} \right] - \sqrt{\frac{1}{32 \pi}} \sin \theta \cos \theta \sin \beta$$

$$\left\{ \sin^2 \left( \frac{\beta}{2} \right) \left\{ \sin(2(\alpha - \phi) - \gamma) - \cos(2(\alpha - \phi) - \gamma) \right\} \right\} - \cos^2 \left( \frac{\beta}{2} \right) \left\{ \sin(2(\alpha - \phi) + \gamma) \right\}$$

$$- \cos(2(\alpha - \phi) + \gamma) \right\} \right] - \sqrt{\frac{1}{32 \pi}} \cos \theta \left[ (1 + 2 \cos \beta)(1 - \cos \beta) \left\{ \cos(\alpha - \phi - \gamma) - \sin(\alpha - \phi - \gamma) \right\} \right]$$

$$+ \left\{ 2 \cos \beta(\cos \beta + 1) \left\{ \sin(\alpha - \phi + \gamma) - \cos(\alpha - \phi + \gamma) \right\} \right\}$$

$$- \sqrt{\frac{1}{32 \pi}} \sin \theta \cos \theta \sin 2 \theta \cos \gamma \right\}].$$

$$E_{\phi_c} = \left[ -\sqrt{\frac{1}{32 \pi}} \sin \theta \sin \beta \cos^2 \left( \frac{\beta}{2} \right) \left\{ -\cos(2(\alpha - \phi) + \gamma) + \sin(2(\alpha - \phi) + \gamma) \right\} \right]$$

$$+ \sin^2 \left( \frac{\beta}{2} \right) \left\{ + \cos(2(\alpha - \phi) - \gamma) - \sin(2(\alpha - \phi) - \gamma) \right\} \right] - \sqrt{\frac{1}{32 \pi}} \sin \theta \sin \beta$$

$$\left\{ \sin^2 \left( \frac{\beta}{2} \right) \left\{ -\cos(2(\alpha - \phi) - \gamma) - \sin(2(\alpha - \phi) - \gamma) \right\} \right\} + \cos^2 \left( \frac{\beta}{2} \right) \left\{ \right\}$$

contá..
\[
\cos(2(\alpha-\phi)+\gamma)+\sin(2(\alpha-\phi)+\gamma)\right] - \frac{1}{2} \sqrt{\frac{1}{32\pi}} \cos\theta\left[(2\cos\beta-1)(\cos\theta+1)\right.
\]
\[
-\cos(\alpha-\phi+\gamma)+\sin(\alpha-\phi+\gamma)\right] + (1+2\cos\beta)(1-\cos\beta)\left\{ \cos(\alpha-\phi-\gamma)\right.
\]
\[
-\sin(\alpha-\phi-\gamma)\right\} - \frac{1}{2} \sqrt{\frac{1}{32\pi}} \cos\theta\left[(1+2\cos\beta)(1-\cos\beta)\right\} \left\{ \cos(\alpha-\phi-\gamma)\right.
\]
\[
-\sin(\alpha-\phi-\gamma)\right\} + (2\cos\beta-1)(\cos\beta+1)\left\{ \cos(\alpha-\phi+\gamma)+\sin(\alpha-\phi+\gamma)\right\}
\]
\[
+ \text{Zero}\right].
\]

For the transition V:

Field amplitudes of transition V are similar to that of the I where constants $b_1$ and $b_2$ need replacement (defined in c) by $b_3$ and $b_4$ respectively.

We have thus obtained the expressions for the components of the electric vector in terms of the spherical polar coordinates. This representation is ideal for the calculation of the polarization at any angle of emission with respect to an arbitrary coordinate system fixed by the Euler angles $\alpha$, $\beta$, $\gamma$ and the polar angles $\theta$ and $\phi$. In the Mössbauer spectroscopy, one generally deals with either the pure M1 or the pure E2 radiations. However, the expressions derived above are valid even for the mixed M1 and E2 transitions with appropriate admixture coefficient $\delta$. The radiation fields derived here for each distinct energy transition although complicated yet they are necessary to evaluate the hyperfine field parameters in experiments using single crystals. In the foregoing chapter we utilize these results to calculate the intensity and polarization distribution of the hyperfine split Mössbauer radiation. The expressions derived here will be directly utilized in the next chapter for the evaluation of several observable parameters.