CHAPTER I

PREAMBLE TO THE LITERATURE OF TOPOLOGY

Topology originates from the Greek word “τόπος”, meaning place, and “λόγος” meaning study, thus topology amounts to the mathematical study of surfaces. Topology developed as a field of study out of geometry and set theory, through analysis of concepts as space, dimension, and transformation. There are various subfields in topology. Point-set topology deals with the foundational issues of topology and studies topological properties inherent to spaces which are invariant under homeomorphisms. Algebraic topology uses tools of algebra, especially group structures, to study topological spaces; it can be seen as a realization of categorical adjunctions between the categories of topological spaces and categories of groups. Geometric topology deals with mathematical objects called manifolds and embeddings into other manifolds. In a nutshell general topology lays the foundation for several areas of research in topology such as fuzzy topology, bitopology, ideal topology and digital topology and finds many applications in engineering problems, information systems, computational topology and mathematical sciences.

The recent years have witnessed a rich flourishing topology where many fundamental problems were solved and new avenues of research emerged, where in topological methods penetrated into many other domains of mathematics.

This thesis addresses the challenges of new types of sets namely $\tau_p^+g$-closed set, $b\Gamma^*$ open set and fuzzy $b\Gamma^*$ open set in the light of simple extension topology. This chapter partitions itself to explain the basic concepts used in this thesis. Section 1, explains the notion of simple extension topology. The second section deals with the weaker and stronger forms of open and closed sets. The basics of various types of continuity and irresoluteness is discussed in section 3. Section 4 brings out the concept of contra continuity in topological spaces. Closed(open) maps and homeomorphism are discussed in section 5. Section 6 and section 7 marches on with the insights of ideal and fuzzy topology. Section 8 outlines the contributions of the author. Throughout the thesis X, Y and Z denote topological spaces under simple extension, on which no separation axioms are assumed unless or otherwise explicitly mentioned.
1.1 SIMPLE EXTENSION TOPOLOGY

Hewitt [70] in 1943 constructed a topology $\tau'$ finer than $\tau$ on X using a subset $B \notin \tau$. $\tau'$ is generated by $\tau \cup \{B\}$. Levine [89], 1963 defined $\tau(B) = \{O \cup [(O' \cap B)] / O, O' \in \tau\}$ and called it simple expansion of $\tau$ by B. A.M. Kozae and M.S. Bakry [84] proved that the Hewitt’s extension and simple extension of Levine are equivalent. Further this concept was improved by researchers like Carlos Boges, A.M. Kozae in [26] and [85].

1.2 WEAKER AND STRONGER FORMS OF OPEN AND CLOSED SETS

Mashhour et al[104], Levine[88],[90], Njastad [131], Abd El-Monsef [1], Arya and Nour[13], Bhattacharyya and Lahiri[17], Palaniappan and Rao [132], Maki et al [100,103] Dontchev [41] and D. Andrijevic[9] have respectively introduced pre-open sets, semi-open sets and g-open sets, $\alpha$-open sets, $\beta$-open sets, generalized semi-open sets, semi-generalized open sets, regular generalized open sets, generalized pre-open sets, generalized semi pre-open sets and b-open sets which belong to the category of the weaker form of sets. The set was named as b-open and sp-open by El-Atik [56] and Doncthev and Przemski [41], respectively.

Cameron [27] has introduced regular semi-open sets. Nakaoka and Oda [112,113] introduced and studied minimal open and maximal open sets. We give the definitions of some of them which are used in our present study. Note that complements of the various types of closed (open) sets defined here are open (closed) sets of respective types.

DEFINITION 1.2.1

A subset $A$ of a space $(X, \tau)$ is called

(i) a semi-open set [88] if $A \subset \text{cl}(\text{int}(A))$ and a semi-closed set if $\text{int}(\text{cl}(A)) \subset A$.

(ii) a pre-open sets [104] if $A \subset \text{int}(\text{cl}(A))$ and a pre-closed set if $\text{cl}(\text{int}(A)) \subset A$.

(iii) a regular open set [132] if $A = \text{int}(\text{cl}(A))$ and regular closed set if $A = \text{cl}(\text{int}(A))$.

(iv) a regular semi-open set [27] if there exists a regular open set $U$ such that $U \subset A \subset \text{cl}(U)$. The complement of a regular semi-open set is a regular semi-closed set.

(v) an $\alpha$- open set [131] if $A \subset \text{int}(\text{cl}(\text{int}(A)))$ and an $\alpha$-closed set if $\text{cl}(\text{int}(\text{cl}(A))) \subset A$.

(vi) a $\beta$ - open set [1] if $A \subset \text{cl}(\text{int}(\text{cl}(A)))$ and $\beta$ - closed set if $\text{int}(\text{cl}(\text{int}(A))) \subset A$. 
The pre-interior of a subset $A \subseteq X$ (briefly $\text{pint}(A)$) is the union of all pre-open sets contained in $A$. The pre-closure of $A$ (denoted by $\text{pcl}(A)$) is the intersection of all pre-closed sets that contain $A$. Also $\text{pcl}(A) = A \cup \text{cl(int}(A))$ and $\text{pint}(A) = A \cap \text{int(cl}(A))$ as proved in Andrijevic [8]. The semi-closure of $A$ and semi-preclosure of $A$ are analogously defined.

**DEFINITION 1.2.2**

A subset $A$ of a space $(X, \tau)$ is called

(i) a generalized closed set (briefly g-closed) [90] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open.

(ii) a semi-generalized closed set (briefly sg-closed) [17] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open.

(iii) a generalized semi-closed set (briefly gs-closed) [13] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open.

(iv) a generalized $\alpha$-closed set (briefly $g\alpha$-closed) [101] if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$-open.

(v) a $\alpha$-generalized closed set (briefly $\alpha g$-closed) [101] if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open.

(vi) a regular generalized closed set (briefly rg-closed) [132] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open.

(vii) a generalized pre-closed set (briefly gp-closed) [129] if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open.

**DEFINITION 1.2.3**

A space $(X, \tau)$ is called

(i) a door space [38] if every subset of $X$ is either open or closed in $X$.

(ii) extremely disconnected [36] if the closure of each open subset of $X$ is open.

(iii) a submaximal space [37] if every dense subset of $X$ is open in $X$.

(iv) a $T_{1/2}$ space [47] if every $g$-closed subset of $X$ is closed in $X$.

**1.3 STRONG AND WEAK FORMS OF CONTINUOUS AND IRRESOLU TE MAPS**

This section deals with the strong and weak forms of continuous maps which were introduced by several mathematicians [10,15,32,41,91,119,133,147,153]. The strong
forms of continuous maps have been discussed by Noiri [127], Levine [87], Arya and Gupta [12] Munshi and Bassan [109] and Reilly and Vamanamurthy [140]. They have proposed strong continuous maps, strongly $\theta$-continuous maps, completely continuous maps, super continuous maps and clopen continuous maps. Noiri and Kang [126] introduced a new class of maps called almost strongly $\theta$-continuous maps which contains the class of all strongly $\theta$-continuous maps. Biswas[20], Fomin[59], Hussain [71], Noiri [123], Mashour et al [105], Abd El – Monsef et al [1], Tong [151] and Kar et al [81 ], U. Sengul [145] have introduced and investigated simple continuity, $\theta$-continuity, almost continuity, weak continuity, $\alpha$ continuity, $\beta$ continuity, semi-weak continuity and weak almost continuity and weakly semi-continuity, weakly $b$ continuity. Continuity using pre-open sets have been studied by various authors [41],[43],[60],[64],[157]. Balachandran et al [15] and Arockiarani et al [10] have defined and studied $g$-continuous maps and $rg$ continuous maps. Jain [75], Nour [130] and Veerakumar[157] introduced totally continuous functions, totally semi continuous and strongly continuous functions and totally pre-continuous functions and studied their properties.

Crossley and Hildebrand [31] introduced and studied irresolute maps which are stronger than semi-continuous maps and independent of continuous maps. Maio and Noiri [96], Cammaroto and Noiri [28], Maheshwari and Thakur [93], Faro [57], Dontchev et al [43], Balachandran et al [15], Devi et al [35] and Arockiarani[11] have introduced and studied quasi-irresolute and strongly irresolute maps, weak irresolute and $\theta$ irresolute maps, $\alpha$ irresolute maps, strongly $\alpha$ irresolute maps, $\delta g$ – irresolute maps, gc –irresolute maps, $\alpha g$ irresolute maps, $g \alpha$ irresolute maps and $rg$ irresolute maps respectively. Here we present some definitions useful for our later study.

**DEFINITION 1.3.1**

Let $f : (X, \tau) \rightarrow (Y,\sigma)$ be a map. Then $f$ is said to be

(i) **pre-continuous** [104] if $f^{-1}(V)$ is pre-closed in $X$ for every closed set $V$ of $Y$.

(ii) **semi-continuous** [88] if $f^{-1}(V)$ is semi-closed in $X$ for every closed set $V$ of $Y$.

(iii) **$g$-continuous** [15] if $f^{-1}(V)$ is $g$-closed in $X$ for every closed set $V$ of $Y$.

(iv) **$gp$-continuous** [11] if $f^{-1}(V)$ is $gp$-closed in $X$ for every closed set $V$ of $Y$.

(v) **gpr-continuous** [64] if $f^{-1}(V)$ is gpr-closed in $X$ for every closed set $V$ of $Y$.

(vi) **$\alpha$-continuous** [105] if $f^{-1}(V)$ is $\alpha$-closed in $X$ for every closed set $V$ of $Y$.

(vii) **$\beta$-continuous** [1] if $f^{-1}(V)$ is $\beta$-closed in $X$ for every closed set $V$ of $Y$. 

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(viii) regular-continuous [132] if \( f^{-1}(V) \) is regular-closed in \( X \) for every closed set \( V \) of \( Y \).

(ix) clopen-continuous [140] if for each \( x \in X \) and each neighbourhood \( V \) of \( f(x) \), there exists a clopen neighbourhood \( U \) of \( X \) such that \( f(U) \subseteq V \).

(x) totally-continuous [93] if the inverse image of each open subset of \( Y \) is a clopen subset of \( X \).

(xi) totally pre-continuous[157] if the inverse image of every open set of \( Y \) is a pre-clopen subset of \((X, \tau)\).

(xii) strongly pre-continuous[157] if the inverse image of every subset of \( Y \) is a pre-clopen subset of \((X, \tau)\).

(xiii) \( b \)-continuous[50] (or \( \gamma \)-continuous) if the inverse image of each open set in \( Y \) is \( b \)-open set in \( X \).

(xiv) almost b-continuous (briefly a.b.c.) [145] if for each \( x \in X \) and each \( V \in \text{RO}(Y) \) containing \( f(x) \), there exists \( U \in \text{BO}(X) \) containing \( x \) such that \( f(U) \subseteq V \).

(xv) weakly b-continuous (briefly w.b.c.) [146] if for each \( x \in X \) and each open set \( V \) in \( Y \) containing \( f(x) \), there exists \( U \in \text{BO}(X) \) containing \( x \) such that \( f(U) \subseteq \text{cl}(V) \).

**DEFINITION 1.3.2**

Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a map. Then \( f \) is said to be

(i) irresolute [31] if \( f^{-1}(V) \) is semi-open in \( X \) for every semi-open set \( V \) of \( Y \).

(ii) pre-irresolute [139] if \( f^{-1}(V) \) is pre-closed in \( X \) for every pre-closed set \( V \) of \( Y \).

(iii) gc-irresolute [15] if \( f^{-1}(V) \) is gc closed in \( X \) for every gc closed set \( V \) of \( Y \).

(iv) sg-irresolute [148] if \( f^{-1}(V) \) is sg closed in \( X \) for every sg closed set \( V \) of \( Y \).

(v) \( \alpha \)-g-irresolute [35] if \( f^{-1}(V) \) is \( \alpha \)-g closed in \( X \) for every \( \alpha \)-g closed set \( V \) of \( Y \).

(vi) \( \alpha \)-irresolute [93] if \( f^{-1}(V) \) is \( \alpha \)-closed in \( X \) for every \( \alpha \)-closed set \( V \) of \( Y \).

(vi) \( \beta \)-irresolute [1] if \( f^{-1}(V) \) is \( \beta \)-closed in \( X \) for every \( \beta \)-closed set \( V \) of \( Y \).

(vi) \( b \)-irresolute [50] if \( f^{-1}(V) \) is \( b \)-closed in \( X \) for every \( b \)-closed set \( V \) of \( Y \).

**1.4 OPEN (CLOSED) MAPS AND GENERALIZED HOMEOMORPHISMS**

Sen and Bhattacharyya [144] coined and studied pre-closed mapping. Malghan[98] introduced and investigated some properties of generalized closed maps. Biswas [20], Das [33] and Noiri [122], Mashhour er al [104,105], Mrsevic et al [111],
Crossley et al [31] and Arockiarani [11] have defined and studied semi-open maps and semi-closed maps, \( \alpha \)-open maps, pre-open maps and weak pre-open, \( \delta \)-open maps and \( \delta \)-closed maps, pre-semi-open maps and \( \delta g \)-closed maps and rg-closed maps respectively. Several researchers have generalized the notion of homeomorphisms in topological spaces [11,19,31], Biswas [20] and Crossley and Hilderbrand [31] introduced semi-homeomorphisms which are strictly weaker than homeomorphisms. Neubrunn [118] has proved that the semi-homeomorphisms of [31] need not imply the semi-homeomorphisms of [19]. Piotrowski [137] has proved the semi-homeomorphisms of [15] need not imply semi-homeomorphisms of [31]. Semi-generalized homeomorphisms and generalized semi-homeomorphisms, \( \alpha \)-homeomorphisms and \( \beta \)-homeomorphisms, somewhat homeomorphisms, \( g \)-homeomorphisms and gc-homeomorphisms and rg-homeomorphisms have been respectively discussed by Devi et al [34], Tadros and Abd Allah [149], Gentry and Hoyle [62] and Arockiarani [11]. The following definitions are used in our study.

**DEFINITION 1.4.1:**

Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a map, then \( f \) is said to be

(i) pre-closed [144] if \( f(V) \) is pre-closed in \( Y \) for every closed set \( V \) of \( X \).

(ii) semi-closed [123] if \( f(V) \) is semi-closed in \( Y \) for every closed set \( V \) of \( X \).

(iii) regular closed [132] if \( f(V) \) is regular closed in \( Y \) for every closed set \( V \) of \( X \).

(iv) \( g \)-closed [98] if \( f(V) \) is \( g \)-closed in \( Y \) for every closed set \( V \) of \( X \).

(v) \( \alpha \)-open [105] if \( f(V) \) is \( \alpha \)-open in \( Y \) for every open set \( V \) of \( X \).

(vi) \( \beta \)-open [1] if \( f(V) \) is \( \beta \)-open in \( Y \) for every open set \( V \) of \( X \).

**DEFINITION 1.4.2:**

i) A map \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be generalized homeomorphism [148] if \( f \) is \( g \)-continuous, \( g \)-open and bijective.

ii) A map \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be gc-homeomorphism[148] if \( f \) is \( g \)-continuous, gc-irresolute and its inverse is also gc-irresolute.

iii) A map \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be semi-homeomorphism (B) [20] if \( f \) is continuous, semi-open and bijective.

iv) A map \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be semi-homeomorphism (C.H) [31] if \( f \) is irresolute, pre-semi-open and bijective.
1.5 CONTRA CONTINUOUS FUNCTIONS

In 1996, Dontchev [43] proposed contra-continuous functions. Jafari and Noiri [72,73] introduced and studied contra-precontinuous functions. Nasef [116] devised and studied contra-\( \gamma \) continuous functions as well as introduced and investigated contra b-continuous functions. Ahmad Al-Omari, Mohd. Salmi and Md. Noorani [5] investigated some more properties of contra-b-continuous and defined almost contra-b-continuous functions. Caldas and Jafari [25] have coined and studied contra-\( \beta \)-continuous functions while Jafari and Noiri [72,73] have introduced and investigated the notions of contra-super continuity, contra-pre-continuity and contra-\( \alpha \)-continuity.

**Definition 1.5.1:**

A function \( f : X \to Y \) is called contra-continuous [43] (resp. contra-pre continuous [72], contra-\( \beta \)-continuous [25], contra-b-continuous [116]) if \( f^{-1}(V) \) closed (resp. pre-closed, \( \beta \)-closed, b-closed) in \( X \) for each open set \( V \) of \( Y \).

1.6 IDEAL TOPOLOGICAL SPACES

Ideals in topological spaces have been considered since 1930. This topic has won its importance by the paper of Vaidyanathaswamy [156]. It was the works of Newcomb [120], Rancin [138], Samuels [141], Erdal Ekici [55] and Hamlet and Jankovic [76,77,78,79,80] which motivated the research in applying topological ideals to generalize the most basic properties in general topology. In 1990, Jankovic and Hamlett [78] introduced the notion of I-open sets in ideal topological spaces. El-Monsef et al. [2] investigated I-open sets and I continuous functions. In 1996, Dontchev [45] introduced pre-I open sets and obtained its decomposition of I continuity. The notion of semi I open sets to obtain decomposition of continuity was introduced by Hatir and Noiri [66], [67]. In addition to this, Caskusu Guler and Aslim [21] have introduced the concept of bI sets and bI continuous functions and further research was done by Metin Akdag [108] on these sets. [155] contributes to contra-\( \alpha \)I continuous functions.

An ideal is defined as a non-empty collection \( I \) of subsets of \( X \) satisfying the following two conditions.

1. If \( A \in I \) and \( B \subset A \), then \( B \in I \).
2. If \( A \in I \) and \( B \in I \), then \( A \cup B \in I \).
An ideal topological space is a topological space \((X, \tau)\) with an ideal \(I\) on \(X\) and it is denoted by \((X, \tau, I)\). For a subset \(A \subset X\), \(A^*(I) = \{ x \in X: U \cap A \notin I \text{ for each neighbourhood } U \text{ of } x \}\) is called the local function of \(A\) with respect to \(I\) and \(\tau\). We simply write \(A^*\) instead of \(A^*(I)\) to be brief. For every ideal topological space \((X, \tau, I)\), there exists a topology \(\tau^*(I)\), finer than \(\tau\). \(X^*\) is often a proper subset of \(X\). It is well-known that \(\text{Cl}^*(A) = A \cup A^*\) defines a Kuratowski closure operator for \(\tau^*(I)\) which is finer than \(\tau\). A subset \(A\) of \((X, \tau, I)\) is called \(\tau^*\)-closed if \(A^* \subseteq A\).

**DEFINITION 1.6.1:**

A subset \(A\) of an ideal topological space \((X, \tau, I)\) is said to be

(i) I-open [76] if \(A \subset \text{int}(A^*)\).

(ii) \(\alpha I\) open [76] if \(A \subseteq \text{int}(\text{cl}^*(\text{int}(A)))\).

(iii) Pre-I-open [40] if \(A \subseteq \text{int}(\text{cl}^*(A))\).

(iv) Semi I-open [66] if \(A \subseteq \text{cl}^*(\text{int}(A))\).

(v) \(bI\) open [21] if \(A \subseteq \text{int}(\text{cl}^*(A)) \cup \text{cl}^*(\text{int}(A))\).

(vi) \(\beta I\) open [66] if \(A \subseteq \text{cl}^*(\text{int}(\text{cl}^*(A)))\).

(vii) almost-I-open [3] if \(A \subset \text{cl}(\text{int}(A^*))\).

**DEFINITION 1.6.2:**

A function \(f : (X, \tau, I) \rightarrow (Y, \sigma)\) is said to be almost I continuous [3] if \(f^{-1}(V)\) is almost I open in \((X, \tau, I)\) for each open set \(V\) of \((Y, \sigma)\).

**DEFINITION 1.6.3:**

A function \(f : (X, \tau, I) \rightarrow (Y, \sigma)\) is called bI-continuous[21] if the inverse image of each open set in \(Y\) is bI open set in \(X\).

**DEFINITION 1.6.4:**

A function \(f : (X, \tau, I) \rightarrow (Y, \sigma)\) is said to be \(\beta I\) continuous [66] if \(f^{-1}(V)\) is \(\beta I\) open in \((X, \tau, I)\) for each open set \(V\) of \((Y, \sigma)\).

**LEMMA 1.6.5:**[76]

Let \((X, \tau, I)\) be an ideal topological space and \(A, B\) be subsets of \(X\) such that \(B \subset A\). Then \(B^*(\tau|A, I|A) = B^*(\tau, I) \cap A\).
**DEFINITION 1.6.6:** [66]
A function $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is said to be $\beta$I-irresolute if $f^{-1}(V)$ is $\beta$I-open for every $\beta$J-open set $V$ of $(Y, \sigma, J)$.

### 1.7 FUZZY TOPOLOGICAL SPACES


Mahmoud [95], Sarkar [143] defined a nonempty collection of fuzzy sets $I$ of a set $X$ as a fuzzy ideal on $X$ if and only if

1. $A \in I$ and $B \leq A$, then $B \in I$ (heredity),
2. if $A \in I$ and $B \in I$, then $A \vee B \in I$ (finite additivity).

The triple $(X, \tau, I)$ means fuzzy ideal topological space with a fuzzy ideal $I$ and fuzzy topology $\tau$. Sarkar [144] in 1997 defined that for $(X, \tau, I)$ the fuzzy local function of $A \leq X$ with respect to $\tau$ and $I$ is denoted by $A^* (\tau, I)$ (briefly $A^*$). The fuzzy local function $A^* (\tau, I)$ of $A$ is the union of all fuzzy points $x_\lambda$ such that if $U \in N_q (x_\lambda)$ and $E \in I$ then there is at least one $y \in X$ for which $U(y) + A(y) - 1 > E(y)$. Fuzzy closure operator of a fuzzy set $A$ in $(X, \tau, I)$ is defined as $Cl^* (A) = A \vee A^*$. In $(X, \tau, I)$, the collection $\tau^*$ ($I$) means an extension of fuzzy topological space than $\tau$ via fuzzy ideal which is constructed by considering the class $\beta = \{ U,E : U \in \tau, E \in I \}$ as a base.
**DEFINITION 1.7.1:**

A fuzzy subset $A$ of a fuzzy ideal topological space $(X, \tau, I)$ is said to be

(i) fuzzy $I$-open set [115], if $A \leq \text{Int}(A^*)$.

(ii) fuzzy $\alpha$-$I$-open set [160], if $A \leq \text{Int} (\text{Cl}^* (\text{Int}(A)))$.

(iii) fuzzy pre-$I$-open set [117], if $A \leq \text{Int} (\text{Cl}^* (A))$.

(iv) fuzzy semi-$I$-open set [69], if $A \leq \text{Cl}^* (\text{Int}(A))$.

(v) fuzzy $\beta I$-open set [160], if $A \leq \text{Cl} (\text{Int} (\text{Cl}^* (A)))$.

(vi) fuzzy $\beta I$-open set [161], if $A \leq \text{Cl}^* (\text{Int}(A)) \lor \text{Int}(\text{Cl}^* (A))$.

The family of all fuzzy $I$-open (resp. fuzzy $\alpha$-$I$-open, fuzzy pre-$I$-open, fuzzy semi-$I$-open, fuzzy $\beta$-$I$-open) sets is denoted by $\text{FIO}(X)$ (resp. $\text{F} \alpha \text{IO}(X)$, $\text{FP} \text{IO}(X)$, $\text{FS} \text{IO}(X)$, $\text{F} \beta \text{IO}(X)$). The complement of a fuzzy $I$-open set (resp. fuzzy $\alpha$-$I$-open set, fuzzy pre-$I$-open set, fuzzy semi-$I$-open set, fuzzy $\beta$-$I$-open set) is said to be fuzzy $I$-closed set (resp. fuzzy $\alpha$-$I$ closed set, fuzzy pre-$I$-closed set, fuzzy semi-$I$-closed set, fuzzy $\beta$-$I$-closed set).

**DEFINITION 1.7.2:**

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called fuzzy $b$ continuous[16] if the inverse image of each fuzzy open set in $Y$ is fuzzy $b$ open in $(X, \tau)$.

**DEFINITION 1.7.3:**

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called fuzzy $b$ irresolute[16] if the inverse image of each fuzzy $b$ open set of $Y$ is fuzzy $b$ open in $(X, \tau)$.

**DEFINITION 1.7.4:**

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called fuzzy $b$ open[16] (resp. fuzzy $b$ closed) if the image of each open (resp. closed) set of $X$ is fuzzy $b$ open (resp. fuzzy $b$ closed) set in $(Y, \sigma)$.

**DEFINITION 1.7.5:**

A function $f : (X, \tau, I) \rightarrow (Y, \sigma)$ is called fuzzy $b$-$I$-continuous [161] if the inverse image of each fuzzy open set in $Y$ is fuzzy $b$-$I$-open in $(X, \tau, I)$.
1.8 CONTRIBUTIONS OF THE AUTHOR

In the elucidation of the above depiction, the author has some significant results on the following topics.

1. Generalization of pre-open sets in extended topology.
2. Extended generalized mappings.
3. Various continuities via $bl^*$ open sets.
5. Weaker and Stronger forms of $bl^*$ open sets in fuzzy setting.