Chapter 1

INTRODUCTION
Interest in the microscopic study of nucleus-nucleus scattering at medium and high energies may be traced back to the late sixties when Glauber multiple scattering model¹ for hadron-nucleus scattering was generalized to study nucleus-nucleus scattering involving very light nuclei²⁻⁵. These studies were soon followed by the pioneering work of Czyz and Maximon⁶ relating to general nucleus-nucleus scattering using full Glauber multiple scattering series. Czyz and Maximon demonstrated that the number of multiple interactions in the general case of nucleus-nucleus scattering grows so rapidly with the mass number of the colliding nuclei that the evaluation of the full multiple scattering series becomes extremely difficult even for the simplest model for the two colliding nuclei. By neglecting certain class of multiple scattering terms in the full series these authors also derived an approximate expression for the Glauber amplitude involving only the ground state densities of the nuclei in collision. This approximate expression is known in the literature as the optical limit result.

One of the most attractive features of the Glauber multiple scattering model for nucleon-nucleus scattering (which provides the basis for generalization to nucleus-nucleus scattering) is that it reduces the complicated many-body problem to a sequence of two-body problems in a form.
that leads to a mathematically tractable description of N-nucleus scattering in terms of the nucleon-nucleon (NN) scattering amplitude and the target wavefunction. Thus if sufficient information regarding the structure of the target nucleus is available one can, in principle, use N-nucleus scattering experiments to determine the elementary NN amplitude, and conversely knowing the NN amplitude one may extract useful information regarding the structure of the target nucleus.

Much confidence in the applicability of Glauber model to nuclear scattering problems at high energies was gained in early seventies when it was used to interpret the high precision p-nucleus scattering experiments of the Saclay\(^7\) and other groups\(^8-10\). Analyses of the elastic and inelastic scattering data showed that high energy p-nucleus scattering experiments are very useful for extracting information regarding nucleon ground state and transition densities in nuclei\(^2,11-13\).

Encouraged by the usefulness of high energy p-nucleus scattering experiments for nuclear structure studies the Saclay group also performed a series of \(\alpha\)-nucleus scattering experiments\(^14-17\) at 1.37 GeV. The main idea of undertaking these experiments was to use high energy \(\alpha\) particle as a complementary probe for nuclear structure studies. However, it was soon realized that analysis of \(\alpha\) scattering
experiments in terms of the Glauber model, as generalized to nucleus-nucleus scattering, is not as straightforward as for N-nucleus scattering\textsuperscript{18-21}. The evaluation of the full Glauber scattering amplitude in this case is beset with serious difficulties as has been mentioned earlier.

Over the past about 15 years, the Saclay scattering data\textsuperscript{16} have been analyzed by several workers\textsuperscript{6,19,22-26}. All these analyses show that the so-called optical limit result of Czyz and Maximon\textsuperscript{6} fails seriously to account for the data even in the low momentum transfer region. Thus in order to make the working of the Glauber model for nucleus-nucleus scattering transparent one has to consider either the full series or to search for a better approximation scheme than is involved in arriving at the optical limit. Noting that, evaluation of the full Glauber series is, in general, mathematically intractable because of very slow convergence of the usual expansion of the Glauber series and the presence of higher order densities and multidimensional integrals, several workers have proposed alternative expansions in an attempt to have a rapidly convergent series expansions for the Glauber amplitude. Notable attempts in this direction are: phase expansion method by Franco and Varma\textsuperscript{19}, correlation expansion method by Alkhazov\textsuperscript{25} and the effective profile expansion method by Ahmad\textsuperscript{22}. The first two give infinite series while the last one gives a series of finite
number of terms which may be put in one-to-one correspondence with the usual expansion in terms of the NN profile function. All these expansions converge more rapidly than the usual one, yet not so rapidly as to make the evaluation of the nucleus-nucleus S-matrix or the scattering amplitude sufficiently simple.

In the phase expansion approach of Franco and Varma\textsuperscript{19} the total elastic phase of nucleus-nucleus scattering is expanded as an infinite series, the first term of which involves the one-body densities of the colliding nuclei, the second term involves the two-body densities of the colliding nuclei, and so on. Since evaluation of the higher order terms with realistic wavefunction becomes mathematically intractable these authors used the independent particle model with the Gaussian wavefunction to study the convergence of the phase expansion series. In the case of $^4\text{He}$-$^4\text{He}$ scattering at 2.1 GeV/n these authors showed that consideration of at least first four terms of the expansion is to get the same result as with the full Glauber series. Still, the fact, that consideration of the first four terms is necessary implies that not much has been gained on the computational front specially when realistic wavefunctions of the colliding nuclei are to be used. Most of the remarks made above also apply to the correlation expansion approach proposed by Alkhazov\textsuperscript{29}.
In the effective profile expansion method as proposed by Ahmad\textsuperscript{22} instead of the elastic phase one expands the elastic S-matrix. This differs from the usual expansion in that the S-matrix is now expanded in terms of an effective profile function (to be discussed in detail later) rather than the NN profile function. The advantage of this approach is that all such multiple scattering terms of the full Glauber series in which both the colliding nuclei always remain in the ground state are lumped together in the first term. The higher order terms of the expansion involve only those scatterings in which at least one of the nuclei is excited. Using this expansion, Ahmad analysed the $\alpha-^{40}$Ca elastic scattering data at 1.37 GeV using realistic densities with fair success. Although, Ahmad could not go beyond the second order term in the expansion because of the computational difficulties, yet the finiteness of the expansion makes the effective profile approach promising and worthy of further study.

Efforts have also been made to interpret the nucleus-nucleus scattering data using the so called "rigid projectile approximation"\textsuperscript{14,23,27-30}. This model assumes that the incoming projectile is a single rigid entity such that nucleus-nucleus scattering problem may be treated in the same manner as the N-nucleus scattering problem with the N-projectile amplitude as elementry. Studies of $\alpha-^{12}$C and
\( ^{40} \text{Ca} \) scattering data in the framework of this model at 1.37 GeV show that the model gives much better results than the optical limit approximation particularly in the forward angle region. However, at large momentum transfers the predictions of the model deviate from experiments considerably. This led Ahmad and Alvi\(^{31}\) to propose a semiphenomenological method of analysis within the framework of the rigid projectile model using a phenomenological effective N\(\alpha\) amplitude instead of the free N\(\alpha\) amplitude. This method satisfactorily explains the experimental data even at large momentum transfers over a wide energy range with only one free parameter. However, further studies are needed to delineate the Physics behind the success of the semiphenomenology.

In this work we present some studies in nucleus-nucleus elastic scattering at intermediate energies with two main motivations. One concerns the study of the working of the Glauber model for nucleus-nucleus scattering. It has been already pointed out that whereas the Glauber model for N-nucleus scattering has been extensively studied and has been found useful as a theoretical basis for analyzing relevant experiments, its generalization to nucleus-nucleus scattering has not undergone a similar test. This is mainly because of the computational difficulties involved in evaluating the full Glauber series. The only situation when the full series can be evaluated without recourse to any approximation is
that of the single gaussian description of the colliding nuclei. Knowing that the single particle Gaussian model does not work well specially in situations involving high momentum components of nuclear wavefunction, a comparison of the prediction of the full Glauber series evaluated using the Gaussian model with corresponding experiment is unlikely to shed enough light on the working of the model for nucleus-nucleus scattering. This is mainly because discrepancies, if any, may be attributed either to the poor description of the colliding nuclei or to the failure of the model. Therefore, keeping in view the increasing interest in the application of the Glauber model for nucleus-nucleus scattering to a variety of situations, it is highly desirable to undertake the full Glauber model calculation with realistic densities (or wavefunctions) of the colliding nuclei. In the present work we have undertaken such calculations for the $^4\text{He}-^4\text{He}$ elastic scattering although the technique used is not practable when any one of the nuclei involved in the collision has large mass number.

The other motivation of the present work is to examine at length certain approximation schemes for the Glauber scattering amplitude. The main consideration for undertaking the study is that unless there is some simple and reliable method of analysis (such as we have for N-nucleus scattering), the nucleus-nucleus scattering experiments may not be used to
extract nuclear structure or any other information. We start by giving two expansions of the Glauber S-matrix. One is such that the first term of the expansion gives the rigid projectile model (RPM) and the remaining terms provide corrections of various order to it. The other is the expansion of the S-matrix in terms of the Bell's polynomial. The two expansions have been used to study respectively, corrections to the RPM and the convergence of the effective profile expansion of the Glauber S-matrix.

The contents of the thesis may be summarized as follows. In chapters 2 and 3 we briefly review respectively the development of the Glauber model for nucleus-nucleus scattering at medium and high energies and the various notable approaches or approximation schemes that have been used so far to evaluate the Glauber amplitude.

In chapter 4 we apply Monte Carlo method of evaluating the multidimensional integrals to calculate the full Glauber amplitude for $^4$He-$^4$He elastic scattering at intermediate energies without recourse to any approximation using Gaussian and the double Gaussian densities for $^4$He. The latter density is taken to be consistent with electron scattering experiments over the momentum transfer range covered so far. The results of this chapter highlight the importance of using realistic densities in nucleus-nucleus scattering calculations.
The computational technique developed in chapter 4 is further employed to calculate \(^4\text{He}-^4\text{He}\) elastic scattering with the best available variational wavefunction with very encouraging results. These calculations are presented and discussed in chapter 5.

In chapter 6 we present an expansion of the Glauber S-matrix such that the first term gives the RPM and successive terms provide corrections to it. We calculate the correction to the RPM as provided by the second term of the expansion and find that its contribution, though in right direction, is small.

The chapter 7 presents an expansion of the Glauber S-matrix in terms of Bell's polynomial. We use this expansion technique to study the convergence of the effective profile expansion series of the Glauber S-matrix for \(^4\text{He}-^4\text{He}\) elastic scattering.

Finally, a summary of the present study is presented in chapter 8.
References


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