CHAPTER I

INTRODUCTION
Historical development:

One of the fundamental challenges we usually encounter with in theoretical nuclear physics is to understand the properties of nuclei and nuclear reactions in terms of fundamental interaction. In this context, the interaction between a nucleon and a nucleus is of basic importance. The study of the nucleon-nucleus interaction has quite a long history. As long ago as 1935, Bethe\(^1\) calculated the scattering of nucleons by a purely real potential and found marked resonances that are not observed experimentally. Le Levier and Saxon\(^2\) showed that these are damped if the potential is allowed to become complex, and that such potentials are able to reproduce quite well the differential cross-sections for the elastic scattering of the medium energy protons by nuclei. This work was extended to neutron scattering by Feshbach, Porter and Weisskopf\(^3\) who examined the total and the reaction cross-sections for the interaction of the neutrons with nuclei over a wide range of energies and nuclei. Feshbach, Porter and Weisskopf\(^3\) also used a complex potential, and this is referred to as the optical model by analogy with the interaction of light with a medium that is both refractive and absorptive. Just as this can be treated by allowing the refractive index to become complex, so the scattering and absorption of nucleons by nuclei can be represented by a complex potential. In their calculations they
used a spherically symmetric potential with simple square well form and they were able to reproduce very well the overall features of the measured total and reaction cross-sections. Subsequently, it was realised that this success is partly due to the relatively poor energy resolution of the early measurements. Later, the improvements in the experimental techniques showed that the cross-sections of reaction passing through the compound nucleus fluctuate as the incident energy varies, but if the energy spread of the incident beam is greater than the mean width of the fluctuations an averaged cross-section is obtained that generally varies quite smoothly with energy. Detailed analyses showed that this energy averaged differential cross-section is the sum of compound nucleus and direct reaction components. The former may be calculated using the theory of Houser and Feshbach \(^4\) and subtracted from the measured cross-section. As the energy increases the contribution of the compound nucleus processes falls rapidly and soon becomes negligible. For most nuclei, the compound nucleus contribution to the elastic scattering cross-section is negligible for energies above 15 MeV, so that the data may be analysed directly using the optical model. Summarizing the results of last three decades one may say that the nucleon optical model has proved to be a very useful tool in understanding a large variety of nuclear reactions.

In recent years very many analyses of neutron and proton
interactions with nuclei have been made with optical potentials of increasing sophistication. The nucleon optical potential can be determined either by the phenomenological analyses of the experimental data or by a more fundamental calculation (often called microscopic calculation) starting from the nucleon-nucleon interaction. The former uses rather simple physical arguments to establish the form of the potential and the appropriate values of its parameters, and then relies on comparisons with experimental data to fix these parameters more precisely. In the conventional optical model phenomenology (i.e. the standard optical model), both the real and imaginary central potentials are parametrized in Woods-Saxon (two-parameter Fermi) form. The standard spin-orbit potential is considered to be the conventional Thomas form which involves the derivatives of a Woods-Saxon function. These potentials are then inserted into the Schrödinger equation which involves relativistic kinematics. The potentials that one finds in a conventional, standard optical model analyses in the energy region 20-800 MeV, exhibit several characteristic features: First, the real central potential becomes repulsive above about 600 MeV and since in this parametrization the potential has a monotonic radial dependence, it is obviously either attractive everywhere or repulsive everywhere. Second, the imaginary part of the central potential increases monotonically with energy.
Third, the real spin-orbit term is attractive while the imaginary part is repulsive. Generally, the real spin-orbit potential decreases with increasing energy, while the imaginary spin-orbit potential grows with increasing energy, with the exception that the real spin-orbit potential at 500 MeV is found to be larger than at 200 MeV\(^5\).

The conventional optical model phenomenology poses several severe problems of peculiarity. For instance, the root mean square radius of the real central potential (for \(^{208}\text{Pb}\) as target) in the intermediate energy region, exhibits a peculiar non-monotonic behaviour\(^5\), indicating that the geometry of the real central potential appears to be changing quite substantially with energy. At high energies, one finds a root mean square radius which is considerably smaller than at lower energies, indicating that the range of the repulsive potential is shorter than that of the attractive potential at low energies. Other peculiar behaviour that one usually encounters with in the conventional optical model phenomenology is concerned with the spin-orbit potential. With increasing energy, the volume integral of the real spin-orbit potential, \(K^\Re_{\text{so}}\), falls sharply and seems to have a minimum near 200 MeV before resuming its decrease beyond 400 MeV. Similarly the volume integral of the imaginary spin-orbit potential, \(K^\Im_{\text{so}}\), peaks at 200 MeV, decreases rapidly again and even changes sign near 400 MeV.
The above mentioned difficulties are associated with the use of smooth Woods-Saxon geometry for the radial behaviour of the potential over a wide energy range of the projectile. It also presents a serious problem of fitting the differential cross-sections and analyzing powers above 200 MeV. Subsequently, the need for a more flexible parametrization, certainly of the real central potential, was emphasized. Various non-Woods-Saxon form factors have in fact been proposed. It was realised that above 200 MeV the interior of the nucleus, in terms of real central potential, becomes repulsive while the tail region remains attractive (up to around 700 MeV). Recently, the success of Dirac phenomenology \(^{6-8}\) indicates a non-Woods-Saxon (wine-bottle-bottom) shape for the real central potential. Further, at higher energies one still finds a small attractive tail with a strongly repulsive interior for the real part of the potential. This type of potential gives excellent fits to the elastic scattering data, which are greatly superior to any fit with standard Woods-Saxon potentials.
(1.2) Microscopic calculation of nucleon-nucleus optical potentials

The microscopic calculations of the optical potential are computationally more difficult. However, the numerical techniques used have now been developed\textsuperscript{9-15} to give reliable quantitative results. We shall confine our discussion to the microscopic calculation of nucleon-nucleus optical potential within the framework of the Brueckner theory of infinite nuclear matter. The essential ingredient in the microscopic approach is the energy and density dependant complex NN effective interaction (\(t\)-matrix) which is obtained by solving the Bethe-Goldstone equation. This effective interaction is then folded with the ground state target nuclear density to yield a nucleon-nucleus optical potential using some prescription for folding. In fact, the evaluation of the effective interaction needs a realistic NN interaction which must in principle be determined ultimately by the underlying dynamics of quarks and gluons, namely quantum chromodynamics (QCD). However, due to the non-perturbative character of QCD in the low energy regime relevant for nuclear physics, we are far away from quantitative understanding of the NN interaction in this way. Consequently, one looks forward for an alternative approach of constructing an NN interaction. A variety of quantitative realistic NN interactions empirically determined and based on the meson exchange are now available.
in the literature. Well known examples of such NN interactions are Paris potential¹⁶), Bonn potential¹⁷), Reid hard/soft core potential¹⁸), Hamada-Johnston hard core potential¹⁹) and Urbana V14 soft core potential²⁰). However, throughout the present work, we make use of only Urbana V14 soft core potential and have done calculation using Hamada-Johnston hard core potential also.
(1.3) Outline of the present work:

In Chapter II we discuss in detail the calculational techniques to obtain the nuclear matter optical potential in a self-consistent manner. We also describe the calculation of binding energy and incompressibility of infinite nuclear matter using first order Brueckner theory, starting from both the Hamada-Johnston hard core \(^{19}\) and Urbana V14 soft core \(^{20}\) realistic interactions. Finally we discuss the results of our calculation and compare with the earlier calculations. We would see here that the calculated nuclear matter optical potentials using Urbana V14 interaction are qualitatively similar to the one using Hamada-Johnston interactions, except that the use of Urbana V14 interaction gives a real nuclear matter optical potential which is more attractive as compared with the results using Hamada-Johnston interaction. We would also see that the first order Brueckner theory with the use of V14 interaction predicts an overbound infinite nuclear matter of large saturation density as compared with the empirical value, whereas that with the use of Hamada-Johnston interaction predicts an underbound infinite nuclear matter of saturation density closer to empirical one. Further, the lowest order Brueckner theory with the use of V14 interaction predicts a nuclear matter incompressibility which is in fair agreement with the empirical value and also with the results using variational approach, whereas that with the use of
Hamada-Johnston interaction predicts a nuclear matter incompressibility which is quite low as compared with the empirical value.

Chapter III describes the procedure for obtaining optical potential for finite nuclei from the infinite nuclear matter potential within the frame work of the first order Brueckner theory starting from a realistic interaction. Here we first systematically present the formalism for obtaining different components (central direct, central exchange, spin-orbit direct and spin-orbit exchange terms) of the nucleon-nucleus optical potential using some folding prescription in local density approximation. We also present a new formulation of obtaining the effective mass correction to the optical potential, which slightly differs from others. We would see that our new formulation not only modifies the central imaginary part but also gives a feedback term in the real central part and modifies the real and imaginary parts of spin-orbit optical potential. We then give the results of our calculation of nucleon-nucleus optical potential using both the Urbana V14 soft core\textsuperscript{20} and Hamada-Johnston hard core\textsuperscript{19} interactions for use in studying the elastic scattering of protons from $^{40}$Ca nucleus at low and intermediate energies. Finally, we analyse the experimental data of differential cross-sections and spin observables (analyzing powers and spin rotation functions) for $p^{40}$Ca elastic scattering at about 18
energies in the low and intermediate energy regions. We would see that the calculated potentials are in reasonable agreement with the ones required to fit the scattering data. In particular we find that the real part of our calculated optical potential resembles in shape with the wine-bottle bottom type of potential at an incident energy in the transition region. The radial shape of real optical potential changes substantially with increasing energy. Our calculation shows a mild surface enhancement in the imaginary part of the optical potential at low incident energies. At high incident energies the imaginary potential has no surface peaking in the calculated potential. However, there are several discrepancies. Firstly, the calculated imaginary central potentials at low energies are larger (by about 30-50 %) than the ones required by the experimental data. Secondly, the surface enhancement in the calculated central imaginary potential at low energies is insignificant as compared with that observed in the empirical potentials. Finally, significant discrepancies are observed in the calculated and empirical spin-orbit potential. The reasons for these shortcomings will be explored in detail in the succeeding chapters IV and V.

In Chapter IV we investigate the approximations made in Chapter III for the calculation of the central part of nucleon-nucleus optical potential. Here we propose a new
method for the calculation of the central part of nucleon-nucleus optical potential. We first give the relevant formal derivation. We then discuss the results of our exact calculations. We also compare these results with the corresponding results of earlier calculations described in Chapter III. We shall show that the exchange part of the central potential can be written as a series whose first term corresponds to the commonly used expression for the potential. We have been able to calculate the first three terms of this series. One of the consequences of these additional exchange terms is the enhancement of the surface peaking in the central imaginary part of the calculated potential at low incident nucleon energies. The effect of these new terms is very small at high energies. The real part of the central optical potential is only marginally affected with the inclusion of these additional exchange terms.

In Chapter V we investigate the approximations generally used (as also described in chapter III) in calculating the spin-orbit part of the nucleon-nucleus optical potential. We have shown that the spin-orbit part of the potential can be expressed as a series. All the earlier calculations have only used the first term of this series. An interesting fact that comes out is that the second order term of this series can be as large as the first term and that the full series gives results which are quite close to the first term. After
describing the formalism for obtaining this series expansion we discuss the results of our calculations which use the full series. In the later part of this chapter we discuss the results of our exact calculations using improved exchange part of the central potential and spin-orbit part calculated without making any approximation and then compare these results with our older calculations.

Finally Chapter VI provides the conclusion of the present work.
References:

2) R. E. LeLevier and D. S. Saxon; Phys. Rev. 87(1952)40.
4) U. Houser and H. Feshbach; Phys. Rev. 87(1952)366.