CHAPTER III

NUCLEON-NUCLEUS OPTICAL POTENTIAL
(3.1) Introduction

Over the past few years, the experimental data of proton scattering from nuclei have become abundant, covering various target nuclei and a wide energy range. Most of these experiments are of high accuracy and include measurements of not only the differential cross sections but also analyzing powers and spin rotation functions. The increase of such data of high accuracy may be helpful in eliminating the ambiguities and reveals new aspect of the potential model. For instance, experimental data at high momentum transfer may provide information on the internal shape of the optical model potential. Some analyses of these data have indicated the failure of the traditional Woods-Saxon potential. Microscopic optical potentials have become a focus of attention due to their ability to connect the fundamental NN information with the many-body situation. In the previous chapter we have observed that the strength and the energy variation of both the real and imaginary parts of the nucleon-nucleus optical potential can be calculated in infinite nuclear matter as a function of matter density starting from a realistic internucleon interaction within the framework of first order Brueckner theory. However, the construction of a reliable optical potential for the scattering of nucleons from the finite nuclei is somewhat difficult since not only the strength but also the geometry must be correctly obtained in
order to reproduce the experimental data. This task of constructing optical potential for finite nuclei has been accomplished by a number of authors\textsuperscript{1-12) using various forms of local density approximation\textsuperscript{1-8). However, we shall follow the approach suggested by Brieva and Rook\textsuperscript{3-5). It should be noted that we do not use the generalized reference spectrum\textsuperscript{3-5) method but solve the integral equation using matrix inversion technique described in the previous chapter. The basic idea behind this approach is that an effective internucleon interaction, the t-matrix, is obtained by solving the Bethe-Goldstone equation. This effective interaction is then folded with the nucleon density inside the nucleus to yield a nucleon-nucleus optical potential. The optical potential obtained in this way is used to predict the differential cross sections, analyzing powers and spin rotation functions for the elastic scattering of nucleons.

In section 3.2 we describe the method of calculation of different components (direct central, exchange central, direct spin-orbit and exchange spin-orbit parts) of the nucleon-nucleus optical potential. In section 3.3 we present a new formulation of obtaining the effective mass correction to the optical potential, which slightly differs from others. We would see that our new formulation not only modifies the central imaginary part but also gives a feedback term in the real central part and modifies the real and imaginary parts of
spin-orbit optical potential. Section 3.4 discusses the results of our calculated optical potential for $p^{-40}\text{Ca}$. We have performed calculations using both the Urbana V14 soft core\textsuperscript{13} and Hamada-Johnston hard core\textsuperscript{14} realistic interactions. Other basic input is the nucleon density in the target nucleus. For $^{40}\text{Ca}$ nucleus we use LRAY density\textsuperscript{15}. In section 3.5 we make a detailed analysis of $p^{-40}\text{Ca}$ elastic scattering data at low and intermediate energies using these calculated optical potentials.
Method of calculation:

The nucleon-nucleus optical potential, \( M \), is written as the sum of a local direct term and a non-local exchange term \(^{3-5}\), namely

\[
M(r_4, r_4', E) = \delta(r_4 - r_4') \sum_n \int \phi^*_n(r_2) t^D(r_4, r_2; E) \phi_n(r_2) \, dr_2 + \sum_n \phi^*_n(r_4') t^{EX}(r_4, r_4'; E) \phi_n(r_4'), \tag{3.1}
\]

where \( r_4 \) and \( r_2 \) refer to the radial coordinates of the incident and the bound nucleons respectively, \( \phi_n(r_2) \) is the bound-state single-particle wavefunction with \( n \) representing the appropriate quantum numbers and \( t^D \) and \( t^{EX} \) are the direct and the exchange effective nucleon-nucleon interactions. Both \( t^D \) and \( t^{EX} \) have essentially the following structure:

\[
t(r_4, r_2; E) = t^D_0(r_4, r_2; E) + t^{EX}(r_4, r_2; E) L S + \text{other terms}, \tag{3.2}
\]

that is, a central plus a spin-orbit components (and also tensor components etc.) of the NN effective force. From eq. (3.1), it is convenient to define a local equivalent optical potential, \( U \), by

\[
U(r_4, E) \psi(r_4) = \int M(r_4, r_4'; E) \psi(r_4') \, dr_4'. \tag{3.3}
\]
where $\psi(r_{\text{inc}})$ is the scattering wavefunction of the incident nucleon. The nucleon-nucleus optical potential can now be written in the standard form (neglecting the tensor part assuming the target to be spherical):

$$U(r_{\text{inc}}, E) = U_{c}(r_{\text{inc}}, E) + U_{so}(r_{\text{inc}}, E), \quad (3.4)$$

where

$$U_{c}(r_{\text{inc}}, E) = -V(r_{\text{inc}}, E) - iW(r_{\text{inc}}, E) \quad (3.5)$$

and

$$U_{so}(r_{\text{inc}}, E) = -\left[V_{so}(r_{\text{inc}}, E) + iW_{so}(r_{\text{inc}}, E)\right] l_{\sigma} \quad (3.6)$$

refer to the central and the spin-orbit components of the nucleon-nucleus optical potential. In eq. (3.6), $l_{\sigma}$ and $s_{\sigma}$ ($=\frac{1}{2} \sigma$) are the incident nucleon orbital angular momentum and spin respectively.

The evaluation of the effective NN interaction, $t^D$ and $t^{EN}$, in finite nuclei is quite difficult. However, a hypothesis is made that these effective interactions in finite nuclei can be approximated by the local, and the density and energy dependant effective interactions calculated in infinite
nuclear matter, that is

\[ t_{gso}^{D,EX}(r_a, r_s; E) \approx t_{gso}^{D,EX}(|r_a - r_s|; \rho(R), E), \]  

(3.7)

where \( \rho(R) \) is the nuclear matter density at

\[ R = (r_a + r_s)/2. \]  

(3.8)

Using eqs. (3.1), (3.3) and (3.7) we can obtain the local equivalent nucleon-nucleus optical potential, \( U(r_a, E) \).

(3.2.1) Direct part of the central optical potential:

In this subsection, we discuss the calculation of direct component of the central optical potential. The expression for the direct central optical potential is given by

\[ U^D(r_a, E) = \sum_n \int \phi^*(r_s) t^D_{g}(|r_a - r_s|; \rho(R), E) \phi_n(r_s) \, dr_s. \]  

(3.9)

Eq. (3.9) can be written, in terms of the single-particle density distribution in the target nucleus, as

\[ U^D(r_a, E) = \int \rho(r_s) t^D_{g}(|r_a - r_s|; \rho(R), E) \, dr_s, \]  

(3.10)

where
\[ \rho(z) = \sum_n \phi_n^*(z) \phi_n(z) \]  

is the single-particle density distribution. We now incorporate in eq. (3.10) the difference between neutron and proton matter densities and the difference between pp and pn effective interactions. The expression for the direct component of the central optical potential for incident proton can thus be written as

\[ U_D^{pp}(r_z, E) = \int \rho_p(r_z) t_D^{pp}(|r_z - r_z|; \rho(R), E) \, dr_z \]

\[ + \int \rho_n(r_z) t_D^{pn}(|r_z - r_z|; \rho(R), E) \, dr_z, \]  

(3.12)

The quantities \( t_D^{pp} \) and \( t_D^{pn} \) in eq. (3.12) are the direct part of the central pp and pn effective interactions respectively and are defined by

\[ t_D^{pp} = \frac{1}{4} (t^{00} + 3t^{44}) \]  

(3.13)

and

\[ t_D^{pn} = \frac{1}{8} (3t^{40} + t^{04} + t^{00} + 3t^{44}) \]  

(3.14)

Here the complex quantities \( t^{\alpha \beta} \) occurring in eqs. (3.13) and
(3.14) are the central effective interactions in the spin-isospin \((S,T)\) states of the two-nucleon system. Eq. (3.12) coupled with eqs. (3.13) and (3.14) is used to calculate the direct part of the central component of the optical potential.

(3.2.2) Exchange part of the central optical potential:

The exchange part of the central optical potential can be written as

\[
U^{\text{EX}}_c(r, E)\psi(r) = \sum \int \phi_n^*(r_z) t^{\text{EX}}_c(|r_z - r|; \rho(R), E) \phi_n(r) \psi(r_z) \phi_z.
\]

(3.15)

We now use the local momentum approximation\(^{3-5}\) and factorize out the incident nucleon wavefunction, \(\psi(r)\). We discuss these approximations in detail in chapter IV, where we show that the exchange part can be written as the sum of a series whose first term is used in the following. The expression generally used\(^{3-5}\) under these approximations is given by the following equation:

\[
U^{\text{EX}}_c(r, E) = \int \rho(r_z, R) t^{\text{EX}}_c(|r_z - r|; \rho(R), E) j_0(k|r_z - r|) \phi_z.
\]

(3.16)

where

\[
\rho(x, y) = \sum \phi_n^*(x) \phi_n(y),
\]

(3.17)
is the single-particle mixed density, $j_o(x)$ is a spherical Bessel function of order zero and $k$ in the argument of Bessel function occurring in eq. (3.16) is the local momentum defined by

$$E = \frac{\hbar^2 k^2}{2m} + V(r, E), \quad (3.18)$$

with $V(r, E)$ being total (sum of direct and exchange) real central potential.

In order to include the difference between proton and neutron matter densities and the difference between pp and pn central effective interactions, we write the above eq. (3.16) in the following form

$$U^{**}(r, E) = j_{pp}(r, E) C^{**}(r, E) j_{oo}(k |r - r|) d\tau.$$  

$$U^{**}(r, E) = \int \rho_p(r, E) t_c^{pp} (|r - r|; \rho(R), E) j_{oo}(k |r - r|) d\tau + \int \rho_n(r, E) t_c^{pn} (|r - r|; \rho(R), E) j_{oo}(k |r - r|) d\tau. \quad (3.19)$$

Here the quantities $t_c^{pp}$ and $t_c^{pn}$ in eq. (3.19) are the exchange part of the central pp and pn effective interactions respectively and are defined by

$$t_c^{pp} = \frac{1}{4} (t^{oo} - 3t^{**}). \quad (3.20)$$
and

\[
\begin{align*}
t_{\text{c}}^{\text{EX, pn}} &= \frac{1}{8} (3t^{00} + t^{04} - t^{00} - 3t^{44}). \tag{3.21}
\end{align*}
\]

The proton and neutron single-particle mixed densities in eq. (3.19), in the first approximation, is given by the first term of an expansion proposed by Negele and Vautherin \(^{16}\), i.e.

\[
\rho_{\text{c,n,n}}(r_1, r_2) \approx \rho_{\text{c,n,n}} \left[ \frac{r_1 + r_2}{2} \right] \frac{3}{(sk_p)^3} \left[ \sin(sk_p) - sk_p \cos(sk_p) \right], \tag{3.22}
\]

with \(s = |r_1 - r_2|\), and \(k_p\) being the magnitude of fermi momentum.

(3.2.3) Direct part of the spin-orbit optical potential:

In this subsection we present the commonly used prescription \(^4-12\) for obtaining the direct part of the spin-orbit potential under the short range approximation. However, as shown in chapter V one can now easily avoid making this approximation and calculate the spin-orbit potential exactly. The direct part of the spin-orbit optical potential in the approximation of ref. \(^4\) is given by
The product \( l.m \) in eq. (3.23) can be written as

\[
l.m = r \times p.m = \frac{1}{2} (r_4 - r_s) \times (p_4 - p_s), \quad (s_4 + s_s) \tag{3.24}
\]

where \( p_4 (p_s) \) and \( s_4 (s_s) \) refer to the momentum and spin vectors of the incident (bound) nucleons. Changing the integration variable in eq. (3.23) to \( x = r_s - r_4 \) we obtain

\[
U^b_{SO}(r_4, E) = -\frac{1}{2} \int \rho(|r_4 + x|) \ t^b_{SO}(x; \rho, E) \times
\]

\[
x \times (p_4 - p_s), \quad (s_4 + s_s) \ dx, \tag{3.25}
\]

where

\[
\rho(|r_4 + x|) = \sum_n \phi_n^{*}(r_4 + x) \phi_n(r_4 + x). \tag{3.26}
\]

The integration over \( p_s \) vanishes, since no direction is preferred, while the sum over the bound nucleon spin \( s_s \) is zero for a spin zero nucleus. This gives us

\[
U^b_{SO}(r_4, E) = -\frac{1}{2} \int \rho(|r_4 + x|) \ t^b_{SO}(x; \rho, E) \times dx \times p_4 \cdot s_4. \tag{3.27}
\]
The expression given by eq. (3.27) is exact. It can be approximated in the coordinate space provided the direct part of spin-orbit force is of sufficiently short range. Nuclear density can be expanded around \( x = 0 \)

\[
\rho(r_x + x) = \rho(r_x) + x \cdot [\nabla_x \rho(r_x + x)]_{x=0} + \ldots \tag{3.28}
\]

Replacing eq. (3.28) in eq. (3.27) we get to the first order in the derivative of the density

\[
\mathcal{U}^D_{so}(r_x, E) = -\frac{2\pi}{3} B^D(\rho, E) \frac{1}{r_x} \frac{\partial}{\partial r_x} \rho(r_x) \, L_x, \tag{3.29}
\]

with \( B^D(\rho, E) \) is given by

\[
B^D(\rho, E) = \int t^D_{so}(x; \rho, E) \, x^2 \, dx. \tag{3.30}
\]

If we include the difference between the proton and neutron densities, we obtain the following expression for the direct part of the spin-orbit optical potential for incident proton as

\[
\mathcal{U}^{D,P}_{so}(r_x, E) = -\frac{2\pi}{3} \left[ \int t^{D,PP}_{so}(x; \rho, E) x^4 \, dx \frac{1}{r_x} \frac{\partial}{\partial r_x} \rho_p(r_x) + \right.
\]

\[
\left. \int t^{D,PN}_{so}(x; \rho, E) x^4 \, dx \frac{1}{r_x} \frac{\partial}{\partial r_x} \rho_n(r_x) \right].
\]
Here $t_{so}^{D,pp}$ and $t_{so}^{D,pn}$ are the direct part of the spin-orbit pp and pn effective interactions respectively and are defined by

$$t_{so}^{D,pp} = t_{so}^{T}$$

(3.32)

and

$$t_{so}^{D,pn} = \frac{1}{2} \left( t_{so}^{T} + t_{so}^{P} \right).$$

(3.33)

The complex quantities $t_{so}^{ST}$ in eq. (3.32) and (3.33) are the spin-orbit NN effective interactions in the spin-isospin $(S,T)$ states of the two-nucleon system.

(3.2.4) Exchange part of the spin-orbit optical potential:

In this subsection we present the commonly used prescription \(^{4-12}\) for obtaining the exchange part of the spin-orbit potential under the short range approximation. However, as shown in chapter V one can now easily avoid making this approximation and calculate the spin-orbit potential exactly. The exchange part of the spin-orbit optical potential in the approximation of ref. \(^{4}\) is given by
Using eq. (3.24) for the product l.s., changing variable to \( x = r_s - r_4 \) and Fourier transforming the incident nucleon scattering wavefunction \( \psi(r_4) \) in eq. (3.34), we obtain

\[
\begin{align*}
U^{\text{EX}}(r_4, E)\psi(r_4) &= \frac{1}{2(2\pi)^{3/2}} \int t^{\text{EX}}(x; \rho, E)\, d^3x \\
&\quad \times \left[ i(\mathbf{\nabla} \rho(r_4 + x, r_4, y) + \rho(r_4 + x, r_4, y)) \right] \psi(r) \\
&\quad \times \exp(ik \cdot x) \exp(ik \cdot r_4) \psi(k) \, dk \, dx,
\end{align*}
\]

where

\[
\rho(x, y) = \sum_n \phi_n^*(x)\phi_n(y)
\]

is the single particle mixed density and

\[
\psi(k) = (2\pi)^{-3/2} \int \exp(-ir \cdot k)\psi(r) \, dr.
\]

The sum over the bound nucleon spin, \( s_z \), does not appear in eq. (3.35) as it averages out to zero for a spin-saturated nuclei. We now expand the single-particle mixed density around \( x = 0 \) and obtain an approximate expression to the first order.
in the gradient of the matter density,

\[
U^\text{EX}_{\text{so}}(r_\perp, E) \psi(r_\perp) = \frac{1}{2} \left(2\pi\right)^{-3/2} \int t^\text{EX}_{\text{so}}(x; \rho, E) \times \times
\]

\[
\left[ \rho(r_\perp k + i(\nabla^\rho)(r_\perp, r_\parallel, y))_{y=0} + x . (\nabla^\rho)(r_\perp y, r_\parallel)_{y=0} k \right] s_\perp \times
\]

\[
\exp(ik \cdot x) \exp(ik \cdot r_\perp) \psi(k) dk dx.
\]  

Integrating over \(x\) in eq. (3.36) we get

\[
U^\text{EX}_{\text{so}}(r_\perp, E) \psi(r_\perp) = \left(2\pi\right)^{-3/2} \frac{2\pi}{3} \int R^\text{EX}(k; \rho, E) \times
\]

\[
\nabla_\perp \rho(r_\perp) \times \rho_\perp \times \exp(ik \cdot r_\perp) \psi(k) dk,
\]  

where \(R^\text{EX}(k; \rho, E)\) is given by

\[
R^\text{EX}(k; \rho, E) = \frac{3}{k} \int t^\text{EX}_{\text{so}}(x; \rho, E) j_4(kx) x^\parallel dx.
\]  

For spherical density distribution, we obtain the following expression for exchange part of the spin-orbit potential

\[
U^\text{EX}_{\text{so}}(r_\perp, E) = \frac{2}{3} \left[ \frac{3}{k} \int t^\text{EX}_{\text{so}}(x; \rho, E) j_4(kx) x^\parallel dx \right] \times
\]
When the difference between the proton and neutron densities and the difference between exchange part of the pp and pn spin-orbit effective interactions are included, the expression for the exchange spin-orbit optical potential for the incident proton can be written as

$$U_{\text{SO}}^{\text{EX,P}}(r, E) = \frac{2\pi}{3} \left[ \int t_{\text{SO}}^{\text{EX,pp}}(x; \rho, E) j_4(k_x) x^3 dx \left( \frac{1}{r} \frac{\partial}{\partial r} \rho_p(r) \right) \right] \frac{\partial}{\partial r} \rho_n(r_4) \right] l_4 \cdot \hat{z} \cdot (3.40)$$

Here, $t_{\text{SO}}^{\text{EX,pp}}$ and $t_{\text{SO}}^{\text{EX,pn}}$ are the exchange part of the spin-orbit pp and pn effective interactions respectively and they are defined by

$$t_{\text{SO}}^{\text{EX,pp}} = -t_{\text{SO}}^{44} \quad (3.41)$$

and

$$t_{\text{SO}}^{\text{EX,pn}} = \frac{1}{2} (t_{\text{SO}}^{44} - t_{\text{SO}}^{40}) \quad (3.42)$$
(3.2.5) Procedure for obtaining radial dependence of t-matrices:

In order to calculate the radial dependence of the nucleon-nucleon effective interaction or t-matrix we follow the approach proposed by Siemens\(^\text{17}\). But instead of imposing the requirement that the approximate t should reproduce the binding energy of nuclear matter when it is used in lowest Born approximation we define t so as to reproduce the average single particle complex potential. This condition allows us to obtain not only a density dependent t but also its energy dependence.

We assume that the energy E and momentum k of a nucleon moving in an infinite system of nuclear matter density \(\rho_{\text{nm}}\) is related by

\[
E = \frac{k^2}{2m} + \text{Re} \left[ U(k_F; k, E) \right], \tag{3.43}
\]

where m is the nucleon mass and \(U(k_F; k, E)\) is the average complex potential felt by the incident nucleon. This nucleon collides with a bound nucleon with momentum p with \(|p| \leq k_F\). We introduce the total and relative momentum for the nucleon pair,

\[
K_0 = k + p, \tag{3.44a}
\]
\( k_0 = \frac{(k - p)/2}{2} \), \hspace{1cm} (3.44b)

and recall \( r \) their relative coordinate. We denote radial part of the correlated wavefunction of the two nucleons by \( \psi_{LL',\alpha}^{J,M}(r) \), where \( L, S \) and \( J \) refer to the orbital angular momentum, total spin and total angular momentum respectively of the nucleon pair. Angular momentum \( L' \) allows for the tensor coupling in the internucleon interaction and \( \alpha \) represents the dependence of the wavefunction on \( E, k, p \) and \( \rho_{NM} \).

A diagonal representation of \( t \) in coordinate space is easily obtained from

\[
\langle \phi | t | \phi \rangle = \langle \phi | V | \psi \rangle ,
\]

(3.45)

where \( |\phi\rangle \) is a plane wave state characterized by the relative momentum of the pair, \( V \) is the realistic internucleon potential and \( |\psi\rangle \) is the correlated two-nucleon wavefunction. We obtain, in states of \( L, S, J \) quantum numbers,

\[
t_{LL'}^{J,M}(r; \rho_{NM}, E) = \frac{\sum_{p<k_F} \sum_{L''=-|J-S|}^{J+S} \frac{1}{k_0} l_{L}(r) \psi_{LL'',\alpha}^{J',M}(r) \psi_{LL',\alpha}^{J',S}(r)}{\sum_{p<k_F} \frac{1}{k_0^2} l_{L}(r)} ,
\]

(3.46)
where \( I_L(r) = k_0 r j_L(k_0 r) \), with \( j_L(x) \) the spherical Bessel function of order \( L \), and \( V_{LL'}^j(r) \) are the reduced matrix elements of the realistic internucleon force. For singlet states, the sum over \( L' \) in eq. (3.46) does not apply. In triplet states it is convenient to have a \( J \)-independent interaction, namely

\[
\Sigma_{J=|L-L'|}^{L+1} \frac{[2J + 1] t_{L}^{J,J'=\delta}(r; \rho_{NM}, E)}{3[2L + 1]}. \tag{3.47}
\]

For practical purposes and computational simplicity an \( L \)-independent effective interaction can be defined in states of spin \( S \) and isospin \( T \),

\[
t^{ST}(r; \rho_{NM}, E) = \frac{\Sigma_{L} [2L + 1] t_{L}^{ST}(r; \rho_{NM}, E) \Sigma_{p<k_F} \frac{1}{k_F} I_L^{z}(r)}{\Sigma_{L} [2L + 1] \Sigma_{p<k_F} \frac{1}{k_F} I_L^{z}(r)} \tag{3.48}
\]

where the sum over \( L \) is over even or odd values so as to have negative total parity. The \( t^{ST} \) effective interactions are complex and dependant on density and energy.
(3.3) Effective mass correction:

In this subsection, we describe the calculation of effective mass correction to the optical potential. This treatment of the effective mass correction slightly differs from that generally used\(^9,18-19\). We show that both the real and imaginary parts of the central and the spin-orbit components of the optical potential get modified.

The optical potential \(U(k) = V(k) + iW(k)\) for a nucleon of energy \(E\) satisfies the following relation

\[
\frac{\hbar^2 k^2}{2m} + V(k) + i W(k) = E. \tag{3.49}
\]

Here, in eq. (3.49), we have suppressed the spin and other variables for the sake of convenience only.

The local momentum \(k_o\), at which the \(t\)-matrix calculations are performed to determine \(U(k)\), is calculated using only the real part in the following equation

\[
\frac{\hbar^2 k_o^2}{2m} + V(k_o) = E. \tag{3.50}
\]

This leads to an error when one uses the optical potential calculated at \(k_o\) in eq. (3.49). If the potential \(U(k_o)\) does not vary strongly with \(k\), the error in using \(U(k_o)\) in eq. (3.49) can be easily calculated and is called the effective
mass correction. The results may be obtained as follows.

If we expand $V(k)$ and $U(k)$ around $k_0$ and retain only first order terms, we get from eqs. (3.49) and (3.50)

$$\frac{\hbar^2 k^2}{2m} + V(k_0) + \frac{i U(k_0)}{1 + \left( \frac{\delta V}{\delta E} + i \frac{\delta U}{\delta E} \right)_{k = k_0}} = E. \quad (3.51)$$

Comparison of eqs. (3.49) and (3.51) readily gives us

$$U_{\text{opt}}(k) = V(k_0) + \frac{i U(k_0)}{1 + \left( \frac{\delta V}{\delta E} + i \frac{\delta U}{\delta E} \right)_{k = k_0}}. \quad (3.52)$$

The above treatment can easily be extended to include spins. Treating the spin-orbit potential $(V_{so} + i U_{so})_{s.l}$ on the same footing as the imaginary central part and neglecting the terms like $\frac{\delta V}{\delta E}$ (which are expected to be small) we get

$$U_{\text{opt}}(k) = V(k_0) + \frac{i U(k_0) + [V_{so}(k_0) + i U_{so}(k_0)]_{s.l}}{1 + \left( \frac{\delta V}{\delta E} + i \frac{\delta U}{\delta E} \right)_{k = k_0}}. \quad (3.53)$$

Simplifying the above expression, we can write down different components (i.e. real central, $U^r_c(k)$, imaginary central, $U^i_c(k)$, real spin-orbit, $U^r_{so}(k)$ and imaginary
spin-orbit, \( U_{so}^{I}(k) \) parts) of the optical potential as

\[
U_{c}^{R}(k) = V(k_{0}) + \frac{\partial U}{\partial E} \left[ \left( 1 + \frac{\partial V}{\partial E} \right)^{2} + \left( \frac{\partial U}{\partial E} \right)^{2} \right],
\]

(3.54)

and

\[
U_{c}^{I}(k) = \frac{U(k_{0}) \left( 1 + \frac{\partial V}{\partial E} \right)}{\left[ \left( 1 + \frac{\partial V}{\partial E} \right)^{2} + \left( \frac{\partial U}{\partial E} \right)^{2} \right]},
\]

(3.55)

\[
U_{so}^{R}(k) = \frac{V_{so} \left( 1 + \frac{\partial V}{\partial E} \right) + U_{so} \frac{\partial U}{\partial E}}{\left[ \left( 1 + \frac{\partial V}{\partial E} \right)^{2} + \left( \frac{\partial U}{\partial E} \right)^{2} \right]},
\]

(3.56)

and

\[
U_{so}^{I}(k) = \frac{U_{so} \left( 1 + \frac{\partial V}{\partial E} \right) - V_{so} \frac{\partial U}{\partial E}}{\left[ 1 + \frac{\partial V}{\partial E} \right] + \left( \frac{\partial U}{\partial E} \right)^{2}}.
\]

(3.57)

From eqs. (3.54)-(3.57) we see that not only the central imaginary part is modified (as has been considered by various authors\(^{6,9}\)) but the effective mass correction should also be included in the real central, real spin-orbit and imaginary spin-orbit parts of the calculated optical potential. The calculations of elastic scattering using eqs. (3.54)-(3.57)
shall henceforth be denoted by $m$. 
(3.4) Results of calculated optical potentials

Using equations 3.12, 3.19, 3.31 and 3.40 described in the previous section 3.2, one can easily calculate the nucleon-nucleus optical potential. The calculation of optical potential mainly involves two steps. In the first step we calculate complex NN effective interactions (t-matrices) in nuclear matter within the framework of lowest order Brueckner theory starting from a realistic interaction. These t-matrices, obtained in a local density approximation, depend on the coordinate describing the internucleon separation distance as well as on the density and incident nucleon energy. In the next step we fold these t-matrices over the ground state target nuclear density using the folding procedure described in section 3.2. We have performed the calculation of nucleon-nucleus optical potential at several energies to describe elastic scattering of protons from $^{40}$Ca over a wide range of energies from 21 MeV to 800 MeV, using both the Urbana V14 soft core$^{13)}$ and Hamada-Johnston hard core$^{14)}$ realistic interactions. For density distribution of $^{40}$Ca we use LRAY density$^{15)}$. In the present section we describe various features of the calculated nucleon-nucleus optical potential. A detailed analysis of $p-^{40}$Ca elastic scattering data making use of these calculated potential will be presented in the next section 3.5.
(3.4.1) Real part of central optical potential:

Figs. 1(a) and 1(b) show respectively the radial behaviour of real part of the calculated central optical potential for $p-^{40}\text{Ca}$ elastic scattering at low energies ($E_p = 21, 30, 40, 48, 65$ and $80$ MeV) and at intermediate energies ($E_p = 135, 160, 181, 200, 300, 362, 400, 497$ and $800$ MeV), using Urbana V14 realistic interaction$^{13}$. Fig. 1(a) shows that at low incident energies ($E_p \leq 80$ MeV) the real part of central optical potential is very similar to the empirical optical model potential. However, with increasing energy the strength of this attractive potential in the nuclear interior decreases and then the shape can no longer be described by simple Saxon-Woods function. The potential in the nuclear interior changes rapidly with energy and it becomes repulsive at higher energies (see fig. 1(b)). In the transition region the real part of central optical potential shows a wine-bottle bottom shape, with reduced attractive strength in the interior region of the nucleus. This characteristic feature - the development of wine-bottle bottom shape of real central potential in the transition region is mainly due to cancellation of direct and exchange parts of the optical potential. At energies above $300$ MeV the real part of the central potential is repulsive in the interior and contains a small pocket of attraction in the surface region of nuclei. This energy dependence of the real central potential
is also observed in the Dirac phenomenology. Similar results have also been obtained in the microscopic potential of Li and Zhuo, and Chen and Mackeller using Bonn realistic potential in the Dirac Brueckner Hartree Fock approach.

Figs. 1(c) and 1(d) show respectively the radial shape of calculated real central optical potential at low and intermediate energies, using Hamada-Johnston (HJ) realistic interaction. The results of our calculation indicate that the use of HJ interaction gives rise to a real central optical potential which is qualitatively similar to the one obtained when V14 interaction is used (figs. 1(a) and 1(b)). However, the two potentials quantitatively differ in the following respects. Firstly, the real central optical potential using V14 interaction in the interior region is more attractive (by about 8 MeV) at low energies as compared with that using HJ interaction. Secondly, at low energies the real central optical potential using V14 interaction decreases smoothly with the radial distance, whereas that using HJ interaction shows a small enhancement at a radial distance around \( r = 4 \) F. Thirdly, the real central optical potential obtained by using V14 interaction remains attractive throughout the whole region of the nucleus up to incident energy \( E_p = 300 \) MeV, whereas that obtained using HJ interaction is attractive only up to incident energy \( E_p = 200 \) MeV. At incident energies \( E_p \geq 300 \) MeV the real...
central optical potential calculated using HJ interaction becomes repulsive in the interior region and contains an attractive pocket in the surface region. Finally, the real central optical potential obtained from V14 interaction shows a milder wine-bottle bottom shape in the transition region as compared with that obtained from HJ interaction.

Fig. 2(a) (curve labelled \( V^a \)) shows the energy variation of our calculated real central optical potential depth using V14 interaction. The curve indicates that the depth of real central potential decreases monotonically with increasing energy. This energy variation of potential depth can be fairly well described by the following quadratic expression

\[
V(E) = V_0 + \alpha E + \beta E^2,
\]

with

\[
V_0 = (60.9 \pm 0.4) \text{ MeV},
\]
\[
\alpha = -(0.242 \pm 0.003)
\]
and

\[
\beta = (0.000177 \pm 0.000005) \text{ MeV}^{-4}.
\]

The energy dependence of the depth of our calculated real central potential is in fair agreement with empirical data (see fig. 1 of ref. 25).

The energy variation of the depth of real central potential resulting from HJ interaction (see curve labelled \( V^a \))
in fig. 2(b) can also be represented by similar quadratic expression but with different parameter values as given below

\[
V_0 = (52.4 \pm 0.4) \text{ MeV},
\]
\[
\alpha = -(0.241 \pm 0.003)
\]
and
\[
\beta = (0.000207 \pm 0.000005) \text{ MeV}^{-4}.
\]

In fig. 2(c) (curve labelled \(J_c^R\)) we have shown the energy dependence of the volume integral per nucleon for the real part of calculated central potential using V14 interaction. This energy dependence can be represented by the following quadratic equation

\[
J_c^R = J_0 + pE + qE^2,
\]
with
\[
J_0 = (461.8 \pm 0.4) \text{ MeV-F}^2,
\]
\[
p = -(1.643 \pm 0.003) \text{ F}^2
\]
and
\[
q = (0.001221 \pm 0.000005) \text{ MeV}^{-4}\text{-F}^2.
\]

The decrease in volume integral with energy is more rapid at lower energies than that at higher energies. This is due to the wine-bottle bottom shape of the real central potential at higher energies which gives increased contribution to volume
integral compensating for the loss due to decrease of potential in the interior region.

The volume integral of real central potential \( J_c^R \) resulting from HJ interaction also shows a similar kind of quadratic energy dependence (see fig. 2(d) curve labelled \( J_c^R \)). However, volume integrals resulting from two realistic interactions differ in following respects. First, for a given incident energy \( J_c^R \) obtained from HJ interaction is less than that obtained from V14 interaction. This is expected, as the HJ interaction compared with V14 interaction gives rise to a more attractive real central potential. Second, the rate at which \( J_c^R \) decreases with energy is slow for HJ interaction. This is due to the fact that HJ interaction compared with V14 interaction gives rise to a comparatively pronounced surface enhancement in the real part of central potential in the transition region.

(3.4.2) Imaginary part of central optical potential:

We now describe features of the imaginary part of our calculated central optical potential. Our calculation shows that the imaginary central potentials also exhibit strong energy and radial dependence. The radial behaviour of imaginary central potential, obtained from V14 interaction, for \( p-^{40}\text{Ca} \) elastic scattering at low energies \( (E_p = 21, 30, 40, 48, 65 \text{ and } 80 \text{ MeV}) \) is shown in fig. 3(a), whereas that at intermediate energies \( (E_p = 135, 160, 181, 200, 300, 362, 400 \text{ MeV}) \).
and 497 MeV) is shown in fig. 3(b). These potentials exhibit the following features:

(i) The imaginary central potential is always attractive and its strength in the interior of nucleus increases with the increasing incident energy.

(ii) The imaginary central potential shows a surface enhancement at low incident energy. As the incident energy increases the position of surface peak slowly shifts towards nuclear interior. This surface enhancement is nowhere close to the empirical results. The surface enhancement is completely washed out at incident energy around $E_p = 80$ MeV.

(iii) At high incident energies the imaginary central potential shows a smooth radial dependence which resembles the shape of Woods-Saxon form.

Figs. 3(c) and 3(d) show the corresponding curves for HJ interaction at low and at intermediate energies. Comparison of fig. 3(c) with 3(a) and 3(d) with 3(b) indicates that most of the features of imaginary central potentials obtained from HJ interaction resemble those of the corresponding potentials obtained from V14 interaction. However, the two potentials differ in the following respects:

(i) At low incident energies the imaginary central potential obtained from V14 interaction in the nuclear interior is more attractive (by about 4 MeV at $E_p = 21$ MeV and by about 2 MeV at $E_p = 80$ MeV) than that obtained from HJ
interaction. This difference in their attractive strengths is substantial even at radial distances, \( r \approx 4 \, F \).

(ii) At incident energy \( E_p = 200 \, \text{MeV} \) both the interactions give rise to almost identical imaginary central potentials.

(iii) At incident energies \( E_p \geq 300 \, \text{MeV} \) the strength of imaginary central potential resulting from HJ interaction is more than that obtained from V14 interaction.

(iv) At low incident energies HJ interaction gives a pronounced surface enhancement, whereas V14 interaction gives a milder surface enhancement in the imaginary central potential.

The energy variation of the depth of imaginary central potential resulting from V14 interaction is shown in fig. 2(a) (curve labelled \( V^{I}_c \)) and that resulting from HJ interaction is shown in fig. 2(b) (curve labelled \( V^{I}_c \)). The energy dependence of depth of potentials can be described by following linear equation

\[
U = U_o + \gamma E,
\]

With

\[
U_o = (10.2 \pm 0.4 \, \text{MeV}), \quad \text{for V14 interaction}
\]

\[
U_o = (7.5 \pm 0.4 \, \text{MeV}), \quad \text{for HJ interaction}
\]

and

\[
\gamma = (0.075 \pm 0.003), \quad \text{for V14 interaction}
\]
However, two curves show following differences also.

(i) The depth of imaginary central potential calculated using V14 interaction is more attractive than that using HJ interaction at low energies.

(ii) The depth of imaginary central potential obtained from V14 interaction is nearly equal to that obtained from HJ interaction at incident energy $E_p = 200$ MeV.

(iii) At incident energies $E_p \geq 300$ MeV the imaginary central potential obtained from HJ interaction is greater than that obtained from V14 interaction in the nuclear interior.

The energy dependence of volume integrals of imaginary central potential using V14 interaction is shown by the curve labelled $J_c^V$ in fig. 2(c). The main feature of this curve is that with increasing incident energy, the volume integral of imaginary central potential first decreases sharply, it attains a minimum around $E_p = 80-135$ MeV and then it increases for further increase in incident energy. This behaviour is due to the following reason: At low incident energies the surface enhancement of the imaginary potential gives rise to large volume integral. As the incident energy increases this enhancement in the calculated potential disappears leading to a decrease of the volume integrals. At incident energies $E_p > 135$ MeV it is the volume absorption which dominates over the
surface absorption. The volume integral of imaginary central potential obtained from HJ interaction (see curve labelled $J^1$ in fig. 2(d)) also shows a similar kind of energy dependence.

(3.4.3) Real part of spin-orbit optical potential:

The real part of our calculated spin-orbit potential using V14 interaction for $p^{-40}\text{Ca}$ at low incident energies ($E_p = 21-80 \text{ MeV}$) is shown in fig. 4(a) and that at intermediate energies ($E_p = 135-497 \text{ MeV}$) is shown in fig. 4(b). Prominent features of our calculated real spin-orbit potential are the following:

(i) Radial shape of the calculated real spin-orbit potential is of Thomas form at low as well as at intermediate energies.

(ii) The strength of real spin-orbit potential decreases very slightly with incident energy.

(iii) The peak value of real spin-orbit potential also shows an energy dependence. It decreases with the increase in incident energy. Our calculation shows that this energy dependence is mainly due to the energy dependence of exchange part of spin-orbit potential. The direct part of spin-orbit potential is marginally affected with variation in incident energy. For instance, the peak value of direct part of real spin-orbit potential increases by a factor of 1.11, while that of exchange part of real spin-orbit potential decreases by a factor of 3.15 when incident energy increases from 21 MeV to
The real part of spin-orbit potential obtained from HJ interaction for p-^{40}Ca elastic scattering at low and intermediate energies (see figs. 4(c) and 4(d)) exhibits behaviour similar to that obtained using V14 interaction. However, the two calculated potentials have minor differences also.

(i) At a given incident energy, the real part of spin-orbit potential obtained from HJ interaction is smaller in magnitude, in both the interior and surface regions of the nucleus, than the one obtained from V14 interaction, e.g., at incident energy \( E_p = 21 \) MeV, the real spin-orbit potential resulting from HJ interaction shows a minimum at \( r = 0.65 \) F and \( V_{so} = 0.309 \) MeV and a maximum at \( r = 3.35 \) F and \( V_{so} = 1.131 \) MeV, whereas that resulting from V14 interaction shows a minimum at \( r = 0.65 \) F and \( V_{so} = 0.349 \) MeV and a maximum at \( r = 3.35 \) F and \( V_{so} = 1.258 \) MeV (see figs. 4(a) and 4(c)).

The energy dependence of the volume integrals of real spin-orbit potentials obtained from V14 and HJ interactions can be described by linear equation. This energy dependence is very weak.

(3.4.4) Imaginary part of spin-orbit optical potential:

The imaginary part of our calculated spin-orbit optical potential using V14 interaction for p-^{40}Ca at low incident energies \( (E_p = 21\text{--}80 \) MeV) is shown in fig. 5(a) and that at
intermediate energies \( E_p = 135-497 \text{ MeV} \) is shown in fig. 5(b). The important features of the imaginary spin-orbit potential are the following:

(i) The radial shape of the calculated imaginary spin-orbit potential is also of the Thomas form.

(ii) The strength of imaginary spin-orbit potential increases with increasing incident energy.

(iii) Radial shape of imaginary spin-orbit potential also shows a minimum in the nuclear interior and a maximum in the surface region, e.g., at incident energy \( E_p = 21 \text{ MeV} \), the radial shape of imaginary spin-orbit potential contains a minimum at \( r = 0.60 \text{ F} \) and \( V_{so}^r = 0.0216 \text{ MeV} \) and a maximum at \( r = 3.75 \text{ F} \) and \( V_{so}^r = 0.118 \text{ MeV} \).

(iv) The peak value of imaginary spin-orbit potential shows a slow energy dependence. It increases very slowly with the increasing incident energy. Energy variation of direct and exchange imaginary spin-orbit potentials is shown in fig. 7(b). Since the magnitude of imaginary spin-orbit potential is very small, the analysis of scattering data (as shown in the next section 3.5) does not determine this part of potential uniquely.

The imaginary part of our calculated spin-orbit potential using HJ interaction for \( p-^{40}\text{Ca} \) elastic scattering at low incident energies \( (E_p = 21-80 \text{ MeV}) \) is shown in fig. 5(c) and that at intermediate energies \( (E_p = 135-497 \text{ MeV}) \) is shown in
The potentials obtained from V14 and HJ interactions are very similar to each other, except that the potential arising from HJ interaction is smaller in magnitude than that obtained using V14 interaction not only in the nuclear interior but also in the surface region.

Volume integral of the imaginary spin-orbit potential from V14 interaction (see curve labelled $J_{so}^i$ in fig. 2(c)) is greater than that obtained from HJ interaction (see curve labelled $J_{so}^i$ in fig. 2(d)). Volume integrals arising from both interactions show linear decrease with increasing incident energy.
(3.5) Analysis of p-^{40}Ca elastic scattering data:

In this section we present a detailed analysis of p-^{40}Ca elastic scattering differential cross sections and polarization data at incident energies in the low (21-80 MeV) and intermediate (135-800 MeV) energy regions using calculated optical potential (as described in section 3.4) obtained from first order Brueckner theory using both the Urbana V14 soft core\textsuperscript{13} and Hamada-Johnston hard core\textsuperscript{14} interactions.

We first obtain the different components of nucleon-nucleus optical potential using folding procedure as described in section 3.2 (i.e. using eqs. 3.12, 3.19, 3.31 and 3.40). The calculated potential consists of real and imaginary parts of central and spin-orbit optical potential. These potentials are rescaled by multiplying with the corresponding normalization parameters and best fit to the experimental data is obtained by minimizing $\chi^2$-values with the help of standard search package MINUIT\textsuperscript{26}. The parameters which multiply the real central, imaginary central, real spin-orbit and real spin-orbit potentials are respectively $\lambda_c^r$, $\lambda_c^i$, $\lambda_{so}^r$ and $\lambda_{so}^i$. These normalization parameters are collectively denoted by $\tilde{\lambda}$. The ideal values of $\tilde{\lambda}$ must be unity indicating that the calculated potentials are in 100% agreement with the ones required to fit the experimental data. Thus their deviation from unity provides the measure of a disagreement between the calculated and empirical potentials.
In tables 1-19 we show the normalization parameters $\chi$ and the $\chi^2$-values corresponding to the simultaneous best fit to experimental differential cross section and polarization data at various energies in the low and intermediate energy regions. Symbols $\chi^2_{\text{om}}$, $\chi^2_{\text{p}}$ and $\chi^2_{\text{sr}}$ in these tables correspond to $\chi^2$-values of the best fit to differential cross section, polarization and spin-rotation function data respectively, whereas $\chi^2_{T}$ and $\chi^2_{\text{pdf}}$ represent $\chi^2$-values corresponding to the total and per degree of freedom respectively. The normalization parameters $\chi$ and $\chi^2$ values labelled V14/HJ in these tables indicate that fitting is obtained for the calculated optical potential using Urbana V14 soft core/Hamada-Johnston hard core interaction. Similarly labels m and no m respectively represents the calculated optical potential with and without effective mass correction as described in section 3.3.

Normalization parameters at low incident energies ($E_p = 21-80$ MeV):

We now describe the general features of the normalization parameters (effective mass correction included) obtained for $p-^{40}\text{Ca}$ elastic scattering at low energies $E_p = 21-80$ MeV (see tables 1-11). Calculated differential cross sections and polarization compared with experimental data at low incident energies are shown in figs. 8-18.

(i) Values of normalization parameter $\chi^2_{\text{om}}$ at low incident
energies ($E_p \leq 80$ MeV) usually lie in between 0.9 and 1. This indicates that real part of our calculated central optical potential is only about 10% larger as compared with the empirical one. Further, the optical potential resulting from HJ interaction at low energies yields slightly greater values of $\lambda^r_c$ as compared with that resulting from V14 interaction. This is reasonable because the calculated optical potential using V14 is more attractive than that using HJ interaction.

(ii) The values of normalization parameter $\lambda^i_c$ lie in between 0.5 and 0.7 at low incident energies, except at energy $E_p = 65$ MeV where $\lambda^i_c$ is 0.88 for V14 interaction and 0.85 for HJ interaction. This indicates that at low incident energies the calculated imaginary central optical potentials are about 30-50% larger in strength than the empirically required ones. Further, the values of $\lambda^i_c$ for optical potential obtained from HJ interaction are slightly greater as compared with that obtained from V14 interaction.

(iii) The values of normalization parameter $\lambda^m_{so}$ show a large irregular variation ranging from 0.7 to 1.8 at low incident energies. For potentials using V14 interaction the values of $\lambda^m_{so}$ lie in between 0.84 and 0.96 at incident energies $E_p = 21.0, 40.0, 45.5, 48.0, 61.4$ and 80.2 MeV, whereas the values of $\lambda^m_{so}$ at energies $E_p = 26.3, 28.0, 30.3, 34.8$ and 65.0 MeV lie in between 1.25 and 1.79. Similarly for potentials using HJ interaction the values of $\lambda^m_{so}$ lie in
between 1.16 and 1.73 at low incident energies, except at $E_p = 40.0, 48.0, 61.4$ and $80.2$ MeV where the values of $\lambda_{so}$ are $0.70$ to $0.99$.

(iv) In order to investigate the sensitivity of fit to $\lambda_{so}$ we have performed a $x^2$-fit to the data at $30.3$ MeV by keeping $\lambda_{so} = 0$. A comparison of $x^2$ in tables 4(a) and 4(b) shows that there is hardly any effect of the imaginary part of the spin-orbit.

Normalization parameters at intermediate energies ($E_p = 135-800$ MeV):

We now describe the general features of the normalization parameters (effective mass correction included) obtained for $p-^{40}$Ca elastic scattering at intermediate energies $E_p = 135-800$ MeV (see tables 12-19). Calculated differential cross sections and polarization compared with experimental data at intermediate energies are shown in figs. 19-26. We see that the values of all the four normalization parameters for the potentials obtained from both the interactions are closer to unity in the energy region $E_p = 135-200$ MeV (except at $152$ MeV for which $\lambda_{so}$ is 1.63 for V14 interaction and 1.69 for HJ interaction and $\lambda_{so}$ is 0.176 for V14 interaction and 0.76 for HJ interaction). Further, our calculated optical potential using V14 interaction as compared with HJ interaction yields better values of normalization parameters at intermediate energies. At incident energies $E_p \geq 300$ MeV the normalization
parameters $\lambda_{0}$ is much smaller than unity, indicating that the calculated potential is much larger than that empirically required.

Thus we see that the calculated potentials are in reasonable agreement with the ones required to fit the experimental data for $E_p < 300$ MeV. However, there are several discrepancies. Firstly, the calculated imaginary central potentials at low energies are larger (by about 30-50 \%) than the ones required by the experimental data. Secondly, the surface enhancement in the calculated central imaginary potential at low incident energies is insignificant as compared with that observed in the empirical potentials. Finally significant discrepancies are observed in the calculated and empirical spin-orbit potentials. These problems are present in all microscopically calculated imaginary potentials at low energies. Thus we are unable to solve this problem here. However, in chapters IV and V we have made an attempt to improve the situation.
**TABLE-1**

λ and $\chi^2$ values for p-$^{40}$Ca elastic scattering at 21.0 MeV

<table>
<thead>
<tr>
<th></th>
<th>V14</th>
<th></th>
<th>HJ</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no m²</td>
<td>m</td>
<td>no m²</td>
<td>m</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>0.844</td>
<td>0.788</td>
<td>0.927</td>
<td>0.901</td>
</tr>
<tr>
<td>$\lambda^1_c$</td>
<td>0.444</td>
<td>0.535</td>
<td>0.476</td>
<td>0.662</td>
</tr>
<tr>
<td>$\lambda_{e0}$</td>
<td>0.733</td>
<td>0.837</td>
<td>0.893</td>
<td>1.336</td>
</tr>
<tr>
<td>$\lambda^3_{e0}$</td>
<td>-1.446</td>
<td>1.795</td>
<td>-3.099</td>
<td>-8.026</td>
</tr>
<tr>
<td>$\chi^2_T$</td>
<td>40.99</td>
<td>133.15</td>
<td>83.90</td>
<td>156.94</td>
</tr>
<tr>
<td>$\chi^2_{e0}$</td>
<td>50.30</td>
<td>165.38</td>
<td>103.90</td>
<td>182.55</td>
</tr>
<tr>
<td>$\chi^2_p$</td>
<td>13.52</td>
<td>38.06</td>
<td>24.87</td>
<td>81.39</td>
</tr>
<tr>
<td>$\chi^2_{PDF}$</td>
<td>43.174</td>
<td>140.250</td>
<td>88.371</td>
<td>165.311</td>
</tr>
</tbody>
</table>
### TABLE-2

λ and $\chi^2$ values for $p-^{40}\text{Ca}$ elastic scattering at 26.3 MeV

<table>
<thead>
<tr>
<th></th>
<th>V14</th>
<th></th>
<th>HJ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no m</td>
<td>m</td>
<td>no m</td>
</tr>
<tr>
<td>$\lambda_{c}^m$</td>
<td>0.870</td>
<td>0.858</td>
<td>0.960</td>
</tr>
<tr>
<td>$\lambda_{c}^1$</td>
<td>0.438</td>
<td>0.498</td>
<td>0.465</td>
</tr>
<tr>
<td>$\lambda_{s0}^m$</td>
<td>1.092</td>
<td>1.429</td>
<td>1.200</td>
</tr>
<tr>
<td>$\lambda_{s0}^1$</td>
<td>-0.185</td>
<td>-2.123</td>
<td>-3.128</td>
</tr>
<tr>
<td>$\chi_T^m$</td>
<td>53.26</td>
<td>113.18</td>
<td>131.03</td>
</tr>
<tr>
<td>$\chi_{cs}^m$</td>
<td>49.32</td>
<td>143.08</td>
<td>149.62</td>
</tr>
<tr>
<td>$\chi_p^m$</td>
<td>62.02</td>
<td>46.73</td>
<td>89.72</td>
</tr>
<tr>
<td>$\chi_{PDF}^m$</td>
<td>55.826</td>
<td>118.633</td>
<td>137.349</td>
</tr>
<tr>
<td></td>
<td>V14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>-----</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td></td>
<td></td>
<td>no m*</td>
<td>m*</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>0.868</td>
<td>0.873</td>
<td>0.966</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.418</td>
<td>0.505</td>
<td>0.477</td>
</tr>
<tr>
<td>$\chi^2_{\chi_0}$</td>
<td>0.968</td>
<td>1.246</td>
<td>1.103</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>-1.034</td>
<td>-0.784</td>
<td>-1.598</td>
</tr>
<tr>
<td>$\chi^2_T$</td>
<td>67.38</td>
<td>85.26</td>
<td>107.42</td>
</tr>
<tr>
<td>$\chi^2_{\chi_0}$</td>
<td>32.02</td>
<td>58.12</td>
<td>75.99</td>
</tr>
<tr>
<td>$\chi^2_p$</td>
<td>146.54</td>
<td>146.01</td>
<td>177.76</td>
</tr>
<tr>
<td>$\chi^2_{PDF}$</td>
<td>71.595</td>
<td>90.592</td>
<td>144.136</td>
</tr>
</tbody>
</table>
### TABLE-4(a)

<table>
<thead>
<tr>
<th></th>
<th>V14</th>
<th>HJ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no $m^*$</td>
<td>$m^*$</td>
</tr>
<tr>
<td>$\lambda_C^m$</td>
<td>0.860</td>
<td>0.853</td>
</tr>
<tr>
<td>$\lambda_C^x$</td>
<td>0.459</td>
<td>0.530</td>
</tr>
<tr>
<td>$\lambda_{SO}^m$</td>
<td>1.015</td>
<td>1.316</td>
</tr>
<tr>
<td>$\lambda_{SO}^x$</td>
<td>-0.815</td>
<td>-0.717</td>
</tr>
<tr>
<td>$\chi_T^x$</td>
<td>171.47</td>
<td>262.67</td>
</tr>
<tr>
<td>$\chi_{OS}^x$</td>
<td>61.94</td>
<td>99.13</td>
</tr>
<tr>
<td>$\chi_P^x$</td>
<td>432.41</td>
<td>652.27</td>
</tr>
<tr>
<td>$\chi_{PDF}^x$</td>
<td>177.653</td>
<td>272.133</td>
</tr>
</tbody>
</table>

$\lambda$ and $\chi^2$ values for $p-^{40}$Ca elastic scattering at 30.3 MeV
### TABLE 4(b)

λ and χ² values for p-\(^{40}\)Ca elastic scattering at 30.3 MeV

<table>
<thead>
<tr>
<th></th>
<th>V14</th>
<th></th>
<th>HJ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no m</td>
<td>m</td>
<td>no m</td>
</tr>
<tr>
<td>(\lambda^2_\alpha)</td>
<td>0.872</td>
<td>0.858</td>
<td>0.963</td>
</tr>
<tr>
<td>(\lambda^1_\alpha)</td>
<td>0.478</td>
<td>0.547</td>
<td>0.512</td>
</tr>
<tr>
<td>(\lambda^2_{\pi 0})</td>
<td>0.870</td>
<td>1.105</td>
<td>0.952</td>
</tr>
<tr>
<td>(\lambda^1_{\pi 0})</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>(\chi^2_T)</td>
<td>213.28</td>
<td>295.77</td>
<td>358.80</td>
</tr>
<tr>
<td>(\chi^2_{CS})</td>
<td>53.24</td>
<td>80.98</td>
<td>97.26</td>
</tr>
<tr>
<td>(\chi^2_P)</td>
<td>594.55</td>
<td>807.50</td>
<td>981.88</td>
</tr>
<tr>
<td>(\chi^2_{PDF})</td>
<td>218.992</td>
<td>303.696</td>
<td>368.408</td>
</tr>
<tr>
<td></td>
<td>V14</td>
<td>HJ</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>-----</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>no m*</td>
<td>m*</td>
<td>no m*</td>
</tr>
<tr>
<td>$\lambda^m_{\alpha}$</td>
<td>0.885</td>
<td>0.849</td>
<td>0.947</td>
</tr>
<tr>
<td>$\lambda^x_{\alpha}$</td>
<td>0.509</td>
<td>0.609</td>
<td>0.575</td>
</tr>
<tr>
<td>$\lambda^m_{s0}$</td>
<td>1.115</td>
<td>1.423</td>
<td>1.241</td>
</tr>
<tr>
<td>$\lambda^x_{s0}$</td>
<td>0.107</td>
<td>1.817</td>
<td>2.941</td>
</tr>
<tr>
<td>$\chi^2_T$</td>
<td>3.81</td>
<td>7.57</td>
<td>12.78</td>
</tr>
<tr>
<td>$\chi^2_{AM}$</td>
<td>3.81</td>
<td>7.57</td>
<td>12.78</td>
</tr>
<tr>
<td>$\chi^2_F$</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>$\chi^2_{PDF}$</td>
<td>4.9</td>
<td>9.729</td>
<td>16.432</td>
</tr>
</tbody>
</table>
### TABLE-6

$\lambda$ and $x^2$ values for $p-^{40}$Ca elastic scattering at 40.0 MeV

<table>
<thead>
<tr>
<th></th>
<th>V14</th>
<th></th>
<th>HJ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no $m^*$</td>
<td>$m^*$</td>
<td>no $m^*$</td>
</tr>
<tr>
<td>$\lambda_C^1$</td>
<td>0.872</td>
<td>0.872</td>
<td>0.948</td>
</tr>
<tr>
<td>$\lambda_C^{\pi}$</td>
<td>0.488</td>
<td>0.553</td>
<td>0.514</td>
</tr>
<tr>
<td>$\lambda_{SO}^{\pi}$</td>
<td>0.712</td>
<td>0.939</td>
<td>0.713</td>
</tr>
<tr>
<td>$\lambda_{SO}^{\pi}$</td>
<td>-0.106</td>
<td>-0.050</td>
<td>-0.077</td>
</tr>
<tr>
<td>$\chi_T^2$</td>
<td>16.29</td>
<td>15.06</td>
<td>33.82</td>
</tr>
<tr>
<td>$\chi_{CS}^2$</td>
<td>18.05</td>
<td>16.73</td>
<td>47.19</td>
</tr>
<tr>
<td>$\chi_P^2$</td>
<td>13.94</td>
<td>12.83</td>
<td>16.01</td>
</tr>
<tr>
<td>$\chi_{PDF}^2$</td>
<td>16.893</td>
<td>15.615</td>
<td>35.077</td>
</tr>
</tbody>
</table>
### TABLE-7

$\lambda$ and $\chi^2$ values for $p-^{40}\text{Ca}$ elastic scattering at 45.5 MeV

<table>
<thead>
<tr>
<th></th>
<th>V14</th>
<th></th>
<th>HJ</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no $m^*$</td>
<td>$m^*$</td>
<td>no $m^*$</td>
<td>$m^*$</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>0.871</td>
<td>0.878</td>
<td>0.981</td>
<td>0.995</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>0.442</td>
<td>0.493</td>
<td>0.491</td>
<td>0.592</td>
</tr>
<tr>
<td>$\lambda_{so}$</td>
<td>0.765</td>
<td>0.960</td>
<td>0.915</td>
<td>1.156</td>
</tr>
<tr>
<td>$\lambda_{so}$</td>
<td>-1.484</td>
<td>-1.245</td>
<td>-3.876</td>
<td>-4.096</td>
</tr>
<tr>
<td>$\chi^2_T$</td>
<td>54.30</td>
<td>52.99</td>
<td>58.19</td>
<td>68.33</td>
</tr>
<tr>
<td>$\chi^2_{OS}$</td>
<td>80.39</td>
<td>78.42</td>
<td>75.17</td>
<td>76.88</td>
</tr>
<tr>
<td>$\chi^2_F$</td>
<td>15.74</td>
<td>15.42</td>
<td>33.11</td>
<td>55.69</td>
</tr>
<tr>
<td>$\chi^2_{PDF}$</td>
<td>56.364</td>
<td>55.008</td>
<td>60.409</td>
<td>70.930</td>
</tr>
<tr>
<td></td>
<td>V14</td>
<td>HJ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------------------</td>
<td>--------------</td>
<td>--------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lambda^R_{c} )</td>
<td>0.895</td>
<td>0.962</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lambda^I_{c} )</td>
<td>0.521</td>
<td>0.506</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lambda^R_{so} )</td>
<td>0.749</td>
<td>0.817</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lambda^I_{so} )</td>
<td>-0.268</td>
<td>-1.084</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \chi^R_{T} )</td>
<td>70.95</td>
<td>124.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \chi^R_{CS} )</td>
<td>80.01</td>
<td>133.98</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \chi^R_{P} )</td>
<td>49.20</td>
<td>101.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \chi^R_{PDF} )</td>
<td>75.382</td>
<td>132.067</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>V14</td>
<td>HJ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>------</td>
<td>------</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>no m*</td>
<td>m*</td>
<td>no m*</td>
<td>m*</td>
</tr>
<tr>
<td>$\lambda^R_\alpha$</td>
<td>0.887</td>
<td>0.880</td>
<td>0.998</td>
<td>1.009</td>
</tr>
<tr>
<td>$\lambda^I_\alpha$</td>
<td>0.534</td>
<td>0.570</td>
<td>0.763</td>
<td>0.651</td>
</tr>
<tr>
<td>$\lambda^R_{\pi\pi}$</td>
<td>0.703</td>
<td>0.899</td>
<td>0.399</td>
<td>0.697</td>
</tr>
<tr>
<td>$\lambda^I_{\pi\pi}$</td>
<td>0.045</td>
<td>-0.129</td>
<td>-6.530</td>
<td>-4.347</td>
</tr>
<tr>
<td>$\chi^2_T$</td>
<td>43.00</td>
<td>44.27</td>
<td>39.37</td>
<td>64.92</td>
</tr>
<tr>
<td>$\chi^2_{\pi\pi}$</td>
<td>43.00</td>
<td>44.27</td>
<td>39.37</td>
<td>64.92</td>
</tr>
<tr>
<td>$\chi^2_P$</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>$\chi^2_{PDF}$</td>
<td>48.552</td>
<td>49.987</td>
<td>44.447</td>
<td>73.299</td>
</tr>
</tbody>
</table>
\textbf{TABLE-10}

$\lambda$ and $\chi^{2}$ values for $p-^{40}$Ca elastic scattering at 65.0 MeV

<table>
<thead>
<tr>
<th></th>
<th>\text{V14}</th>
<th>\text{HJ}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no $m^{*}$</td>
<td>$m^{*}$</td>
</tr>
<tr>
<td>$\lambda_{s_{0}}$</td>
<td>0.894</td>
<td>0.890</td>
</tr>
<tr>
<td>$\lambda_{s_{2}}$</td>
<td>0.754</td>
<td>0.875</td>
</tr>
<tr>
<td>$\lambda_{s_{4}}$</td>
<td>1.407</td>
<td>1.794</td>
</tr>
<tr>
<td>$\lambda_{s_{6}}$</td>
<td>-0.692</td>
<td>-0.693</td>
</tr>
<tr>
<td>$\chi_{T}^{2}$</td>
<td>79.99</td>
<td>137.74</td>
</tr>
<tr>
<td>$\chi_{s_{0}}^{2}$</td>
<td>136.63</td>
<td>227.81</td>
</tr>
<tr>
<td>$\chi_{s_{2}}^{2}$</td>
<td>23.35</td>
<td>47.66</td>
</tr>
<tr>
<td>$\chi_{s_{4}}^{2}$</td>
<td>85.703</td>
<td>147.575</td>
</tr>
</tbody>
</table>
### Table 11

$\lambda$ and $\chi^2$ values for $p-^{40}\text{Ca}$ elastic scattering at 80.2 MeV

<table>
<thead>
<tr>
<th></th>
<th>V14</th>
<th>HJ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no $m^*$</td>
<td>$m^*$</td>
</tr>
<tr>
<td>$\lambda_C^2$</td>
<td>0.874</td>
<td>0.894</td>
</tr>
<tr>
<td>$\lambda_C^2$</td>
<td>0.593</td>
<td>0.719</td>
</tr>
<tr>
<td>$\lambda_{so}^m$</td>
<td>0.695</td>
<td>0.847</td>
</tr>
<tr>
<td>$\lambda_{so}^m$</td>
<td>0.667</td>
<td>0.688</td>
</tr>
<tr>
<td>$\chi_T^2$</td>
<td>72.71</td>
<td>69.85</td>
</tr>
<tr>
<td>$\chi_{CS}^2$</td>
<td>84.83</td>
<td>83.38</td>
</tr>
<tr>
<td>$\chi_P^2$</td>
<td>47.83</td>
<td>42.15</td>
</tr>
<tr>
<td>$\chi_{PDF}^2$</td>
<td>77.333</td>
<td>74.281</td>
</tr>
</tbody>
</table>
TABLE-12

$\lambda$ and $\chi^2$ values for $p-^{40}\text{Ca}$ elastic scattering at 135 MeV

<table>
<thead>
<tr>
<th></th>
<th>V14</th>
<th></th>
<th>HJ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no m</td>
<td>m</td>
<td>no m</td>
</tr>
<tr>
<td>$\lambda^a_{G}$</td>
<td>0.910</td>
<td>0.945</td>
<td>0.796</td>
</tr>
<tr>
<td>$\lambda^2_{G}$</td>
<td>0.809</td>
<td>0.862</td>
<td>1.124</td>
</tr>
<tr>
<td>$\lambda^a_{SO}$</td>
<td>0.500</td>
<td>0.650</td>
<td>0.812</td>
</tr>
<tr>
<td>$\lambda^1_{SO}$</td>
<td>1.932</td>
<td>1.104</td>
<td>2.579</td>
</tr>
<tr>
<td>$\chi^2_T$</td>
<td>98.01</td>
<td>126.84</td>
<td>100.05</td>
</tr>
<tr>
<td>$\chi^2_{CS}$</td>
<td>98.01</td>
<td>126.84</td>
<td>100.05</td>
</tr>
<tr>
<td>$\chi^2_P$</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>$\chi^2_{PDF}$</td>
<td>108.068</td>
<td>139.846</td>
<td>110.316</td>
</tr>
<tr>
<td></td>
<td>V14</td>
<td>HJ</td>
<td></td>
</tr>
<tr>
<td>----------------</td>
<td>-----------</td>
<td>-----------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>no m*</td>
<td>m*</td>
<td>no m*</td>
</tr>
<tr>
<td>( \lambda_c )</td>
<td>1.612</td>
<td>1.626</td>
<td>1.722</td>
</tr>
<tr>
<td>( \lambda_i )</td>
<td>0.303</td>
<td>0.176</td>
<td>0.544</td>
</tr>
<tr>
<td>( \lambda_{so} )</td>
<td>0.106</td>
<td>0.026</td>
<td>0.401</td>
</tr>
<tr>
<td>( \chi_{so} )</td>
<td>3.897</td>
<td>3.586</td>
<td>3.875</td>
</tr>
<tr>
<td>( \chi_T )</td>
<td>39.60</td>
<td>51.07</td>
<td>56.19</td>
</tr>
<tr>
<td>( \chi_{OS} )</td>
<td>38.89</td>
<td>54.15</td>
<td>76.08</td>
</tr>
<tr>
<td>( \chi_P )</td>
<td>40.72</td>
<td>46.19</td>
<td>24.60</td>
</tr>
<tr>
<td>( \chi_{PDF} )</td>
<td>43.555</td>
<td>56.182</td>
<td>61.811</td>
</tr>
</tbody>
</table>

**TABLE-13**

\( \lambda \) and \( \chi^2 \) values for p-\(^{40}\)Ca elastic scattering at 152 MeV
### TABLE-14

λ and $\chi^2$ values for p-$^{40}$Ca elastic scattering at 160 MeV

<table>
<thead>
<tr>
<th></th>
<th>V14</th>
<th></th>
<th>HJ</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no $m^*$</td>
<td>$m^*$</td>
<td>no $m^*$</td>
<td>$m^*$</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>0.931</td>
<td>0.960</td>
<td>0.825</td>
<td>0.811</td>
</tr>
<tr>
<td>$\lambda^z_c$</td>
<td>0.856</td>
<td>0.897</td>
<td>1.144</td>
<td>1.177</td>
</tr>
<tr>
<td>$\lambda^x_{so}$</td>
<td>0.513</td>
<td>0.653</td>
<td>0.799</td>
<td>0.942</td>
</tr>
<tr>
<td>$\lambda^z_{so}$</td>
<td>1.862</td>
<td>1.112</td>
<td>2.008</td>
<td>1.630</td>
</tr>
<tr>
<td>$\chi^2_T$</td>
<td>50.44</td>
<td>71.02</td>
<td>73.89</td>
<td>81.91</td>
</tr>
<tr>
<td>$\chi^2_{CS}$</td>
<td>65.03</td>
<td>94.02</td>
<td>89.38</td>
<td>99.61</td>
</tr>
<tr>
<td>$\chi^2_P$</td>
<td>11.20</td>
<td>9.20</td>
<td>32.28</td>
<td>34.33</td>
</tr>
<tr>
<td>$\chi^2_{PDF}$</td>
<td>54.104</td>
<td>76.181</td>
<td>79.268</td>
<td>87.862</td>
</tr>
</tbody>
</table>
**TABLE-15**

λ and $\chi^2$ values for $p^{40}\text{Ca}$ elastic scattering at 181.3 MeV

<table>
<thead>
<tr>
<th></th>
<th>V14</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no m*</td>
<td>m*</td>
<td></td>
<td>no m*</td>
</tr>
<tr>
<td>$\lambda_c^m$</td>
<td>0.827</td>
<td>0.826</td>
<td></td>
<td>0.809</td>
</tr>
<tr>
<td>$\lambda_c^s$</td>
<td>0.841</td>
<td>0.844</td>
<td></td>
<td>1.068</td>
</tr>
<tr>
<td>$\lambda_{so}^m$</td>
<td>0.714</td>
<td>0.792</td>
<td></td>
<td>0.898</td>
</tr>
<tr>
<td>$\lambda_{so}^s$</td>
<td>1.033</td>
<td>0.727</td>
<td></td>
<td>1.027</td>
</tr>
<tr>
<td>$\chi_T^2$</td>
<td>48.00</td>
<td>56.83</td>
<td></td>
<td>91.06</td>
</tr>
<tr>
<td>$\chi_{cs}^2$</td>
<td>65.85</td>
<td>78.09</td>
<td></td>
<td>103.85</td>
</tr>
<tr>
<td>$\chi_P^2$</td>
<td>30.15</td>
<td>35.57</td>
<td></td>
<td>78.26</td>
</tr>
<tr>
<td>$\chi_{PDF}^2$</td>
<td>51.310</td>
<td>60.750</td>
<td></td>
<td>97.336</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th></th>
<th>V14 no m^</th>
<th>V14 m^</th>
<th>HJ no m^</th>
<th>HJ m^</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ_σ</td>
<td>0.832</td>
<td>0.824</td>
<td>0.816</td>
<td>0.811</td>
</tr>
<tr>
<td>λ_σ^1</td>
<td>0.975</td>
<td>0.986</td>
<td>1.176</td>
<td>1.023</td>
</tr>
<tr>
<td>λ_σ^2</td>
<td>0.749</td>
<td>0.824</td>
<td>0.921</td>
<td>1.070</td>
</tr>
<tr>
<td>λ_σ^3</td>
<td>0.938</td>
<td>0.680</td>
<td>0.868</td>
<td>1.306</td>
</tr>
<tr>
<td>χ_T^2</td>
<td>19.75</td>
<td>21.70</td>
<td>41.03</td>
<td>16.68</td>
</tr>
<tr>
<td>χ_CS^2</td>
<td>22.20</td>
<td>24.66</td>
<td>44.05</td>
<td>21.55</td>
</tr>
<tr>
<td>χ_P^2</td>
<td>17.29</td>
<td>18.74</td>
<td>38.00</td>
<td>11.80</td>
</tr>
<tr>
<td>χ_PDF^2</td>
<td>21.156</td>
<td>23.247</td>
<td>43.957</td>
<td>17.868</td>
</tr>
<tr>
<td></td>
<td>V14</td>
<td>HJ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
<td>------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>no m</td>
<td>m</td>
<td>no m</td>
<td>m</td>
</tr>
<tr>
<td>$\lambda_0^c$</td>
<td>0.743</td>
<td>0.279</td>
<td>0.345</td>
<td>0.206</td>
</tr>
<tr>
<td>$\lambda_0^s$</td>
<td>0.976</td>
<td>0.926</td>
<td>1.032</td>
<td>0.992</td>
</tr>
<tr>
<td>$\lambda_{so}$</td>
<td>0.784</td>
<td>0.840</td>
<td>0.947</td>
<td>1.002</td>
</tr>
<tr>
<td>$\chi_T^2$</td>
<td>8.47</td>
<td>8.67</td>
<td>11.78</td>
<td>8.71</td>
</tr>
<tr>
<td>$\chi_{CS}^2$</td>
<td>13.20</td>
<td>13.58</td>
<td>19.65</td>
<td>13.71</td>
</tr>
<tr>
<td>$\chi_p^2$</td>
<td>3.75</td>
<td>3.76</td>
<td>3.92</td>
<td>3.70</td>
</tr>
<tr>
<td>$\chi_{PDF}^2$</td>
<td>8.644</td>
<td>8.848</td>
<td>12.024</td>
<td>8.883</td>
</tr>
</tbody>
</table>

TABLE-17

$\lambda$ and $\chi^2$ values for $p^{-40}\text{Ca}$ elastic scattering at 300 MeV
<table>
<thead>
<tr>
<th></th>
<th>V14</th>
<th></th>
<th>HJ</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no m^*</td>
<td>m^*</td>
<td>no m^*</td>
<td>m^*</td>
</tr>
<tr>
<td>$\lambda_{G1}$</td>
<td>0.368</td>
<td>0.332</td>
<td>0.408</td>
<td>0.658</td>
</tr>
<tr>
<td>$\lambda_{G}$</td>
<td>0.744</td>
<td>0.714</td>
<td>0.746</td>
<td>0.780</td>
</tr>
<tr>
<td>$\lambda_{SO}$</td>
<td>0.798</td>
<td>0.846</td>
<td>0.810</td>
<td>0.710</td>
</tr>
<tr>
<td>$\lambda_{SD}$</td>
<td>1.290</td>
<td>1.359</td>
<td>0.940</td>
<td>0.077</td>
</tr>
<tr>
<td>$\chi_T^2$</td>
<td>84.57</td>
<td>76.26</td>
<td>201.46</td>
<td>331.85</td>
</tr>
<tr>
<td>$\chi_{CGS}^2$</td>
<td>136.57</td>
<td>123.21</td>
<td>370.29</td>
<td>605.28</td>
</tr>
<tr>
<td>$\chi_p^2$</td>
<td>32.56</td>
<td>29.32</td>
<td>32.63</td>
<td>58.43</td>
</tr>
<tr>
<td>$\chi_{PDF}^2$</td>
<td>85.709</td>
<td>77.292</td>
<td>204.185</td>
<td>336.339</td>
</tr>
</tbody>
</table>
TABLE-19

$\lambda$ and $\chi^2$ values for $p^{-40}\text{Ca}$ elastic scattering at 400 MeV

<table>
<thead>
<tr>
<th></th>
<th>V14</th>
<th>HJ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$^{*}\lambda$</td>
<td>$^{*}\lambda$</td>
</tr>
<tr>
<td>$^{*}\lambda_{SO}$</td>
<td>0.383</td>
<td>0.314</td>
</tr>
<tr>
<td>$^{*}\lambda_{SO}$</td>
<td>0.967</td>
<td>0.919</td>
</tr>
<tr>
<td>$^{*}\lambda_{SO}$</td>
<td>0.797</td>
<td>0.800</td>
</tr>
<tr>
<td>$^{*}\lambda_{SO}$</td>
<td>1.350</td>
<td>1.735</td>
</tr>
<tr>
<td>$^{*}\chi^2_T$</td>
<td>9.82</td>
<td>8.82</td>
</tr>
<tr>
<td>$^{*}\chi^2_{CS}$</td>
<td>10.98</td>
<td>9.30</td>
</tr>
<tr>
<td>$^{*}\chi^2_P$</td>
<td>9.25</td>
<td>8.59</td>
</tr>
<tr>
<td>$^{*}\chi^2_{PDF}$</td>
<td>10.155</td>
<td>9.125</td>
</tr>
</tbody>
</table>
Figure captions:

Figure 1(a):
Energy dependence of calculated real central optical potential at low energies using V14 interaction.

Figure 1(b):
Energy dependence of calculated real central optical potential at intermediate energies using V14 interaction.

Figure 1(c):
Same as in fig. 1(a) but using HJ interaction.

Figure 1(d):
Same as in fig. 1(b) but using HJ interaction.

Figure 2(a):
Energy dependence of strength of calculated real and imaginary central optical potentials using V14 interaction.

Figure 2(b):
Same as in fig. 2(a) but using HJ interaction.

Figure 2(c):
Energy dependence of volume integrals per nucleon of real and imaginary parts of calculated central and spin-orbit optical potentials using V14 interaction.

Figure 2(d):
Same as in fig. 2(c) but using HJ interaction.
Figure 3(a):
Energy dependence of imaginary part of the central optical potential at low energies using V14 interaction.

Figure 3(b):
Energy dependence of imaginary part of the central optical potential at intermediate energies using V14 interaction.

Figure 3(c):
Same as in fig. 3(a) but using HJ interaction.

Figure 3(d):
Same as in fig. 3(b) but using HJ interaction.

Figure 4(a):
Energy dependence of calculated real spin-orbit optical potential at low energies using V14 interaction.

Figure 4(b):
Energy dependence of calculated real spin-orbit optical potential at intermediate energies using V14 interaction.

Figure 4(c):
Same as in fig. 4(a) but using HJ interaction.

Figure 4(d):
Same as in fig. 4(b) but using HJ interaction.

Figure 5(a):
Energy dependence of calculated imaginary spin-orbit optical potential at low energies using V14 interaction.
Figure 5(b):

Energy dependence of calculated imaginary spin-orbit optical potential at intermediate energies using V14 interaction.

Figure 5(c):

Same as in fig. 5(a) but using HJ interaction.

Figure 5(d):

Same as in fig. 5(b) but using HJ interaction.

Figure 6(a):

Calculated real central optical potential with and without effective mass correction at 21 MeV using V14 and HJ interaction.

Figure 6(b):

Calculated imaginary central optical potential with and without effective mass correction at 21 MeV using V14 and HJ interaction.

Figure 6(c):

Calculated real spin-orbit optical potential with and without effective mass correction at 21 MeV using V14 and HJ interaction.

Figure 6(d):

Calculated imaginary spin-orbit optical potential with and without effective mass correction at 21 MeV using V14 and HJ interaction.
Figure 7(a):

Energy dependence of direct and exchange components of calculated real spin-orbit optical potential using V14 interaction.

Figure 7(b):

Energy dependence of direct and exchange components of calculated imaginary spin-orbit optical potential using V14 interaction.

Figures 8(a)-18(b):

Calculated cross sections and polarizations compared with corresponding experimental data for p-\(^{40}\)Ca elastic scattering at low energies.

Figures 19-26:

Calculated cross sections and polarizations compared with corresponding experimental data for p-\(^{40}\)Ca elastic scattering at intermediate energies.
Figure 1(a)

$p-^{40}$Ca Elastic Scattering

$E_p$ (MeV) =

- A - 21
- B - 30
- C - 40
- D - 48
- E - 65
- F - 80

DISTANCE (F)

REAL CENTRAL POTENTIAL (MeV)
Figure: 1(b)

$^p$-$^{40}$Ca Elastic Scattering

$E_p$ (MeV) = 135

$E_p$ (MeV) = 160

$E_p$ (MeV) = 181

$E_p$ (MeV) = 200

$E_p$ (MeV) = 300

$E_p$ (MeV) = 362

$E_p$ (MeV) = 400

$E_p$ (MeV) = 497

$E_p$ (MeV) = 800

DISTANCE (F)

REAL CENTRAL POTENTIAL (MeV)
FIGURE: 1(c)

$p^{40}\text{Ca Elastic Scattering (LRAY, HJ, no m')}$

A \(-E_\text{p} \text{ (MeV)} = 21\)
B \(-30\)
C \(-40\)
D \(-48\)
E \(-65\)
F \(-80\)
Figure 1(d) shows the real central potential (MeV) for different distances (F) in the $p-^{40}$Ca elastic scattering experiment. The curves are labeled with different energies:

- $A - E_p$ (MeV) = 135
- $B - E_p$ = 160
- $C - E_p$ = 181
- $D - E_p$ = 200
- $E - E_p$ = 300
- $F - E_p$ = 362
- $G - E_p$ = 400

The figure emphasizes the variation of the potential with distance.
**FIGURE: 2(a)**

- **STRENGTH OF POTENTIAL (MeV)**
- **INCIDENT ENERGY (MeV)**

**P\(^{44}\)Ca**

(LEAF, V14, no m)
FIGURE 2(b)

\( P^{-40}Ca \) (LARY. HJ. no m')
FIGURE: 2(c)
\textbf{FIGURE: 2 (d)}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2d.png}
\caption{\textit{p-^{40}\text{Ca}} (LRAY, HJ, no m²)}
\end{figure}
FIGURE: 3(a)
\[ A - E_p \text{ (MeV)} = 135 \]

\[ B = 160 \]

\[ C = 181 \]

\[ D = 200 \]

\[ E = 300 \]

\[ F = 362 \]

\[ G = 400 \]

\[ H = 497 \]

**Figure 3(b)**

**p-^{48}Ca Elastic Scattering**

*LRAY, Urbana VI4, no m*
Figure: 3(c)

$p - ^{40}$Ca Elastic Scattering
(LEAY, HJ, no m)

Imaginary central potential (MeV)

Distance (F)

A - $E_p$ (MeV) = 21
B - 30
C - 40
D - 48
E - 65
F - 80
FIGURE 3(d)
**Figure: 4(a)**

**p-^{40}Ca Elastic Scattering**

(LEAY, Urbana V14, no m)

---

**Legend:**

- **A** - \( E_p \) (MeV) = 21
- **B** - 30
- **C** - 40
- **D** - 48
- **E** - 65
- **F** - 80

---

**Axes:**

- **Y-axis:** Real Spin-Orbit Potential (MeV)
- **X-axis:** Distance (F)

---

**Graph Details:**

- The graph shows the real spin-orbit potential vs. distance for the p-^{40}Ca elastic scattering.
- The points A, B, C, D, E, and F are marked with corresponding energies (MeV).
**P-^{40}Ca Elastic Scattering**

*LEHAY, Urbana VI4, no m*

**REAL SPIN-ORBIT POTENTIAL (MeV)**

**DISTANCE (F)**

**FIGURE: 4(b)**
\textbf{p-^{40}Ca Elastic Scattering (LRAY, HJ, no m)}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4c.png}
\caption{Real spin-orbit potential (MeV) vs. distance (f).}
\end{figure}

\begin{align*}
A &- E_p (\text{MeV}) = 21 \\
B &- 30 \\
C &- 40 \\
D &- 48 \\
E &- 65 \\
F &- 80
\end{align*}
**Figure 4(d)**

The diagram illustrates the **p-^{40}Ca Elastic Scattering** as a function of **Real Spin-Orbit Potential (MeV)** and **Distance (F)**. The curves are labeled as follows:

- **A** - $E_p$ (MeV) = 135
- **B** - 160
- **C** - 181
- **D** - 200
- **E** - 300
- **F** - 362
- **G** - 400
- **H** - 497

The graph shows the variation of real spin-orbit potential with distance for different energies, highlighting the scattering characteristics of p-^{40}Ca.
**FIGURE: 5(a)**

- **p-^{40}Ca Elastic Scattering**
- **(LRAY, Urbana V14, no m)**

<table>
<thead>
<tr>
<th>Label</th>
<th>Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>21</td>
</tr>
<tr>
<td>B</td>
<td>30</td>
</tr>
<tr>
<td>C</td>
<td>40</td>
</tr>
<tr>
<td>D</td>
<td>48</td>
</tr>
<tr>
<td>E</td>
<td>65</td>
</tr>
<tr>
<td>F</td>
<td>80</td>
</tr>
</tbody>
</table>
FIGURE: 5(c)

**p-^{40}\text{Ca Elastic Scattering}**

(LRAY, BJ, no m)

<table>
<thead>
<tr>
<th>Label</th>
<th>E (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>21</td>
</tr>
<tr>
<td>B</td>
<td>30</td>
</tr>
<tr>
<td>C</td>
<td>40</td>
</tr>
<tr>
<td>D</td>
<td>48</td>
</tr>
<tr>
<td>E</td>
<td>65</td>
</tr>
<tr>
<td>F</td>
<td>80</td>
</tr>
</tbody>
</table>
The image shows a graph labeled "p-$^{40}\text{Ca}$ Elastic Scattering (LRAY, HJ, no m)". The graph plots the imaginary spin-orbit potential (in MeV) against distance (in fm). The curves A through H are labeled with different energies: A - $E_p$ (MeV) = 135, B - 160, C - 181, D - 200, E - 300, F - 362, G - 400, H - 497. The graph is labeled "FIGURE: 5(d)".
Figure 6(a): Real Central Potential (MeV) vs Distance (F) for p-\(^{40}\)Ca Elastic Scattering (LRAY, 21 MeV). The graph shows the potential for both cases: no \(m^*\) and with \(m^*\). The curves are labeled HJ and V14.
**Figure 6(b)**

**p-^{40}Ca Elastic Scattering**

(LRAY, 21 MeV)

*IMAGINARY CENTRAL POTENTIAL (MeV)*

DISTANCE (F)

---

- **HJ** (solid line)
- **V14** (dashed line)

---

- **no m^***
- **with m^***
Figure 6(c)

$^{14}\text{Ca} \text{ Elastic Scattering}$

($\text{LEAY, 21 MeV}$)

Real Spin-Orbit Potential (MeV)

Distance (F)

No $m^*$

With $m^*$
FIGURE: 6(d)

p-$^{40}$Ca Elastic Scattering
(LRAY, 21 MeV)

IMAGINARY SPIN-ORBIT POTENTIAL (MeV)

DISTANCE (f)
p-$^{40}$Ca Elastic Scattering

(LRAY, Urbana V14)

**FIGURE: 7(a)**
**Figure 7(b)**

**P-^{40}Ca Elastic Scattering**

*L.RAY, Urbana V14*

**Imag. Spin-Orbit Potential (MeV)**

- **A**: \( W_\omega (\text{DIR}) \) at 21 MeV
- **B**: \( W_\omega (\text{DIR}) \) at 200 MeV
- **C**: \( W_\omega (\text{EX}) \) at 21 MeV
- **D**: \( W_\omega (\text{EX}) \) at 200 MeV

**Distance (F)**

- **DISTANCE (F)**
  - 0.0
  - 4.0
  - 8.0

**Legend**

- **A**: \( W_\omega (\text{DIR}) \) at 21 MeV
- **B**: \( W_\omega (\text{DIR}) \) at 200 MeV
- **C**: \( W_\omega (\text{EX}) \) at 21 MeV
- **D**: \( W_\omega (\text{EX}) \) at 200 MeV
Figure 8(a): 

$^{14}$Ca Elastic Scattering at 21.1 MeV

(LRAY, m)

Differential Cross Section (MB/ST)

Scattering Angle (Degree)

---

V14

---

HJ
$^{14}$Ca Elastic Scattering at 21.1 MeV (LRAV, m)

Figure: 8(b)
$p-^{40}\text{Ca} \text{ Elastic Scattering at } 26.3 \text{ MeV}$

\[ \text{(LRAY, m)} \]

**FIGURE: 9(a)**
Figure 9(b)

p-^{40}Ca Elastic Scattering
at 26.3 MeV
(LRAY, m)
$p^{-}{^{40}}Ca$ Elastic Scattering
at 28.5 MeV
(LRAY, m)

Figure: 10(a)
p-$^{40}$Ca Elastic Scattering at 28.5 MeV (LRAY, m)

FIGURE: 10(b)
p-\(^{40}\)Ca Elastic Scattering at 30.3 MeV (LRAY, m)

**Figure: 11(a)**
Figure 11(b): 

$\text{p-}^{40}\text{Ca Elastic Scattering}$

at 30.3 MeV

(LRAY, m)
**Figure 12**

**p-^{40}Ca Elastic Scattering at 34.8 MeV**

- **V14**
- **HJ**

Differential Cross Section (MB/sr)

Scattering Angle (Degree)
FIGURE: 13(a)

$p-{}^{40}\text{Ca}$ Elastic Scattering at 40.0 MeV

(V14, HJ)

Differential Cross Section (MB/σr)

Scattering Angle (Degree)
Figure 13(b) shows the analyzing power for p-$^{40}$Ca elastic scattering at 40.0 MeV, as a function of the scattering angle. The solid line represents the V14 model, while the dashed line represents the HJ model. The data points correspond to experimental measurements. The scatter plot highlights the variation in analyzing power across different scattering angles.
$^{16}$O Elastic Scattering at 45.5 MeV

FIGURE: 14(a)
FIGURE: 14(b)
$^{40}$Ca Elastic Scattering at 48.0 MeV

Figure: 15(a)
p-$^{40}$Ca Elastic Scattering at 48.0 MeV
(LRAY. m)

**FIGURE: 15(b)**
Elastic Scattering at 61.4 MeV (LRAY, m)

\[ p^{40}Ca \text{ Elastic Scattering} \]

\[ \text{at 61.4 MeV} \]

\[ \text{(LRAY, m)} \]

**Figure: 16**
\( p^{14}Ca \) Elastic Scattering at 66.0 MeV

**FIGURE: 17(a)**
**Figure 17(b)**

$p - ^{40}\text{Ca} \text{ Elastic Scattering}
\text{ at } 65.0 \text{ MeV}
\text{(LEAY, m)}$

Graph showing the analyzing power versus the scattering angle (degrees) with data points and curves for different models.
p-$^{40}$Ca Elastic Scattering
at 80.2 MeV
( LRAY, m )

---

Figure: 18(a)
p-$^{40}$Ca Elastic Scattering at 60.2 MeV

**Figure: 18(b)**
p-$^{40}$Ca Elastic Scattering at 135 MeV (LRAY, m)

FIGURE: 19
$p^{40}\text{Ca}$ Elastic Scattering at 152 MeV $(\text{LRAY}, \text{m})$

**Figure 20(a)**
Figure: p-\(^{40}\)Ca Elastic Scattering at 152 MeV (LRAY, m)

**Figure: 20(b)**
p-$^{40}$Ca Elastic Scattering at 160 MeV
(LRAY, m)

FILE: 21(a)
**Figure 21(b)**

**p-⁴⁰Ca Elastic Scattering at 160 MeV**

*LRAY, m*
**Figure 22(a)**

The graph illustrates the differential cross section of $p-{ }^{40}\text{Ca}$ elastic scattering at 181.3 MeV as a function of scattering angle (degree). The data are compared with the theoretical models labeled V14 and HJ.
Figure 22(b)

$^{14}\text{Ca}$ Elastic Scattering at 181.3 MeV

(LEAP, m)

Scattering Angle (Degree)

Analyzing Power

V14
HJ
$p - ^{40}\text{Ca}\text{ Elastic Scattering at } 200.0\text{ MeV (LAB, m)}$

**FIGURE: 23(a)**
Figure: 23(b)

p-$^{40}$Ca Elastic Scattering at 200.0 MeV
(LRAY, m)
$p^{40}\text{Ca Elastic Scattering at 300.0 MeV}$

**Figure: 24(a)**
Figure 24(b) shows the analyzing power for p-\(^{48}\)Ca elastic scattering at 300.0 MeV. The data points represent the experimental measurements, while the solid line with open circles represents the theoretical prediction by V14, and the dashed line with open squares represents the prediction by HJ. The graph plots the scattering angle (degree) against the analyzing power.
**Figure 25 (a)**

p-$^{40}$Ca Elastic Scattering at 362.0 MeV

(DRAY, m)

- \text{V14}
- \text{HJ}

Scattering Angle (Degree)

Differential Cross Section (mb/sr)
Figure 25(b)

$^p$-Ca Elastic Scattering at 362.0 MeV (LRAV, m)
Elastic Scattering
at 400.0 MeV

Differential Cross Section (MB/sr)

V14
HJ

FIGURE: 26(a)
Figure 26(b)

$p^{-40}\text{Ca}$ Elastic Scattering at 400.0 MeV (LRAY, m)

Scattering Angle (Degree)
References:

   C16(1977)80.

2) H. V. von Geramb; in The interaction between medium energy 
   nucleons in nuclei-1982 ( Indiana University Cyclotron 
   Facility), AIP Conf. Proc. No.97, edited by H. O. Meyer 
   (AIP, New York, 1983)44.


6) C. Mahaux; in The interaction between medium energy 
   nucleons in nuclei-1982 ( Indiana University Cyclotron 
   Facility), AIP Conf. Proc. No.97, edited by H. O. Meyer 

7) L. Rikus, K. Nakano and H. V. von Geramb; Nucl. Phys. 


10) N. Yamaguchi, S. Nagata and J. Michiyama; Prog. Theor. 


26) F. James and M. Roos; Comp. Phys. Comm. 10(1975)343.