CHAPTER V

SPIN-ORBIT POTENTIAL
(5.1) Introduction:

The calculation of the nucleon-nucleus optical potential, as described in chapter III, starting from realistic internucleon interactions within the framework of the first order Brueckner theory comprises two steps. The first step involves the calculation of t-matrices using first order Brueckner theory. The second step involves the calculation of optical potential from these t-matrices using folding procedure described in chapter III and refs.1-7). For finite nuclei various approximations are made to evaluate folding expressions for both the central and the spin-orbit parts of the optical potential. We have already investigated these approximations concerning the exchange part of central optical potential in the previous chapter IV. The purpose of the present chapter is to investigate the approximations, other than those in the calculation of the t-matrices, which are expected to affect the calculated values of the spin-orbit part of the potential.

The spin-orbit part of the potential has been calculated by several authors2,8-11) as a series expansion. The earliest formula, for only the direct part of the spin-orbit potential was given by Blin-Stoyle9)

\[ V_\infty = \text{const.} \frac{1}{r} \frac{d\varphi}{dr} , \]  

(5.1)
where $\rho$ is the density distribution of nucleons in the nucleus at a point $r$. The right hand side of this formula is the first term in a series expansion for $V_{\text{so}}$. The full series was given by Greenlees et al.\textsuperscript{10,11}. The expansion parameter in this series is the ratio of two distances. The first is the range of the spin-orbit part of the $t$-matrix and the second is the distance in which $\rho$ changes appreciably. It will be shown that for suitable values of the expansion parameter the second term in the series of $V_{\text{so}}$ can be quite large whereas the full sum, except for some minor differences, is quite similar to the first term, i.e. eq. (5.1).

A further discussion of the spin-orbit potential was given by Scheerbaum\textsuperscript{8)} who gave a formally different series and then used various approximations to sum the series. We shall show that this series and that of Greenlees et al.\textsuperscript{10,11} are formally equivalent. Scheerbaum\textsuperscript{8)} then made further approximations, obtaining finally eq. (5.1) with a different, but energy dependant, multiplying factor. Since our investigations suggest that these approaches are in fact identical, the approximations of refs.\textsuperscript{2,8,10-11} need further investigations.

A further result obtained by Scheerbaum\textsuperscript{8)}, and confirmed by other authors\textsuperscript{2,4)}, is that the exchange terms contribute substantially to the spin-orbit part of the optical potential. The approximations referred to above in connection with the
direct part of the spin-orbit terms are more important for its exchange part. We also investigate the approximations made for this part of the potential.

In section 5.2 we discuss various approximations used to calculate the direct part of the nucleon-nucleus spin-orbit optical potential. We then give our exact results. We also compare our results with the earlier calculation of Brieva and Rook \(^1\)-\(^2\) who used the short range approximation for the \(t\)-matrix and retained only the first non vanishing term in the expansion of the density. We show that as a result of using the short range approximation, the calculation of ref. \(^2\) underestimates the direct part of the spin-orbit potential at short distances and overestimates it at large distances. In section 5.3 we investigate the approximations used to calculate the exchange part of the nucleon-nucleus spin-orbit potential. We first give a new formal derivation of the exchange part of the nucleon-nucleus spin-orbit potential in subsection 5.3.1, while the results of calculations for the exchange part of nucleon-nucleus optical potential are discussed in subsection 5.3.2. Section 5.4 is devoted to the detailed analysis of differential cross section and polarization data for the elastic scattering of protons from \(^{40}\text{Ca}\) in the low and intermediate energy regions using optical potential of the exact calculation.
In this section we investigate in detail the approximations used to calculate the direct part of nucleon-nucleus spin-orbit potential. We also present our exact calculations of the direct part of spin-orbit potential for the elastic scattering of protons from $^{40}\text{Ca}$ using Urbana V14 $^{12}$ and Hamada-Johnston (HJ) hard core $^{13}$ internucleon potentials at several energies in the low and in the intermediate energy regions.

In subsection 5.2.1 we present the formulation for the exact calculation of the direct part of the spin-orbit potential. Here we also present our investigations concerning various series expansions. In particular we show that the first order term is a close approximation to the exact result while the second order term alone is quite sizeable. Subsection 5.2.2 gives the results of an approximate calculation to demonstrate the above mentioned result. In subsection 5.2.3 we present our results of the exact calculation for the direct part of spin-orbit potential and compare with those of older calculations discussed in chapter III and also described in ref. $^2$.

(5.2.1) Formal derivation:

The direct part of the nucleon-nucleus spin-orbit potential in the folding model approach $^2$ is
\[ U^D_{\text{so}}(r_4, E) = \sum \int \phi_n^*(r_2) t^D_{\text{so}} 1.s \phi_n(r_2) dr_2, \]  
\[ (5.2) \]

where \( t^D_{\text{so}} \) is the direct part of the nucleon-nucleon spin-orbit \( t \)-matrix. 1 and \( s \) are respectively the total orbital angular momentum and spin for the nucleon pair. \( \phi_n(r_2) \) is the bound single-particle wavefunction in the target nucleus and the label \( n \) represents the appropriate quantum numbers. The summation in eq. 5.2 is over all the occupied states. We use label 1 for the incident nucleon and label 2 for a typical nucleon in the target. For the product 1.s we take

\[ 1.s = \frac{1}{2}(r_4 - r_2) \times (p_{4_i} - p_{2_i}).(s_{4_i} + s_{2_i}), \]  
\[ (5.3) \]

where \( p_{4_i} \) and \( s_{4_i} \) refer to the momenta and spins of the respective particles. Changing the integration variable in eq. (5.2) to \( x = r_2 - r_4 \) we obtain

\[ U^D_{\text{so}}(r_4, E) = -\frac{1}{2} \int \rho(|r_4 + x|) t^D_{\text{so}} x \times (p_{4_i} - p_{2_i}).(s_{4_i} + s_{2_i}) dx, \]  
\[ (5.4) \]

where

\[ \rho(|r_4 + x|) = \sum \phi_n^*(r_4 + x) \phi_n(r_4 + x) \]  
\[ (5.5) \]

is the density distribution in the target nucleus. We consider
spin-zero nuclei. The integration over $p_\alpha$ vanishes identically, since no direction is specified, while the sum over the bound nucleon spin $s_\alpha$ is similarly zero. This gives us

$$U^D_{SO}(r_\alpha, E) = -\frac{1}{2} \int \rho(|r_\alpha + x|) t^D_{SO} x dx \times p_\alpha \cdot s_\alpha.$$  (5.6)

The integral in eq. (5.6) must be in the $r_\alpha$ direction. Taking the $z$-axis along $r_\alpha$, we finally obtain

$$U^D_{SO}(r_\alpha, E) = \frac{1}{2} A(r_\alpha) l_\alpha \cdot s_\alpha / r_\alpha.$$  (5.7)

where $l_\alpha = r_\alpha \times p_\alpha$ is the orbital angular momentum of the incident nucleon and

$$A(r_\alpha) r_\alpha / r_\alpha = \int \rho(|r_\alpha + x|) t^D_{SO} x dx.$$  (5.8)

For simplicity we assume that the distribution of protons and neutrons is same. Further we consider in this subsection only the real part of $t^D_{SO}$ and drop its dependence on variables other than the internucleon separation. Hence we shall drop all suffices on $t^D_{SO}$. We shall discuss the results of our calculation with full sophistications, in subsection 5.2.3. Eq. (5.7) is now of the familiar\(^2,10-11\) spin-orbit form. Our
remaining discussion of this subsection is based on eq. (5.8).

We can expand \( \rho(|r + x|) \) as a Taylor series in \( x \). This gives

\[
A(r) = 4\pi \sum_{\nu=0}^{\infty} \frac{(2\nu+2)}{(2\nu+3)!} I_{2\nu+4} \frac{d}{dr} \nu^{2\nu} \rho, \tag{5.9}
\]

where

\[
I_n = \int_{0}^{\infty} x^n t(x) \, dx. \tag{5.10}
\]

This is the series given by Greenlees et al.\(^{10,11}\). To make exact comparison we note that

\[
\nu^{2\nu} \rho = \frac{d^{2\nu} \rho}{dr^{2\nu}} + \frac{2\nu}{r} \frac{d^{2\nu-1} \rho}{dr^{2\nu-1}}. \tag{5.11}
\]

The first term of eq. (5.9) gives

\[
A(r) = \frac{4\pi}{3} I_4 \frac{d\rho}{dr}, \tag{5.12}
\]

which when combined with eq. (5.7), gives the results of Blin-Stoyle\(^9\), i.e. eq. (5.1). It is possible to obtain, in place of eq. (5.9), the formula
\[ A(r) = 4\pi \frac{d}{dr} \int_0^\infty \left[ \frac{i}{r} j_4(x) t(x) x^2 \right] dx \varphi. \] 

(5.13)

This is derived by methods similar to those in chapter IV. The quantity in bracket in eq. (5.13) is to be evaluated as a function of \( \nabla^2 \), with this operator acting solely on \( \varphi \). It is important to note for our further discussion that eqs. (5.9) and (5.13) are identical. This is obvious because no approximations are used in their derivation and can be readily verified by expanding Bessel function in eq. (5.13).

A plausible approximation to eq. (5.13) is obtained by replacing \( \nabla/i \) by \( k \) where the latter is a typical momentum (divided by \( h \)) characteristic of the nucleus. Then eq. (5.13) becomes

\[ A(r) = 4\pi \int_0^\infty \frac{1}{k} j_4(kx) t(x) x^2 dx \frac{d\varphi}{dr}. \] 

(5.14)

If \( kx \) is assumed to be small, we recover eq. (5.12) and hence eq. (5.1) for the spin-orbit potential. More generally, assuming \( k \) is independent of \( r \), eq. (5.14) together with eq. (5.7), yields a formula for the spin-orbit potential differing from the first order equation, eq. (5.1), only through a multiplying constant which is energy dependant. The constant in this case is related to the quantity in bracket in eq.
(5.14). This is the result obtained by Scheerbaum\textsuperscript{8}) by a somewhat different method. The essential part of the approximation leading from eq. (5.9) to that of Scheerbaum is that \(k\) is independent of \(r\). We have made a more detailed analysis and shown that this is not correct and that a more careful treatment returns the exact result, eq. (5.8).

In view of the above discussion we conclude that the methods employed by Greenlees et al.\textsuperscript{10,11} and Scheerbaum\textsuperscript{8}) are essentially equivalent and involve the expansion of the nucleon density assuming the short range nature of the internucleon spin-orbit \(t\)-matrix.

(5.2.2) Numerical discussion of eq. (5.9):

In order to investigate the validity of approximations discussed above we have done the following calculations. We have used the series of eq. (5.9) and the exact result of eq. (5.8) to evaluate \(A(r)\) and hence to obtain the spin-orbit potential, eq. (5.7). For \(\rho(r)\) we take the form

\[
\rho(r) = [1 + \exp((r-R)/a)]^{-4},
\]  

(5.15)

with \(a = 0.54\) \(F\) and \(R = 4.0\) \(F\). For the \(t\)-matrix, we take only the triplet odd part (which is the dominant contribution for the spin-orbit potential) the form suggested by Wong\textsuperscript{14,15})
\[ t(x) = 0 \quad \text{for } x < c \]  
\[ = v[1 - \exp(-\gamma(x-c))] \quad \text{for } x \geq c, \]  
(5.16a)  
(5.16b)

where \( c = 0.485 \text{ F}, \gamma = 4.2 \text{ F}^{-1} \). For \( v \) we take the triplet odd part of the Hamada-Johnston\(^{13}\) internucleon potential.

Fig. 1 shows the exact result, eq. (5.8), and the effect of only considering first and second order terms of the series expansion, eq. (5.9). We conclude that the exact and the first order term give very close results while the second order term alone is quite sizeable. This result is consistent with the result of Brieva and Rook\(^2\) who used a slightly different form for \( t \). Further this is also confirmed by our calculations as discussed in the next subsection. All of these results indicate that the use of this type of series expansion of \( \rho \) is rather dangerous and that the series convergence is highly unreliable.

(5.2.3) Results of exact calculation of direct part of spin-orbit potential:

Using eq. (5.7) we have calculated the direct part of the spin-orbit potential for the elastic scattering of protons from \(^{40}\text{Ca}\) in the low (21-80 MeV) and intermediate (135-800 MeV) energy regions. Urbana V14 soft core\(^{12}\) and Hamada-Johnston hard core\(^{13}\) internucleon potentials have been used to solve Bethe-Goldstone equation. For \(^{40}\text{Ca}\) we have used...
the nucleon density from ref. 16).

Fig. 2, curve labelled $V_n$, shows the calculated spin-orbit potential for $p^{^{40}}$Ca at 200 MeV. For comparison we also show, curve labelled $V_b$, the results of Brieva and Rook 2), i.e. using only the first term, i.e. eq. (5.12). We do not show the results at other energies since they are qualitatively similar. Moreover we have also done the calculations mentioned above using the hard core Hamada-Johnston 13) internucleon potential with similar results.

From fig. 2 we see that the use of the first order term alone underestimates the direct part of the spin-orbit potential at short distances and overestimates it at larger distances. These differences remain qualitatively similar throughout the energy region considered here. We have calculated the volume integrals of these potentials and found them very similar. Since the Brueckner theory calculations produce numerical t-matrices, there is no additional labour involved in the exact calculation of the direct part of the spin-orbit potential using, eq. (5.7). However, since our results indicate that the volume integrals of the exact and first order results are not very different, the nucleon-nucleus scattering and polarization data is not likely to distinguish between the two. In summarizing the results of this section we may say that we have outlined a method to calculate the direct part of the spin-orbit potential exactly.
without making any approximation as has been done till now \(^2, 8, 10-11\).
(5.3) Exchange part of the spin-orbit potential:

In this section we discuss various approximations used to calculate the exchange part of nucleon-nucleus spin-orbit potential. Our main objective in the present section is to avoid some of the approximations involved in the calculation of exchange part of spin-orbit potential.

In subsection 5.3.1 we give a formal derivation of the potential. Subsection 5.3.2 describes results of our calculations for the exchange part of the nucleon-nucleus spin-orbit potential for the scattering of protons from $^{40}$Ca in the low and in the intermediate energy regions. We also compare our results with the earlier calculations of Brieva and Rook.$^2$

(5.3.1) Formal derivation:

The local equivalent to the exchange part of the nucleon-nucleus spin-orbit potential as given in refs.$^{2,8}$ is

$$U_{eo}^{ex}(r_x, E)\Psi_{nljm}(1) = \sum_{n' l' j'} \int \phi_{n' l' j'}^{*}(2) t_{eo}^{ex}(2) \Psi_{nljm}(1) \Phi_{n' l' j'}(2) dr_x,$$

where the labels have the same meaning as that used in eq. (5.2) of section 5.2. The t-matrix term $t_{eo}^{ex}$ is the exchange part of the nucleon-nucleon spin-orbit t-matrix as defined in ref.$^2$ and we have suppressed its dependence on internucleon separation, nuclear matter density and energy of the incident
nucleon. The quantity $l.s$ is defined as in eq. (5.3). The function $\psi_{nlmj}(1)$ is the space-spin coupled product of the single nucleon coordinate space wavefunctions $\psi_{nlm_l}(r_4)$ and the two-component spin eigenfunctions $\chi_{am_a}(1)$. In these formulae $n$ is the principal quantum number, while $l$ and $m_l$ are respectively the total orbital angular quantum number and its z projection. The quantity $s$, $m_s$ and $j$, $m$ are defined for the spin and total angular momenta similarly to $l$ and $m_l$. We can write

$$\psi_{nljm}(1) = \sum_{m_l m_s} \frac{(l m l m_s | j m)}{2} \psi_{nlm_l}(r_4) \chi_{am_a}(1),$$  

where $(l m l m_s | j m)$ are the relevant Clebsch-Gordan coefficients. Similar expression may be written down for the bound state $\phi_{nljm}(2)$. Substituting these in eq. (5.17) and using the orthogonality relation for Clebsch-Gordon coefficients we obtain

$$U_{SO}^{EX}(r_4, E)\psi_{nljm}(1) = \sum_{n,l,m_l} \frac{\mu_l \mu_s | j m)}{2} \int \phi^*_{n,l,m_l}(r_4) \chi_{am_a}(2) \times \phi_{n,l,m_l}(r_4) \chi_{am_a}(1) \psi_{nlm_l}(r_4) \chi_{am_a}(2) dr_4.$$

$$\times t_{SO}^{EX}(r_4, \cdot \cdot \cdot) \phi_{n,l,m_l}(r_4) \chi_{am_a}(1) \psi_{nlm_l}(r_4) \chi_{am_a}(2) dr_4.$$  

Considering the product $l.s$, eq. (5.3), and only the
spin-orbit part of eq. (5.19), as in ref. 8, we obtain

$$\sum_{m_e} \chi_{m_e}^{+} (2)[\sigma_{+} + \sigma_{-}] \chi_{m_e} (1) \chi_{m_e}^{+} (2) = \sigma_{+} \chi_{m_e} (2),$$

(5.20)

where $\sigma_{+}$ denotes the Pauli spin vector. Changing the integration variable in eq. (5.19) to $x = r_{s} - r_{a}$, we can simplify the action of the momentum operators on the space part of the wavefunction $\psi(r_{s})$ as below

$$\mathbf{x} \cdot \nabla (p_{a} - p_{s}) \psi(r_{s}) = -2(-i\hbar \nabla_{x}) e^{\mathbf{x} \cdot \mathbf{r}_{a}} \psi(r_{s})$$

(5.21a)

$$= -2p_{a} e^{\mathbf{x} \cdot \mathbf{r}_{a}} \psi(r_{s}).$$

(5.21b)

where $\nabla_{x}$ and $\nabla_{r}$ are the gradient operators with respect to the coordinates $x$ and $r_{a}$ respectively. We have approximated $-i\hbar \nabla_{r}$ with the local momentum $p_{a}$ of the incident nucleon to obtain eq. (5.21b).

Substituting eqs. (5.20) and (5.21b) in eq. (5.19) and recombining the space and spin eigenfunctions to form the scattering state $\psi_{nljm}(1)$ for the incident nucleon we obtain

$$U_{ex}(r_{a}, E) = \frac{1}{2} \left[ \int \rho(|r_{a} + x|, r_{s}) e^{\mathbf{x} \cdot \mathbf{r}_{a}} e^{\mathbf{x} \cdot \mathbf{r}_{a}} \mathbf{x} \cdot \mathbf{r}_{a} \right] \times \mathbf{p}_{a} \cdot \sigma_{a},$$

(5.22)

where $\rho(r_{s}, r_{a})$ is the density matrix as defined in ref. 2.

We now denote by $D$ the integral in square bracket of eq.
We note that since \( D \) is a vector, it can be only a linear function of the vector \( r_4 \) and \( \nabla_4 \). The integral \( D \) depends also on higher order scalar combinations of \( r_4 \) and \( \nabla_4 \), which is of no concern for the calculation of spin-orbit term. Hence we can write

\[
D = \alpha r_4 + \beta \nabla_4, \quad (5.23)
\]

where \( \alpha \) and \( \beta \) must be scalar quantities.

The second term in eq. (5.23) is of no interest since it would not contribute to the spin-orbit potential. In order to determine \( \alpha \) we integrate eq. (5.23) over the angles of \( \nabla_4 \) and identify \( |\nabla_4/i| \) with the local momentum \( k_4 \) of the incident nucleon. This gives us

\[
\alpha = \frac{1}{r_4} \int \rho(|r_4 + x|, r_4) t_{\text{SO}}^{\text{EX}} x \cdot \hat{r}_4 \left( j_{\sigma}(k_4 x) \right) dx. \quad (5.24)
\]

Now choosing the z-axis along the direction of \( r_4 \) and using eqs. (5.23), and (5.24) in eq. (5.22) we finally obtain

\[
U_{\text{SO}}^{\text{EX}}(r_4, E) = \frac{1}{2 r_4} \int \rho(|r_4 + x|, r_4) t_{\text{SO}}^{\text{EX}} x \cos \theta \left( j_{\sigma}(k_4 x) \right) dx \cdot \hat{r}_4, \quad (5.25)
\]

where \( l_4 \) is the orbital angular momentum of the incident nucleon.

Eq. (5.25) is our expression for the exchange part of the
nucleon-nucleus spin-orbit potential. It is important to note that we have made no such approximation as in refs.\textsuperscript{2,8,10,11)}, e.g. the short range nature of the spin-orbit t-matrices. Thus we have presented a method to calculate the exchange part exactly. We can easily generalize eq. (5.25) for the case of incident protons and take into account the differences between neutron and proton matter densities as in ref.\textsuperscript{2).}

(5.3.2) Results of exact calculation of exchange part of spin-orbit potential:

This subsection describes the results of our calculation for the spin-orbit potential using eq. (5.25). We consider the scattering of protons from \textsuperscript{40}Ca in the low and in the intermediate energy regions. We have done calculations using both the Urbana V14 soft core\textsuperscript{12}) and the Hamada-Johnston hard core\textsuperscript{13}) internucleon potentials. However, we show results only for V14 since the use of the hard core potential does not give qualitatively different results. For the single particle mixed neutron density $\rho_n(r_{\Lambda}, r_{\Sigma})$ and proton density $\rho_p(r_{\Lambda}, r_{\Sigma})$ we have used the approximate expression, proposed by Negele and Vautherin\textsuperscript{17), that is eq. 24 of ref.\textsuperscript{1}). Brieva and Rook\textsuperscript{2}) started from eq. (5.17) and expanded the resulting mixed densities $\rho_p(r_{\Lambda}, r_{\Sigma})$ and $\rho_n(r_{\Lambda}, r_{\Sigma})$ as a power series in $x = r_{\Sigma} - r_{\Lambda}$. They retained the lowest non-zero order and used the mixed densities as defined above. We also give the results from this calculation in order to see the effect of the higher order terms implicitly included in our exact calculation but
Fig. 3 shows the results of our calculation of the exchange part of the spin-orbit potential for the scattering of protons from $^{40}$Ca at 30.3 MeV. We do not show the results at other energies since they are qualitatively similar, except that the exchange term becomes negligibly small with the increase in incident energy. The curves labelled $V_2^b$ and $V_2^r$ show respectively the results of Brieva and Rook, ref. 2), and those from eq. (5.25). The immediate conclusion is that the first term of the expansion, as used in ref. 2), is not valid; indeed the results from eq. (5.25) are about one half of those of ref. 2) in the nuclear surface region.

These results however require a rather more critical consideration. To obtain the curve labelled $V_2^r$ in fig. 3 a value of $k_*$ at each radius in the nucleus is required. In order to obtain the curve of fig. 3 we used

$$k_*^2 = \frac{2m(E + V)}{\hbar^2}, \quad (5.26)$$

where $m$ is the nucleon mass, $E$ is the incident nucleon energy and $V$ is the real part of the optical potential derived from a nuclear matter calculation, at the given nuclear density. This leads to possible errors and to investigate the importance of these we proceed as follows. The Bessel function of eq. (5.25) can be expanded as
\[ j_0(kx) \approx 1 - \frac{k^2x^2}{6} \]  
(5.27a)

\[ \approx 1 + \frac{x^2V^2}{6}. \]  
(5.27b)

Calculations using eq. (5.27a) are trivially easy while calculations using eq. (5.27b) can be made by incorporating the \( V^2 \) term into the kinetic energy term of the Schrodinger equation. This is just equivalent to self-consistent treatment of the potential \( V \) in eq. (5.26). In fig. 3 the curves labelled \( V \) and \( V \) refer respectively to the use of eqs. (5.27a) and (5.27b). We see that these curves differ very markedly indicating that a simple treatment of \( k \), as in the curve \( V \), fig. 3, is not fully adequate.

To obtain final answer we note that the curves \( V \) and \( V \) are fairly close. This suggests that a second order treatment of the values of \( k \) should be adequate. Thus our final answer for the exchange part is taken to be

\[ V^{\text{EX}} = V_n + V - V_n. \]  
(5.28)

where \( V_n \), for instance, is taken from the curve labelled \( V \) in fig. 3. This curve is shown in fig. 3 also. Comparing \( V^{\text{EX}} \) and \( V_b \) we see that an overall effect is a reduction of some 30% compared with the earlier results of ref. 2).
(5.4) Analysis of $^4\text{He}^{-40}\text{Ca}$ scattering data:

In this section we describe in detail the analysis of $p^{-40}\text{Ca}$ elastic scattering differential cross section and polarization data from $^{40}\text{Ca}$ at low and at intermediate energies using our calculated optical potential and compare with previous calculations. We use the Urbana V14 soft core interaction\textsuperscript{12}). Different components of the optical potential are calculated as follows. We calculate the direct part of the central potential as described in chapter III and also described in ref.\textsuperscript{1}). The exchange part of the central potential is calculated from eq. (4.17) as described in previous chapter IV. The direct part of the spin-orbit potential is calculated using eqs. (5.7) and (5.8), as discussed in subsection 5.2.1. The exchange part of the spin-orbit potential is obtained from eq. (5.28), as described in subsection 5.3.2. We have also taken into account the effective mass correction\textsuperscript{4)} in both the central and spin-orbit parts of the calculated potential. We denote this optical potential as $V_\text{o}$. The calculated potentials are used to predict the $p^{-40}\text{Ca}$ elastic scattering observables. The normalizations for the real and imaginary parts of the central potential are respectively $\lambda_C^R$ and $\lambda_C^I$, while $\lambda_{so}^R$ and $\lambda_{so}^I$ are the normalizations for the real and imaginary spin-orbit parts of the potential. We use $\lambda$ to denote the above mentioned four normalizations collectively. Values of $\lambda < 1$ denote that the
calculated potential is large while \( \lambda = 1 \) denotes agreement between the calculated and empirical values. For comparison we also show normalizations obtained in analysing the same data using the calculated potential denoted by \( V_b \), following the method of Brieva and Rook \(^2\)), using V14 as in refs. \(^4,5\)).

Though the calculations have been performed at about 18 energies in the low and intermediate energy regions, we show here the results obtained only at 30.3 MeV (as an energy representative of low energy region) and at 200 MeV (as an energy representative of intermediate energy region). Table-1 shows the normalizations obtained at 30.3 MeV and 200 MeV. The values of \( \chi^2 \) show that we are able to obtain fits to the data of similar quality with our potential \( V_n \) as compared with the older potential \( V_b \). However, we do not obtain any improvement.

Consider first the spin-orbit term. We see that the values of \( \lambda_{so}^p \) obtained for \( V_n \) are greater as compared with those obtained for \( V_b \) at both the incident energies, \( E_p = 30.3 \text{ MeV} \) and \( E_p = 200 \text{ MeV} \). This indicates that the real part of spin-orbit optical potential obtained from our exact calculation is smaller as compared with that obtained from the older calculation \(^4,5\)).

Turning now to the central part, the real part is not very different for \( V_n \) and \( V_b \). For the imaginary part we obtain very similar values of \( \lambda_C^x \). In particular we obtain a slight improvement in the values of \( \lambda_C^x \) at 30.3 MeV, but the
calculated potential is still too large. At 200 MeV values of of $\lambda^2$ for both $V_n$ and $V_b$ are closer to unity.

We have also performed the above analysis using calculated potential starting from HJ interaction. However, except for minor differences in the normalization parameters the results are qualitatively similar to those using V14 interaction.
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<tr>
<td>$\lambda_{so}^4$</td>
<td>-1.250</td>
<td>-0.717</td>
<td>0.545</td>
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<td>$\chi_T^2$</td>
<td>146.95</td>
<td>262.67</td>
<td>36.52</td>
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<tr>
<td>$\chi_{os}^2$</td>
<td>55.53</td>
<td>99.13</td>
<td>51.22</td>
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<td>$\chi_p^2$</td>
<td>364.74</td>
<td>652.27</td>
<td>21.80</td>
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<td>$\chi_{PDF}^2$</td>
<td>152.248</td>
<td>272.133</td>
<td>39.129</td>
</tr>
</tbody>
</table>
Figure captions:

Figure 1: The direct part of the calculated real spin-orbit potential, see subsection 5.2.2 for detail.

Figure 2: The direct part of the calculated real spin-orbit potential for $p-^{40}$Ca at 200 MeV. The labels $V_n$ and $V_o$ denote respectively the new and old calculations.

Figure 3: The calculated exchange part of the real spin-orbit potential for $p-^{40}$Ca at 30.3 MeV, see subsection 5.3.2 for detail.
Figure 1

Spin-orbit potential $\times 10$(MeV)

- Exact
- First order
- First+second order

R(fm)
FIGURE: 2

Spin-orbit potential (MeV)

$V_n \quad V_b$

$p^{-40}\text{Ca}$

$R$(fm)
FIGURE: 3
References:
9) R. J. Blin-Stoyle; Phil. Mag. 46(1955)973.


