Abstract

This thesis deals primarily with characterizations and Bayesian estimates of negative exponential and related distributions. It consists of chapters 0, 1, 2, 3 and 4 of which chapter 0 gives a brief outline of Bayesian point estimation. Chapters 1 and 2 are devoted to characterizations of distributions by properties of certain conditional expectations while chapters 3 and 4 report the results obtained on Bayesian estimation of some parametric functions in the Poisson process, two parameter negative exponential and the gamma distributions.

The main result of chapter 1 is a general characterization theorem which identifies those distribution functions for which the regression of a sample observation on the largest ordered observation is linear. This theorem yields some interesting characterizations of negative exponential, Pareto, beta, power function and Weibull distributions. A result of Tanis (1964) is improved by replacing the condition of independence of two statistics by the weaker condition of constant regression. The characterizations reported in chapter 1 have also appeared in Beg and Kirmani (1974).

Chapter 2 has three theorems of which the first two give characteristic properties of the negative exponential distribution while the third one is a general result proving the uniqueness of the conditional expectation \( E(X|X > x) \) for any random variable \( X \). Theorem 1 strengthens and generalizes a result of Galambos (1975) and Theorem 2 proves that a certain conditional variance is constant if and only if the distribution is negative exponential. Theorem 3 which, in fact, extends a result of Cox (1962) to random variables which are not necessarily positive, leads to characterizations of negative exponential, Pareto, geometric, and the "exponential structure" distributions of Laurent (1974).

Chapter 3 is a study of the Bayesian estimation of the scale and
shape parameters and the reliability function for a gamma distribution. Many of the results reported here are generalizations of Bhattacharya (1967). Both proper and improper priors are considered. Special attention is given to the conjugate family of priors and, in this case, Bayes risks of Bayesian and two competitive estimators of the scale parameter are obtained. Simple numerical computations give interesting insight into the superiority (under the assumptions made) of the Bayes estimator over the classical maximum likelihood (ML) cum minimum variance unbiased (MVU) estimator.

The first problem taken up for study in chapter 4 is the estimation of reliability for a Poisson process. Bayes estimator is obtained and its Bayes risk is calculated. Bayes risks of maximum likelihood and minimum variance unbiased estimators are also computed and compared. This comparison supplements the work of Zacks and Even (1966) and Gaver and Hoel (1970). Bayesian estimates of some parametric functions of the location and scale parameter negative exponential distribution are also obtained in chapter 4. The prior adopted is an improper one reflecting prior ignorance. The parametric functions considered include the p-th fractile and the distribution function. The Bayes estimators reduce to the ML and MVU estimators of Epstein and Sobel (1954) and Zacks (1971) in certain situations. Mean-squared errors of the estimators are obtained and compared. Comparison of the mean-squared errors of ML and MVU estimators bring out some important situations where the ML estimator is superior.