CHAPTER V

INVENTORY MODELS USING MONEY INFLATION

5.1 REPLENISHMENT POLICY FOR INVENTORY MODEL WITH MONEY INFLATION AND TIME VALUE OF MONEY

INTRODUCTION

Inflation is an economic condition wherein the price of the goods and services increase steadily measured against standard level of purchasing power, whereas the supply of the goods and services decline along with the devaluation of money.

The definition of inflation undergone lot of changes since 1983 when it appeared in the dictionary for the first time. At that time inflation was thought of as a cause but as time passed by the definition and its significance changed. Economists from different schools differ in their opinion regarding the genesis of inflation. However, it is agreed that inflation occurs due to an unexpected rise in the supply of money which causes devaluation or a decrease in the supply of goods and services. Again, the inflation rate decreases with the increase in the production of goods and with the decrease in the supply of money in the market.

CAUSES OF INFLATION

The following factors can lead to inflation:

- Increases in production costs
- Tax rises
- Declines in exchange rates
- Decreases in the availability of limited resources such as food or oil
- War or other events causing instability.
In Inventory management, studying replenishment policy is a major topic in recent issues. In all the inventory models, the inflation and time value of money were disregarded. It has happened most because of the belief that the inflation and the time value of money would not influence the inventory policy to any significant degree. Hence, while determining the optimal inventory policy, the effects of inflation and time value of money cannot be ignored. Because, in recent marketing trend, the behavior of customers or the waiting nature of customers are decreasing. Hence to avoid missing of regular customers and facing the competition, we consider the effect of inflation and time value of money. Hence finding out the replenishment policy is a hot topic of research in inventory management.

Wu et. al., [94] have developed a replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. They have not considered the effect of inflation and time-value of money. Generally in deterministic inventory models, the demand is either uniform or time dependent and has nothing to do with the status of inventory at hand. Uthayakumar and Geetha [89] have proposed an optimal replenishment policy by considering stock dependent consumption rate for non instantaneous deteriorating items with money inflation and time discounting, in this shortages are allowed and partially backlogged.

We have proposed an optimal replenishment policy by considering time dependent consumption rate for non instantaneous deteriorating items with money inflation with backlogging. Here we have considered the time value of money on replenishment policy under instantaneous replenishment with zero lead-time with reserve inventory. The main aim is to minimize the retailer’s total inventory cost by considering the effect of inflation and time value of money. Thus this model incorporates some realistic features that are likely to be associated with some kinds of inventory. This model is very useful in retail business. Numerical examples are
presented to demonstrate the development of the model. In addition sensitivity analysis of the optimal solution is carried out.

**Assumptions and notations**

- The replenishment rate is infinite and lead time is zero.
- \( t_i \) is the length of time in which the inventory has no shortage.
- \( T \) is the length of order cycle.
- \( Q \) is the order quantity per cycle.
- \( t_i, T \) and \( Q \) are decision variables.
- The demand rate \( D(t) \) at time \( t \) is
  \[
  D(t) = \begin{cases} 
  ae^{bt}, & I(t) > 0 \\
  a I(t) \leq 0 
  \end{cases}
  \]
  where \( a \) and \( b \) are positive constants, \( a > 0 \) and \( 0 \leq b \leq 1 \).

- Shortages are allowed.

- \( t_d \) is the length of time in which the product has no deterioration. After this period a constant fraction \( \theta \) (\( 0 < \theta < 1 \)) of the on-hand inventory deteriorates and there is no repair or replacement of the deteriorated units.

- \( t_d \) and \( \theta \) are given constants.

- \( A, h, d, S, \pi \) denote the ordering cost per order, inventory holding cost per unit time, deteriorating cost per unit, the reserve inventory cost per unit and unit cost of lost sales respectively. All the cost parameters are positive constants.

- \( I_i(t) \) denotes the inventory level at time \( t \) (\( 0 \leq t \leq t_d \)) in which the product has no deterioration.
• $I_2(t)$ is the inventory level at time $t$ ($t_d \leq t \leq t_i$) in which the product has deterioration.

• $I_3(t)$ denotes the inventory level at time $t$ ($t_i \leq t \leq T$) in which the product has shortage.

• $TC(t_1, T)$ is the present value of total relevant inventory cost per unit time of inventory system.

• $r$ denotes the discount rate, representing the time value of money and $i$ denotes the inflation rate, $R = r - i$ represents the net discount rate of inflation and it is a constant.

**MODEL FORMULATION**

The inventory system of the proposed model is described as follows:

$I_m$ units of item arrive at the inventory system at the beginning of each cycle. During the time interval $[0, t_d]$ the inventory level is decreasing only owing to time-dependent demand rate. The inventory level is dropping to zero due to demand and deterioration during the time interval $[t_d, t_i]$. Then the shortage interval keeps to the end of the current order cycle. The whole process is repeated.

The inventory system at any time $t$ can be represented by the following differential equations:

$$\frac{dI_1(t)}{dt} = -ae^{bt} \quad 0 \leq t \leq t_d \quad (5.1.1)$$

$$\frac{dI_2(t)}{dt} + \theta I_3(t) = -ae^{bt} \quad t_d \leq t \leq t_i \quad (5.1.2)$$
\[
\frac{dI_3(t)}{dt} = -a\delta \quad \text{for} \quad t_1 \leq t \leq T
\]  
(5.1.3)

\[I_m\]

\[I_d\]

\[t_1\]

\[t_1\]

\[t_d\]

\[T\]

**Fig. 5.1:** Graphical representation of Inventory system

The solutions of the above differential equations after applying the boundary conditions \( I_1(0) = I_m; \quad I_2(t_1) = 0; \quad I_3(t_1) = 0 \) are,

\[
\frac{dI_1(t)}{dt} = -ae^{bt} \quad 0 \leq t \leq t_d
\]

\[I_1(t) = -\frac{a}{b}e^{bt} + I_m + \frac{a}{b} \]

ie., \( I_1(t) = I_m + \frac{a}{b}[1 - e^{bt}] \)  
(5.1.4)

\[
\frac{dI_2(t)}{dt} + \theta I_3(t) = -ae^{bt}, \quad t_d \leq t \leq t_1
\]

ie., \( I_2(t) = \frac{ae^{-\beta t}}{(\theta + b)}[e^{(\theta+b)(t_1-t)}] \)  
(5.1.5)
\[ \frac{dI_3(t)}{dt} = -a \quad \text{for} \quad t_1 \leq t \leq T \]

ie., \[ I_3(t) = -a(t - t_1) \quad \text{(5.1.6)} \]

Because of the continuity of \( I(t) \) at \( t = t_d \) it follows that \( I_1(t_d) = I_2(t_d) \) which implies that the maximum inventory level for each cycle is,

\[ I_m + \frac{a}{b} e^{b t_d} = \frac{a}{(\theta + b)} e^{-\theta t_d} [e^{(\theta + b)(t_1 - t_d)}] \]

\[ I_m = \frac{a}{(\theta + b)} [e^{(\theta + b)(t_1 - t_d)}] e^{-\theta t_d} + \frac{a}{b} [e^{b t_d} - 1] \quad \text{(5.1.7)} \]

Using (5.1.7) in (5.1.4), we get,

\[ I_1(t) = \frac{a}{(\theta + b)} [e^{(\theta + b)(t_1 - t_d)}] e^{-\theta t_d} + \frac{a}{b} [e^{b t_d} - 1] + \frac{a}{b} [1 - e^{b r}] \]

\[ I_1(t) = \frac{a}{(\theta + b)} [e^{(\theta + b)(t_1 - t_d)}] e^{-\theta t_d} + \frac{a}{b} [e^{b t_d} - e^{b r}] \quad \text{(5.1.8)} \]

The maximum demand of reserve inventory per cycle is given by,

\[ I_s = I_3(T) = -\delta a(T - t_1) \quad \text{(5.1.9)} \]

From equations (5.1.7) and (5.1.9), we obtain the order quantity \( Q \) as,

\[ Q = I_m + I_s \]
= \frac{a}{(\theta + b)} [e^{(\theta+b)(t_1-t_d)}] e^{-\theta t_d} + \frac{a}{b} [e^{b t_d} - 1] - a(T-t_1)

Replenishment is done at the start of the cycle. The present value of the ordering cost per cycle is given by, \( C_r = A \)

The present value of inventory holding cost per cycle is given by,

\[
C_h = h \left[ I_1(t_1)e^{-Rt} dt + h \int_{t_d}^{t_1} I_2(t)e^{-Rt} dt \right]
\]

\[
= h \int_{t_d}^{t_1} \left[ \frac{a}{(\theta + b)} [e^{(\theta+b)(t_1-t_d)}] e^{-\theta t_d} + \frac{a}{b} [e^{b t_d} - e^{b t}] \right] e^{-Rt} dt +
\]

\[
= h a \int_{t_d}^{t_1} \frac{e^{-\theta t}}{(\theta + b)} [e^{(\theta+b)(t_1-t)}] e^{-Rt} dt
\]

\[
= \frac{h a}{(\theta + b)R} \int_{t_d}^{t_1} [e^{\theta t_d} - e^{-(\theta+R)t_d}] + \frac{h a}{R b} [e^{(b-R)t_d} - 1]
\]

\[
+ \frac{h a}{b(\theta - R)} [1 - e^{(b-R)t_d}]
\]

The present value of deterioration cost per cycle is given by,

\[
C_d = d \int_{t_d}^{t_1} \theta I_2(t)e^{-Rt} dt
\]

\[
= d \int_{t_d}^{t_1} \theta \frac{ae^{-\theta t}}{(\theta + b)} [e^{(\theta+b)(t_1-t)}] e^{-Rt} dt
\]

\[
= \frac{d \theta a}{(\theta + b)} e^{(\theta+b)t_d} \int_{t_d}^{t_1} e^{-(b+2\theta+R)t} dt
\]

\[
= \frac{d \theta a}{(\theta + b)(b+R+2\theta)} e^{(\theta+b)t_d} [e^{-(b+2\theta+R)t_d} - e^{-(b+2\theta+R)t_1}] \]
The present value of shortage cost per cycle due to backlogging is given by,
\[ C_S = S \int_{t_i}^{T} -I_s(t) e^{-RT} \, dt \]
\[ = S \int_{t_i}^{T} -a(t-t_i) e^{-RT} \, dt \]
\[ = Sa \left[ \frac{Te^{-RT}}{R} + \frac{e^{-RT}}{R^2} - \frac{e^{-Rt_i}}{R^2} + \frac{t_1 e^{-RT}}{R} \right] \]
\[ = \frac{S \delta a}{R^2} \left[ e^{-Rt_i} - e^{-RT} \left[ R(T-t_i) + 1 \right] \right] \]

The present value of opportunity cost per cycle due to lost sales is given by,
\[ C_0 = \pi \int_{t_i}^{T} a(1-\delta) e^{-RT} \, dt \]
\[ = \frac{\pi a(1-\delta)}{R} \left[ e^{-RT} - e^{-Rt_i} \right] \]

The present value of purchase cost per cycle is given by,
\[ C_p = pl_m + pe^{-RT} I_s \]
\[ = p \left[ \frac{a}{(\theta+b)} e^{(\theta+b)(T-t_i)} e^{-\theta t_i} + \frac{a}{b} e^{b t_i} - 1 \right] + e^{-RT} \left[ \delta a(T-t_i) \right] \]

Therefore the present value of total inventory cost per unit time is given by,
\[ TC(t_i,T) = \left( C_r + C_h + C_d + C_s + C_o + C_p \right) / T \]
\[ = \frac{A}{T} \left\{ \frac{h}{a} e^{(\theta+b)(T-t_i)} \left[ e^{\theta t_i} - e^{-(\theta+R)t_i} \right] + \frac{h}{Rb} [e^{(b-R)t_i} - 1] \right\} \]
\[ + \frac{h}{b(\theta-R)} [1-e^{(b-R)t_i}] + \frac{h e^{(\theta+b)t_i}}{(\theta+b)(2\theta+b+R)} [e^{(b+R+2\theta)t_i} - e^{-(b+R+2\theta)t_i}] \]
For minimizing total average cost per unit time, the optimal values of $t_i$ and $T$ can be obtained by solving the equations,

\[
\frac{\partial TC(t_i, T)}{\partial t_i} = 0 \quad \text{and} \quad \frac{\partial TC(t_i, T)}{\partial T} = 0
\]

ie., \[
\frac{\partial TC(t_i, T)}{\partial t_i} = \frac{a}{T} \left\{ \frac{h}{R} \left[ e^{(\theta+b)(1-\delta)} e^{\theta t_i} - e^{(\theta+b)(1-\delta)} e^{(\theta+R)t_i} \right] + \frac{h}{(\theta+b)(2\theta+b+R)} \left[ e^{(b+R+2\theta)t_i} e^{(\theta+b) t_i} (\theta + b) + e^{(b+R+2\theta)t_i} (b + R + 2\theta) \right] + \frac{d\theta}{(\theta+b)(2\theta+b+R)} \left[ e^{(\theta+b) t_i} e^{-(b+R+2\theta)t_j} (\theta + b) + e^{-(R+\theta)t_j} (\theta + R) \right] + \frac{SD}{R} [e^{-RT} - e^{-Rt_i}] + \pi(1-\delta) e^{-Rt_i} + p [e^{(\theta+b) t_i} e^{-\theta t_i} - e^{-(b+R)t_j} e^{(\theta+R)t_j}] \right\} = 0
\]

\[
\frac{\partial TC(t_i, T)}{\partial T} = -\frac{a}{T^2} \left\{ \frac{h}{R} \left[ e^{(\theta+b)(1-\delta)} e^{\theta t_i} - e^{(\theta+b)(1-\delta)} e^{(\theta+R)t_i} \right] + \frac{h}{(\theta+b)(2\theta+b+R)} \left[ e^{(b+R+2\theta)t_i} e^{(\theta+b) t_i} (\theta + b) + e^{(b+R+2\theta)t_i} (b + R + 2\theta) \right] + \frac{d\theta}{(\theta+b)(2\theta+b+R)} \left[ e^{(\theta+b) t_i} e^{-(b+R+2\theta)t_j} (\theta + b) + e^{-(R+\theta)t_j} (\theta + R) \right] + \frac{SD}{R} [e^{-RT} - e^{-Rt_i}] + \pi(1-\delta) e^{-Rt_i} + p [e^{(\theta+b) t_i} e^{-\theta t_i} - e^{-(b+R)t_j} e^{(\theta+R)t_j}] \right\} = 0
\]
\[
\frac{h}{(\theta + b)(2\theta + b + R)} \left[ e^{(\theta + b)\theta t} e^{(\theta + b)t_0} (\theta + b) + e^{-((R + 2\theta)t)} (b + R + 2\theta) \right] + \\
\frac{d\theta}{(\theta + b)(2\theta + R + b)} \left[ e^{(\theta + b)\theta t} e^{-((b + R + 2\theta)t)} (\theta + b) \right]
\]

\[
+ e^{-(R + \theta)t} (\theta + R) + p \left[ e^{(\theta + b)(t_0 - t_0)} e^{-\theta t_0} \right] \right] +
\]

\[
\frac{a}{T} \left\{ e^{-RT} S \delta (T - t_1) + e^{-RT} \pi (1 - \delta) + p \delta e^{-RT} [1 - R(T - t_1)] \right\} = 0
\]

Since the second order partial derivatives are non-negative, we concluded that the sufficient condition of optimality is satisfied.

**NUMERICAL EXAMPLE**

**Example 1.**

Let us consider the effect of changes in various parameters on the replenishment policy.

Ordering cost Inventory holding cost \( A = 250 \) per order
Holding cost \( h = 0.5 \) per unit time
Deteriorating cost \( d = 1.5 \) per unit
Reserve inventory cost \( S = 2.5 \)
Unit cost of lost sales \( \pi = 2 \)
Purchase cost \( p = 4 \)
Demand rate \( a = 1000 \) and \( b = 0.2 \)
Fraction of on-hand inventory \( \theta = 0.08 \)
Rate of inflation $R = 0.1$

Fresh product time $t_d = 0.0833$

Backlogging parameter $\delta = 0.56$.

**SENSITIVITY ANALYSIS**

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Example 2.

Let us consider the effect of changes in various parameters on the replenishment policy.

Ordering cost Inventory holding cost $A = 250$ per order

Holding cost $h = 0.5$ per unit time

Deteriorating cost $d = 1.5$ per unit

Reserve inventory cost $s = 2.5$

Unit cost of lost sales $\pi = 2$

Purchase cost $p = 4$

Demand rate $a = 600$ and $b = 0.1$

Fraction of on-hand inventory $\theta = 0.08$

Rate of inflation $R = 0.1$

Fresh product time $t_d = 0.0833$

Backlogging parameter $\delta = 0.56$.

**SENSITIVITY ANALYSIS**

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<th>Percentage change in parameter</th>
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The change in the values of parameters may happen due to uncertainties in any decision-making situation. In order to examine the implications of these changes, the sensitivity analysis will be of great help in decision making. The results of sensitivity analysis are summarized below:

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<th>$d$</th>
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The ordering quantity $Q$ and the present value of total cost $TC$ will decrease or increase as the ordering cost $A$ decrease or increase. That is changes in $A$ will lead to the positive changes in $Q$ and $TC$.

When the backlogging parameter $\delta$ and the deterioration rate $\theta$ increases the ordering quantity $Q$ and then present value of the total cost $TC$ increases. That is change in $d$ and $\theta$ lead to a positive change in $Q$ and $TC$.

The change in the deterioration cost $d$ leads to a positive change on the ordering quantity $Q$. Increasing the fresh product time $t_d$ increases the order quantity $Q$ and the total cost $TC$. 

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The effect on inflation and time value of money on the total cost is significant. When the net discount rate of inflation R is increasing, the optimal cost is decreasing.

Changing in carrying cost h does not have any significant effect in the present value of the total cost and it results in a negative change in the ordering quantity Q.

Changes in shortage cost s and the purchase cost p result in a positive change in the present value of total cost TC and a negative change in the ordering quantity Q.

When the opportunity cost \( \pi \) increases, the ordering quantity Q and the present value of total cost TC increases.

CONCLUSION

From the above discussion, a deterministic inventory model has been framed for non-instantaneous deteriorating items with time-dependent consumption rate, shortages are allowed and partially backlogged. The main aim is to minimize the retailers total inventory cost by considering the effect of inflation and time value of money. Sensitivity analysis with respect to various parameters have been carried out. The result implied that, the effect of inflation and time value of money on the present value of total cost is more significant and highlights that the total cost decreases as the inflation rate increases.
5.2 RANDOM ARRIVAL OF SHIPMENTS WITH MONEY INFLATION

A typical inventory situation is considered. Shipments arrive according to a Poisson process with inter arrival time distributed as follows:

\[ f(\tau) = \frac{\lambda}{\lambda} e^{-\lambda \tau}, \quad \tau \geq 0, \]

where \( \frac{1}{\lambda} \) is the expected inter arrival time between consecutive shipments.

Let \( S \) be the predetermined maximum inventory or capacity level. Every time a shipment arrives, an order of size \( Q \) is placed so that the stock on hand becomes \( S \). The arriving shipments are assumed to be sufficient to fulfill the orders, therefore, immediate replenishment is necessary as soon as shipment arrives. \( Q \) is a random variable which depends on the demand between successive arrivals. The demand (\( D \)) per unit time is assumed to be constant throughout the planning horizon. If the demand between arrival of two consecutive shipments is less than \( S \), there is a surplus of inventory at the end of the cycle. The cost of procurement is \( C_p \) per unit and is assumed to be independent of \( Q \). Let the items be sold at \( mC_p \), so that the profit margin per unit sold be \((m-1)C_p\). The cost of ordering (\( C_o \)) is constant and is ignored because it is incurred every time a shipment arrives. A cost of \( C_h \) per unit is incurred for every time an item is carried as inventory.

In traditional inventory models where there are no lost sales, demand is completely satisfied and therefore the minimization of total inventory related costs becomes a valid objective. However in the present case, both the procurement costs and revenue incurred out of demand depend on the order level \( S \), rather than the total demand. The order level \( S \) in turn depends on the fraction of demand actually fulfilled. The objective in this situation is the maximization of net revenue rather than the minimization of total costs. The problem is to find the order level \( S \) that maximizes the total net revenue by satisfying a fraction of the demand between successive arrivals.
Assumptions and notations

$C_1$ - Net revenue generated due to satisfied demand.

$C_2$ - Cost of holding inventory.

$C_3$ - Backordering cost.

$C_4$ - Lost sales cost.

Let $r$ denotes the discount rate, representing the time value of money and $i$ denotes the inflation rate. $R = r-i$ represents the net discount rate of inflation and it is a constant. The inflation rate is $e^{-Rt}$.

Therefore the total expected net revenue per cycle is $E = C_1 - C_2 - C_3 - C_4$, where $S \geq D\tau$ if the procurement (in next period) is $D\tau$ and if $S \leq D\tau$ if the procurement is $S + \beta(D\tau - S)$. Maximum level of backordering is calculated as $(D\tau - S)\beta$. 
If we ignore the cost of ordering, the net revenue is considered to be the difference in the revenue obtained from items that are sold and the cost incurred on items that are procured. Thus the expected net revenue per inventory cycle depends on the order level, the stock on hand at the end of the cycle and the profit margin per unit.

Average revenue per cycle is,

\[ C_1 = (m-1)C_p \left[ \int_0^{\frac{S}{D}} D\tau e^{-(\lambda+R)\tau} d\tau + \int_{\frac{S}{D}}^{\infty} (S + (D\tau - S)\beta)\lambda e^{-(\lambda+R)\tau} d\tau \right] \]

\[ = (m-1)C_p \frac{D}{(\lambda+R)^2} \left[ 1 - (1-\beta)e^{-(\lambda+R)\frac{S}{D}} \right] \]

Inventory holding cost is incurred only when the quantity on hand is positive and is computed on the average inventory level. The expected inventory holding cost per cycle is given by,

Average inventory level per cycle is,

\[ C_2 = C_h \left[ \int_0^{\frac{S}{D}} (S\tau - \frac{D\tau^2}{2}) e^{-(\lambda+R)\tau} d\tau + \int_{\frac{S}{D}}^{\infty} \left( \frac{S^2}{2D} \right)\lambda e^{-(\lambda+R)\tau} d\tau \right] \]

\[ = C_h \left[ S\lambda \int_0^{\frac{S}{D}} \tau e^{-(\lambda+R)\tau} d\tau - \frac{D\lambda}{2} \int_0^{\frac{S}{D}} \tau^2 e^{-(\lambda+R)\tau} d\tau \right] + C_h \left[ \frac{S^2\lambda}{2D} \int_{\frac{S}{D}}^{\infty} e^{-(\lambda+R)\tau} d\tau \right] \]

\[ = C_h \left[ S\lambda \left( \frac{S}{D(\lambda+R)} e^{-(\lambda+R)\frac{S}{D}} - \frac{1}{\lambda} \int_0^{\frac{S}{D}} e^{-(\lambda+R)\tau} d\tau \right) \right] - C_h \frac{D\lambda}{2} \left[ \frac{\tau^2}{-(\lambda+R)} \right]^{\frac{S}{D}}_0 \]

\[ C_h \frac{D\lambda}{2} \left[ \frac{\tau^2}{-(\lambda+R)} \right]^{\frac{S}{D}}_0 \]
The expected backordering cost per cycle is computed based on the average level of backorders and the period of time the customers had to wait.

Average backorders per cycle is,

\[
C_3 = C_b \left[ \int_{\frac{S}{D}}^{\infty} \frac{\beta(D\tau - S)}{2D}(\lambda e^{-(\lambda+R)\tau}) \, d\tau \right]
\]

\[
= C_b \frac{\beta\lambda}{2D} \int_{\frac{S}{D}}^{\infty} (D\tau - S) \, e^{-(\lambda+R)\tau} \, d\tau
\]

\[
= C_b \frac{\beta\lambda}{2(\lambda + R)^2} e^{-(\lambda+R)\frac{S}{D}}
\]

The expected cost of lost demand per cycle is computed based on the total volume of demand that could not be satisfied due to the inventory shortage.

The expected cost of lost sales per cycle is,

\[
C_4 = C_s \left[ \int_{\frac{S}{D}}^{\infty} (1 - \beta)(D\tau - S)\lambda e^{-(\lambda+R)\tau} \, d\tau \right]
\]

\[
= C_s(1 - \beta)\lambda D e^{-(\lambda+R)\frac{S}{D}} \frac{e^{-(\lambda+R)\frac{S}{D}}}{(\lambda + R)^2}
\]

Thus the total expected net revenue per cycle is computed as follows:

\[
E = C_1 - C_2 - C_3 - C_4
\]
\[
\begin{align*}
E &= (m-1)C_p \frac{D}{(\lambda + R)^2} \left[ 1 - (1 - \beta) e^{-\frac{(\lambda + R)S}{D}} \right] - \hspace{1cm} \\
&\quad C_h \left[ \frac{\lambda D}{(\lambda + R)^3} \left\{ \left( \frac{S(\lambda + R)}{D} + 1 \right) e^{-\frac{(\lambda + R)S}{D}} \right\} - 1 \right] - \\
&\quad C_h \frac{\beta \lambda}{2(\lambda + R)^2} e^{-\frac{(\lambda + R)S}{D}} - C_s(1 - \beta) \frac{\lambda D e^{-\frac{(\lambda + R)S}{D}}}{(\lambda + R)^2} \\
&\text{i.e.,} \\
E &= \frac{De^{-\frac{(\lambda + R)S}{D}}}{(\lambda + R)^2} \left[ (m-1)(\beta-1)C_p - \frac{C_h \lambda S}{D(\lambda + R)} - \frac{C_h \lambda}{(\lambda + R)} + \frac{C_h \beta \lambda}{2D} - C_s \lambda (1 - \beta) \right] \\
&\quad + \frac{(m-1)C_p D}{(\lambda + R)^2} - \frac{C_h \lambda}{\lambda + R} \\
\text{The maximum total expected net revenue per cycle } S^* \text{ occurs where the first} \\
\text{derivative } \frac{\partial E}{\partial S} = 0, \\
\text{i.e.,} \\
\frac{\partial}{\partial S} \left[ \frac{De^{-\frac{(\lambda + R)S}{D}}}{(\lambda + R)^2} \left[ (m-1)(\beta-1)C_p - \frac{C_h \lambda S}{D(\lambda + R)} - \frac{C_h \lambda}{(\lambda + R)} + \frac{C_h \beta \lambda}{2D} - C_s \lambda (1 - \beta) \right] \right] \\
&\quad \frac{(m-1)C_p D}{(\lambda + R)^2} - \frac{C_h \lambda}{\lambda + R} = 0 \\
\text{This leads to,} \\
S^* &= \frac{D}{C_h} \left\{ (m-1)(\beta-1)C_p \frac{\lambda + R}{\lambda} + \frac{C_h \beta(\lambda + R)}{2D} \right. \\
&\quad \left. - C_s(\lambda + R)(1 - \beta) + C_h \frac{1 - \lambda - \beta}{\lambda + R} \right\} \\
\end{align*}
\]
At this value of \( S \) the second derivative, \( \frac{\partial^2 E}{\partial S^2} \triangleq 0 \), since \( C_h \) \( \rangle 0 \). Hence the expected net revenue per cycle is maximum, and is given by,

\[
E_{\text{max}} = De^{-\frac{(\lambda + R) S^*}{D}} \left[ (m-1)(\beta - 1)C_p - \frac{C_h \lambda S^*}{D(\lambda + R)} - \frac{C_p \lambda}{(\lambda + R)} + \frac{C_b \beta \lambda}{2D} - C_s \lambda (1 - \beta) \right]
\]

\[
+ \frac{(m-1)C_p D}{(\lambda + R)^2} - \frac{C_p \lambda}{\lambda + R}
\]

**Numerical Example**

**SENSITIVITY ANALYSIS**

<table>
<thead>
<tr>
<th>Set</th>
<th>( \lambda )</th>
<th>( \beta )</th>
<th>( C_b )</th>
<th>( C_S )</th>
<th>( C_h )</th>
<th>( m )</th>
<th>( D )</th>
<th>( C_p )</th>
<th>( R )</th>
<th>( S )</th>
<th>( E_{\text{max}} )</th>
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Consider the sets 1, 2 and 3 from the above analysis, when the arrival rate \( \lambda \) increases, the order level \( S \) decreases, the maximum net revenue \( E_{\text{max}} \) is also decreases, since the inflation rate \( R \) increasing. Similarly consider the set 5 from the above table, if the percentage of stock that is backordered \( \beta \) increases, the
order level decreases and the effect of inflation and time value of money will decrease the maximum net revenue $E_{\text{max}}$.

**CONCLUSION**

From the above result, we have obtained the maximum total expected net revenue comprised of inventory holding cost, backordering cost, lost sales with money inflation. The conclusion is, if the shipments arrive more frequently then the required capacity or order level to meet demand decreases and costs also decreases. Similarly if the inflation rate increases the maximum value of expected net revenue decreases. This problem is unique in itself as inflation and time value of money has been considered. This can be further be extended for variable rate of inflation and time value of money.