3.1 Introduction:

Nuclear reactions data have been investigated primarily to obtain knowledge about nuclear structure and reaction mechanism. The cross-sections in the KeV energy region are useful in the design of fast reactors as well as in the study of cosmological theory of element formation in the universe. To understand clearly the element building formation, neutron capture cross-sections data are required in the KeV energy region. Unfortunately, capture cross-sections in the KeV region are known at isolated energies; and these are also not known for all isotopes because most of these cross-sections have been measured using activation technique. This technique is limited to those cases where the life time of product nucleus is neither very short nor very long. In order to understand nucleosynthesis theory Allen et al used empirical expressions for finding the neutron-capture cross-sections at 30 KeV where they were not known experimentally. For this reason, the need may arise for calculating the cross-sections in those cases when they are not known from experiment.

In this energy region the neutron-capture reaction
takes place mostly through compound nucleus mechanism. The only problem in evaluating the cross-sections for this kind of reaction is the precise knowledge of the parameters involved in the theoretical formula. No systematic calculations of the cross-section have been done so far in this energy region. Miskel et al.\textsuperscript{3}) have calculated the capture cross-sections for $^{180}$Hf, $^{181}$Ta, $^{186}$W, $^{197}$Au and $^{232}$Th nuclei in the energy region of 0.03 to 4 MeV. These workers have taken three different values of the parameter $\xi$ i.e. 1.25 $\xi$, 0.75 $\xi$ (where $\xi = \langle D \rangle / 2\pi \langle \rho \rangle$, $\langle D \rangle$ is the average level spacing and $\langle \rho \rangle$ is the average radiation width) in these calculations. Chaubey and Sehgal\textsuperscript{4}) have calculated the parameter $\xi$ at 24 KeV for a number of cases.

In the present work, neutron total capture cross-sections at 24 KeV for 48 nuclei with mass number $45 \leq A \leq 232$ have been calculated on the basis of statistical theory. We have used the expression of Margolis\textsuperscript{5}) for these calculations. It is not known whether $\xi$ is same for s- and p-wave neutrons. We have performed these calculations assuming (a) that $\langle \rho \rangle / \langle D \rangle$ is same\textsuperscript{5,6}) for s-, p- and d-wave neutrons (b) that $\langle \rho \rangle / \langle D \rangle$ for p- and d-wave is $(2J+1)$ time\textsuperscript{7,8,9}) that of the s-wave, assuming $\langle \rho \rangle$ to be same\textsuperscript{9}) for s-, p- and d-wave neutrons. The present work is devoted to presenting these theoretical values along with the experimental data. It is concluded that $\langle \rho \rangle / \langle D \rangle$ is the same for s- and p-wave neutrons.
3.2 Calculations Based on Statistical Theory:

The statistical theory of nuclear reactions is based on two main assumptions: (a) Bohr picture of the compound nucleus formation holds true, and (b) there is an overlapping of the levels at the excitation energy where the compound nucleus is formed. At higher energies of incident neutrons (\( \gtrsim 1 \text{ MeV} \)) the second assumption will hold good but the first assumption may not be completely valid because of a small contribution due to the non-compound processes as shown in chapter IV. In few hundreds of KeV energy region, both assumptions are expected to be valid and that is why Margolis formula based upon statistical theory has been used in predicting \((n,\gamma)\) cross-sections in preceding chapter. At 24 KeV the first assumption is completely valid whereas the assumption (b) also seems to be valid provided the energy spread of the incident neutron beam is greater than the spacing of the levels so that many compound states are simultaneously excited. Margolis has derived an expression for the \((n,\gamma)\) cross-section based on the statistical theory. A brief outline of Margolis formalism may be given as follows.

Let 'I' be the spin of the target nucleus. This spin combines with the neutron spin to give us a "channel spin"
\[ j = j^z = I + 1/2. \] The components \((m)\) of \( j \) along the Z-axis (the direction of the incident neutron beam) are \(-j, (-j+1)\)....
Following Hauser and Feshbach\textsuperscript{10}, the cross-section for the formation of the compound nucleus with spin $J$ by the incident neutron of angular momentum $\ell$ and energy $E$ in the channel of spin $j$ and Z-component $m$ may be given as

$$\mathcal{J}(\ell,j,J,m,E) = (2\ell + 1)\pi \frac{2}{\lambda} T_\ell(E) \times |\mathcal{C}(\ell j 0 m, Jm)|^2 \quad \ldots \ldots \ldots \ldots \ldots (1)$$

where $\lambda$ is the wavelength of the incident neutron, $\mathcal{C}(\ell j 0 m, Jm)$ is the Clebsch-Gordan coefficient relating to the probability that $\ell$ and $j$ with Z-component 0 and $m$ combine vectorially to give spin $J$ with Z-component $m$, and $T_\ell(E)$ is the transmission coefficient for the neutrons.

Now the partial decay probability of the compound state with spin $J$ and excitation energy $(B+E)$ ($B$ is the last neutron binding energy in the compound nucleus) through $\gamma$-emission may be given as:

$$\mathcal{J}(\ell,j,J,m,E) \times \left[ \frac{\Gamma_\gamma(B+E)}{\Gamma(B+E)} \right] \quad \ldots \ldots \ldots \ldots \ldots (2)$$

where $\Gamma_\gamma(B+E)$ and $\Gamma(B+E)$ are the partial radiation width and total width respectively of the compound state with spin $J$ and excitation energy $(E+B)$. Summing expression (2) over the possible $J$'s and $\ell$'s, and averaging over all $j$ and $m$ one gets

$$\mathcal{J}_{\text{CaF}} = \sum_{\ell=0}^{\infty} T_\ell(E) \left[ \sum_{\ell=0}^{\infty} \frac{\mathcal{J}_{\ellj}}{\Gamma_\gamma(B+E)} \right] \ldots \ldots \ldots \ldots \ldots (3)$$
where \( \epsilon^J_{j_1} = 2 \), if both \( j_1 \) and \( j_2 \) satisfy \( |J - 1| < j^+ \leq J + 1 \)
\( = 1 \), if \( j_1 \) or \( j_2 \) (not both) satisfies \( |J - 1| < j \leq J + 1 \)
\( = 0 \), otherwise

Margolis used a form of level density \( \rho(E) \) at excitation energy \( E \) on the basis of Fermi gas model

\[
\rho(E) = C \exp \left[2(aE)^{1/2}\right] \quad \ldots \quad (4)
\]
to calculate \( \langle T'/n \rangle \). The expression derived by him is given by

\[
\Gamma_n (J')(E')/ \Gamma_n^J (B+E) = f_{\lambda}(E') \sum_{\Delta I} \rho_{\Delta I} (E) \quad \ldots \quad (5)
\]

where \( \Gamma_n (E) \) is the width for the neutron emission of angular momentum \( 1' \) and energy \( E' \) from the compound state of spin \( J \)

and excitation energy \( (B+E) \) and \( \delta + \epsilon \)

\[
f_{\Delta I} = \int_0^{\delta + \epsilon} \epsilon^{2\Delta I + 1} \rho(B-\epsilon)d\epsilon/ \int_0^{\delta + \epsilon} \epsilon^{2\Delta I + 1} \rho(B+E-\epsilon)d\epsilon \ldots \quad (6)
\]

\( \Delta I \) being the multipolarity of the \( \gamma \)-rays emitted and

\[
f^J_{\lambda} = D^J(B)/2\pi \Gamma_j^J(B) \quad \ldots \quad (7)
\]

where \( D^J(B) \) is the spacing of the compound nucleus levels of spin \( J \) at excitation energy \( (B) \).

On the assumption that at the value of the excitation energy concerned compound nucleus can decay by neutron emission or by \( \gamma \)-emission one can write for \((n,\gamma)\) reaction cross-section using equations (3) and (5),

\[
\Omega_{np} (n,\gamma) = \frac{\pi \lambda}{2(2I+1)} \sum_{\ell,\delta} \left\{ T_1 (E) \sum_{\ell,\delta} \epsilon^J_{\ell,\delta} (2J+1) \right\} \ldots \quad (8)
\]
where $E_n$ is the energy of the $n$th excited state of the target nucleus and $j_n = i_n \pm 1/2$, where $i_n$ is the spin of the $n$th excited state of the target nucleus. The sum over $l'$ includes only those values for which parity of the system remains conserved. The sum over $E_n$ includes only those states for which $E_n < E$.

While calculating cross-sections we have included the contribution of angular momentum of neutrons up to $l = 2$, as only s- and p-wave neutrons contribute predominantly to the capture cross-section at $24 \text{ KeV}$ in most of the target nuclei. The neutrons transmission coefficients were calculated using the data of Campbell et al. We have taken the spherical complex well potential with diffuse edges and the value of nuclear radius as $R = (1.25 \ A^{1/3} + 0.5)$ fermis in the calculations of $T_1$. It is assumed that dipole radiation predominates over other higher multipoles, the ratio of the two is of the order of $10^{-5}$. Different values of level density parameter $\alpha$, neutron binding energy $B$ and pairing energy $\Delta$, corresponding to different isotopes have been taken from the results of Gilbert and Cameron and Baba. There was no significant variation in the values of $f_{\Delta I}$ for different isotopes. Most of these values lie between 0.93 to 0.97. However, we have taken different values of $f_{\Delta I}$ for different isotopes in our calculations. We also calculated $f_{\Delta I}$ for $^{197}\text{Au}$ assuming quadrupole radiations. This value of $f_{\Delta I} = 0.975$ is very
close to $f_{\Delta I} = 0.953$ for dipole radiations, therefore, our results are not affected in the presence of a small admixture of quadrupole radiations.

$\xi_J$ is defined as $D_J/2\pi \Gamma^{(\gamma)}_r$ where $\Gamma^{(\gamma)}_r$ is the radiation width and $D_J$ is the level spacing between levels of same spin and parity. We have done two sets of calculations of the capture cross-sections once taking into account $J$-dependence of $\xi$ through $D_J$ and another taking $\xi$ to be independent of $J$. $\xi'$ and $\xi$ denote the value with $J$-dependence and without $J$-dependence.

The parameter $\xi$ was calculated by taking the average value of level spacing $D$ and radiation width $\Gamma^{(\gamma)}_r$ from the recently known resonance parameters. While calculating $\xi$, the average level spacing $D$ for zero spin target nuclei was taken to be equal to the observed level spacing, whereas for non-zero spin target nuclei the average level spacing was taken to be twice that of the observed level spacing. It has been shown that $\langle D \rangle$ and $\langle D \rangle$ do not change significantly up to 100 KeV of incident neutron energy and hence these low energy resonance parameters can be used at 24 KeV.

The s-wave contributions to capture cross-section have been calculated using s-wave resonance parameters available in literature. The p-wave contributions have been calculated following two different approaches:

(a) that $\langle \Delta D \rangle$ is same for s-, p- and d-wave neutrons.
(b) that $\langle \Gamma \rangle/\langle D \rangle$ for p- and d-wave is $(2J + 1)$ times that of the s-wave, assuming $\langle \Gamma \rangle$ to be same for s-, p- and d-wave neutrons.
Table 1. A comparison of theoretical and experimental values of neutron capture cross-sections at 24 keV together with the $\xi_{\text{ave}}$ values.

<table>
<thead>
<tr>
<th>Target nucleus</th>
<th>$\xi_{\text{ave}}$</th>
<th>$(\sigma_{\text{theo}})^t_c$ (mb)</th>
<th>$(\sigma_{\text{exp}})^t_c$ (mb)</th>
<th>$(\sigma_{\text{ave}})^t_c$ (mb)</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{11}$Li</td>
<td>1.54 ± 0.239</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{13}$C</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{14}$N</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{15}$N</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{17}$F</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{18}$O</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{19}$F</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{20}$Ne</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{21}$Ne</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{22}$Na</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{23}$Na</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{24}$Na</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{25}$Na</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{26}$Na</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{27}$Al</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{28}$Si</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{29}$Si</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{30}$Si</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{31}$P</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{32}$S</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{33}$S</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{34}$S</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{35}$Cl</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{36}$Cl</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{37}$Ar</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{38}$Ar</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{39}$K</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
<tr>
<td>$^{40}$Ca</td>
<td>1.54 ± 0.029</td>
<td>6.2 ± 0.17</td>
<td>0.018 ± 0.007</td>
<td>0.109 ± 0.003</td>
<td>0.108 ± 0.002</td>
</tr>
</tbody>
</table>

Notes:
1. Values in parentheses indicate that the uncertainty is larger than the value itself.
2. The uncertainties quoted are the standard deviations of the mean.
3. The values in the last column are the ratios of the theoretical to the experimental cross-sections.
3.3 Results:

The results of the present calculations are summarized in Table I. In column Ist and IIInd are given the target nucleus and the values of $\xi$ for different isotopes. The third and fourth columns contain the computed cross-sections corresponding to $\xi$ and $\xi' (= <D>/2\pi \sum (21+1))$, respectively. The errors in the calculated cross-section values are due to errors present in the value of resonance parameters. Various experimental values \cite{22-30} of total capture cross-sections mostly obtained by activation technique using Sb-Be Photoneutron source, which has an energy spread 5KeV \cite{28}, have been suitably averaged in Vth column of table I. Some of the average values of $\sigma_{\text{expt.}}$ have been taken from ref. 2. The search is not claimed to be exhaustive, but it is believed that no important data have been missed. In $^{79}\text{Br}$, $^{85}\text{Rb}$, $^{103}\text{Pd}$, $^{109}\text{Ag}$, $^{114}\text{Cd}$, $^{115}\text{In}$, $^{133}\text{Cs}$, $^{164}\text{Dy}$, $^{184}\text{W}$, $^{181}\text{Ta}$ and $^{209}\text{Bi}$, values of $\sigma_{\text{expt.}}$ are the sum of the cross-sections for the isomeric and the ground states, and thus are total capture cross-sections. The ratio of the theoretical values to those of experimentally measured cross-sections have been presented in the VIth and VIIth column of the table. Fig.1 illustrates the behaviour of $(\sigma_{\text{theo.}})/\sigma_{\text{expt.}}$ and $(\sigma_{\text{theo.}}')/\sigma_{\text{expt.}}$ versus the neutron number $N$ in the target nucleus.

Practically for all cases given in table I, the contribution of the d-wave to capture cross-section is very
Figure 1

Number \( N \) of the Target Nucleus

Plot of the Ratios \( r_{\text{theo}} / r_{\text{exp}} \) and \( \sigma_{\text{theo}} / \sigma_{\text{exp}} \) vs. the Neutron
small in comparison with s- and p-wave, so it is difficult to say whether \( \langle \gamma \rangle / \langle \sigma \rangle \) for d-wave is same that of s-wave. However, the contribution of p-wave to the capture cross-section is either comparable or more than s-wave contribution in most of the cases, therefore, it is easy to verify whether \( \langle \gamma \rangle / \langle \sigma \rangle \) is the same for s- and p-wave. It is clear from the Fig.1 that most of the points are closer to the line corresponding to the ratio 1, within experimental uncertainties, when \( J \) is taken to be independent of \( J \); thus confirming that \( \langle \gamma \rangle / \langle \sigma \rangle \) is the same for s- and p-wave. From Table it is clear that difference between \( \langle \sigma^- \rangle_{\text{theo}} \) and \( \langle \sigma^- \rangle_{\text{theo}} \) for nuclei \(^{98-100} \text{Mo}, \ 107-109 \text{Ag}, \ 114 \text{Cd}, \ 115 \text{In}, \ 127 \text{I}, \ 133 \text{Cs} \) is not significant and therefore it is difficult to verify the relation \( \langle \gamma \rangle / \langle \sigma \rangle \rangle_{\text{p-wave}} \approx \langle \gamma \rangle / \langle \sigma \rangle \rangle_{\text{s-wave}} \) for these cases.

It may be remarked that \( \langle \gamma \rangle \) and \( \langle \sigma \rangle \) can independently change for p-wave keeping the ratio \( \langle \gamma \rangle / \langle \sigma \rangle \) to be the same. Allen et al.\(^{31} \) have proved for \(^{56} \text{Fe} \) that \( \langle \gamma \rangle / \langle \sigma \rangle \) has the value \( \approx 0.06 \pm 0.0017 \) and \( 0.0428 \pm 0.012 \) for s- and p-wave, respectively. Stieglitz et al.\(^{32} \) have studied p-wave resonances of \(^{52} \text{Cr} \).

They have found that average radiation width \( \langle \gamma \rangle \) for the p-wave resonances is nearly three times smaller than the \( \langle \gamma \rangle \) for the s-wave resonances. Assuming that the average level spacing obeys a law or \( (2J+1)^{-1} \), \( D_J=1 \) will be three times smaller than \( D_J=0 \).

Thus \( \langle \gamma \rangle / \langle \sigma \rangle \) for the p-wave resonances will remain same as that
of s-wave resonances for $^{52}$Cr. Musgrove et al.\cite{33} have shown that $<\langle \rho \rangle / <\Omega >$ is nearly same for s- and p-wave resonances in $^{40}$Ca.

Finally, we would like to point out that the assumption $[<\rho>/<\Omega>_p\text{-wave}] \approx [<\rho>/<\Omega>_s\text{-wave}]$ which has been verified experimentally for many nuclei\cite{16,31-33} except three in 3P region\cite{16,34} is fairly well to reproduce the experimental cross-sections and it may be further used to get information about $[<\rho>/<\Omega>_d\text{-wave}$ with respect to $[<\rho>/<\Omega>_s\text{-wave}$ by fitting the experimental cross-sections at higher energies.
REFERENCES: