Chapter 6

Effects of Hall current on MHD flow through a uniform channel bounded by porous media - Case A

6.1 Introduction

In most of the research works done on MHD flows in the earlier stages, the Hall current was ignored in applying Ohm’s law as it has no marked effect for small magnetic fields. However, to study the effects of strong magnetic fields on the electrically conducting fluid flow, it is known that the influence of the electromagnetic force is noticeable and it causes anisotropic electrical conductivity in the plasma. This anisotropy in the electrical conductivity of the plasma produces a current known as the Hall current. The Hall effect is important in the case of strong magnetic fields where the electron cyclotron frequency is much greater than the electron collision frequency. Hence, the generalized Ohm’s law must be used in solving such problems on MHD channel flows.

Recently, MHD flows with and without thermal effects, through porous media have found considerable applications in many engineering and scientific fields such as MHD generators,
MHD pumps, accelerators, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil, plasma studies, nuclear reactors, geothermal energy extraction, the boundary layer control in the field of aerodynamics, etc. MHD channel flows with Hall effects and their applications to electric power generator, electromagnetic flow meter, electromagnetic accelerators and propulsion systems are receiving a considerable amount of scientific attention.

The effects of Hall current on MHD flow through a channel has been discussed by many researchers in different configurations (Siegel [1958], Gershuno and Zukovitskii [1961], Alpher [1961], Sherman and Sutton [1961], Regirer [1961], Yen [1963], Pop and Soundalgekar [1974], Datta et al., [1976], Vidyanidhi and Narayana [1979], Bharali and Borkakati [1982], Hossain [1986], Hemp [1987], Hossain et al. [1988], Shikin [1994], Mohyuddin et al. [2004] and Hayat et al. [2007]).

Tani [1962] studied the effect of Hall current on the steady motion of electrically conducting viscous fluids in channels. Pop [1971] analyzed the effect of Hall current on hydromagnetic flow near an accelerated plate. The effect of Hall current on the flow of a viscous, incompressible, slightly conducting fluid through a porous straight channel under a uniform transverse magnetic field was studied by Bhaskara Reddy and Bathaiah [1982].


Recently Hayat et al. [2007] studied the effects of Hall current and Heat transfer on the flow in a porous medium using modified Darcy’s law. The flow is analyzed in the presence of a transversely applied uniform magnetic field. In their work, the effect of slip parameter on real and imaginary part of the velocity profile for various values of other parameters were discussed. Mohamed H. Haroun [2007] studied the effects of Hall current on peristaltic transport of hydromagnetic flow through a porous medium. More recently, Mekheimer and El Kot [2008]
analyzed the influence of magnetic field and Hall current on blood flow through a stenotic artery by taking a micropolar model for blood simulation.

In the case of flow past a porous medium Beavers and Joseph [1967] have postulated a slip condition, at the porous boundaries due to transfer of momentum. Many investigators used BJ slip condition to study different flow geometries. Recently, Jat and Santosh Chaudhary [2009] studied MHD boundary layer flow past a porous substrate with BJ slip boundary condition.

**Author’s Contribution**

The main objective of the present work is to study the combined effect of the Hartmann number, porous parameter, slip parameter and Hall parameter on the hydromagnetic flow through a uniform channel covered by porous media using BJ slip condition.

### 6.2 Formulation and Solution of the Problem

To investigate the Hall effects on MHD flow through a uniform channel bounded by porous media, consider a physical configuration as shown in Fig.6.1. A steady laminar flow of a viscous, incompressible, electrically conducting fluid through an infinitely long channel of uniform width $2h$ is considered. It is assumed that the fluid behaves like a homogeneous conducting Newtonian fluid with constant density $\rho$, viscosity $\mu$ and electrical conductivity $\sigma_e$.

It is further assumed that the channel is symmetrical about the $x$-axis. The porous layer is assumed to be homogeneous, isotropic and densely packed so that the usual Darcy law is valid. A uniform magnetic field $H_0$ is applied in the $y$ direction. $B_0 = \mu_e H_0$, is the electromagnetic induction where $\mu_e$ is the magnetic permeability.

Since the channel is infinite in length all physical quantities (except pressure) depend only on $y$. The equation of continuity $\nabla \cdot \vec{q} = 0$ gives $v = 0$ where $\vec{q} = (u, v, w)$. In the case of a strong applied magnetic field, the Hall effect induces an electric current that flows normally to both the magnetic and electric field, which in turn induces a transverse motion of the fluid.
Thus Hall current induces a secondary flow, and so there will be two components of the velocity $u$ and $w$. The solinoidal relation, $\nabla \cdot \vec{H} = 0$ for the magnetic field, gives $H_y = H_0 =$constant everywhere in the fluid where $\vec{H} = (H_x, H_y, H_z)$. The conservation of electric charge $\nabla \cdot \vec{J} = 0$ gives $J_y =$constant , where $\vec{J} = (J_x, J_y, J_z)$. This constant is zero, since $J_y = 0$ on the walls which are electrically non-conducting.

The normal surface bounding the channel on either side is assumed to be represented by the lines $y = \pm h$ where $h$ is positive. It is assumed that the fluid is of small electrical conductivity with magnetic Reynolds number much less than unity (Poots [1961], Roberts [1967]) so that the induced magnetic field can be neglected in comparison with the applied magnetic field, so that $\vec{H} = (0, H_0, 0)$. In the absence of an external electric field, the effect of polarization of the ionized fluid is negligible. We also assume that the applied electric field $\vec{E} = 0$ (Ram [1991]).

Under these assumptions the generalized Ohm’s law is

$$\vec{J} + \frac{\omega_e \tau_e}{H_0} (\vec{J} \times \vec{H}) = \sigma_e \left[ \mu_e \left( \vec{q} \times \vec{H} \right) + \frac{\nabla P_e}{en_e} \right]$$

where $\sigma_e$, $\mu_e$, $\omega_e$, $\tau_e$, $e$, $n_e$ and $P_e$ are respectively the electrical conductivity of the fluid, the magnetic permeability, the cyclotron frequency, the electron collision time, the electric charge, the number density of the electron and the electron pressure. In Eqn.(6.2.1) the ion slip and thermo-electric effects are neglected. Further for weakly ionized gases the electron pressure is
negligible. Thus, the Eqn.(6.2.1) gives

\[ J_x - \omega_e \tau_e J_z = -\sigma_e \mu_e H_0 w \]  \hspace{1cm} (6.2.2)

\[ J_z + \omega_e \tau_e J_x = \sigma_e \mu_e H_0 u \]  \hspace{1cm} (6.2.3)

Solving the eqns. (6.2.2) and (6.2.3), it is seen that

\[ J_x = \frac{\sigma_e \mu_e H_0}{1 + m^2} (mu - w) \]  \hspace{1cm} (6.2.4)

\[ J_z = \frac{\sigma_e \mu_e H_0}{1 + m^2} (u + mw) \]  \hspace{1cm} (6.2.5)

where \( m = \omega_e \tau_e \) is the Hall parameter.

The governing equations (inertial term vanish, since \( v = 0 \) and \( u, w \) are functions of \( y \) alone) of the flow are:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  \hspace{1cm} (6.2.7)

\[ \frac{1}{\rho} \frac{\partial p}{\partial x} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_e \mu_e^2 H_0^2}{\rho (1 + m^2)} (u + mw) \]  \hspace{1cm} (6.2.8)

\[ \frac{1}{\rho} \frac{\partial p}{\partial y} = 0 \]  \hspace{1cm} (6.2.9)

\[ \frac{1}{\rho} \frac{\partial p}{\partial z} = \nu \frac{\partial^2 w}{\partial y^2} + \frac{\sigma_e \mu_e^2 H_0^2}{\rho (1 + m^2)} (mu - w) \]  \hspace{1cm} (6.2.10)

where \( \nu \) the coefficient of kinematic viscosity and \( p \) the pressure at a point \( (x, y) \).

Now, introduce the following dimensionless quantities:

\[ u^* = \frac{u}{U}, \quad w^* = \frac{w}{U}, \quad x^* = \frac{x}{h}, \quad y^* = \frac{y}{h}, \quad p^* = \frac{p}{\rho U^2} \]  \hspace{1cm} (6.2.11)

where \( U \) and \( h \) are respectively the characteristic velocity and the characteristic height. In view
of Eqn.(6.2.11), after dropping the superscripts (*) the Eqns. (6.2.7) to (6.2.10) reduce to

\[ R \frac{\partial p}{\partial x} = \frac{\partial^2 u}{\partial y^2} - \frac{M^2}{1 + m^2} (u + mw) \]  
(6.2.12)

\[ v = 0 \]  
(6.2.13)

\[ \frac{\partial p}{\partial y} = 0 \]  
(6.2.14)

\[ R \frac{\partial p}{\partial z} = \frac{\partial^2 w}{\partial y^2} + \frac{M^2}{1 + m^2} (mu - w) \]  
(6.2.15)

where

\[ M^2 = \frac{\sigma \mu H_0^2 h^2}{\mu_f}, \quad R = \frac{U h}{\nu} \]

Here, \( R \) is the Reynolds number and \( M^2 \) is the square of the Hartmann number. The boundary conditions in dimensionless form are:

\[ \frac{du}{dy} = -\bar{\alpha} \sigma (u_{B_1} - Q_1) \quad \text{at} \quad y = 1 \]  
(6.2.16)

\[ \frac{du}{dy} = \bar{\alpha} \sigma (u_{B_2} - Q_1) \quad \text{at} \quad y = -1 \]  
(6.2.17)

\[ u = u_{B_1} \quad \text{at} \quad y = 1 \]  
(6.2.18)

\[ u = u_{B_2} \quad \text{at} \quad y = -1 \]  
(6.2.19)

\[ \int_{-1}^{1} u(y) dy = n_f \]  
(6.2.20)

\[ v = \epsilon Q_y \]  
(6.2.21)

\[ Q_1 = \frac{R \partial p}{\sigma^2 \partial x} \]  
(6.2.22)

\[ w = 0 \quad \text{at} \quad y = \pm 1 \]  
(6.2.23)

where \( n_f = \frac{n_f}{\nu h} \) is the dimensionless net flux through the channel, \( u_{B_1} \) and \( u_{B_2} \) are the slip velocities, \( k \) is the permeability of the porous material and \( \bar{\alpha} \) is the dimensionless constant called slip parameter and \( \sigma = \frac{h}{\sqrt{k}} \). Here, and \( u_{B_1} = u_{B_2} = u_B \).

Since the channel is of uniform width, we use the approximation that the wall slope is everywhere negligible (Chandrasekhara and Rudraiah [1980]). Eliminating the pressure terms between
(6.2.12), (6.2.14) and (6.2.15), the basic equations in non-dimensional form are as follows:

\[
\frac{d^3u}{dy^3} - F \frac{du}{dy} - mF \frac{dw}{dy} = 0 \quad (6.2.24)
\]

\[
\frac{d^3w}{dy^3} + mF \frac{du}{dy} - F \frac{dw}{dy} = 0 \quad (6.2.25)
\]

since \( u \) and \( w \) are functions of \( y \) alone. Here,

\[ F = \frac{M^2}{1 + m^2} \]

Solving the simultaneous equations (6.2.24) and (6.2.25) subject to the conditions from (6.2.16) to (6.2.23), the expressions for primary and secondary velocity components are given by,

\[
u(y) = A_1 + 2A_2 \cosh(\psi_1 y) \cos(\psi_2 y) + 2A_3 \sinh(\psi_1 y) \sin(\psi_2 y) \quad (6.2.26)
\]

\[
w(y) = \frac{1}{mF(\psi_1^2 + \psi_2^2)} \left[ (\eta_{17}A_2 + \eta_{18}A_3) (\psi_1 \cosh(\psi_1 y) \cos(\psi_2 y) + \psi_2 \sinh(\psi_1 y) \sin(\psi_2 y)) \right]
\]

\[
+ \frac{1}{mF(\psi_1^2 + \psi_2^2)} \left[ (\eta_{17}A_3 - \eta_{18}A_2) (\psi_1 \sinh(\psi_1 y) \sin(\psi_2 y) - \psi_2 \cosh(\psi_1 y) \cos(\psi_2 y)) \right]
\]

\[
- \frac{1}{mF(\psi_1^2 + \psi_2^2)} \left[ \eta_{15} (\eta_{17}A_2 + \eta_{18}A_3) + \eta_{16} (\eta_{17}A_3 - \eta_{18}A_2) \right] \quad (6.2.27)
\]

The expression for slip velocity is,

\[
u_B = \frac{1}{2} \left( \frac{\eta_{3}\eta_{12} - \eta_{1}\eta_{9}}{\eta_{3}\eta_{14} - \eta_{1}\eta_{11} + \tilde{\sigma}} \right) n_f + 2 \left( 1 + \eta_{3}\eta_{13} - \eta_{1}\eta_{10} \right) \tilde{\sigma} \sigma Q_1 \quad (6.2.28)
\]

where

\[
r = \sqrt{1 + m^2}
\]
\[
\theta = \arctan(m)
\]
\[
\psi_1 = \sqrt{Fr} \cos \left( \frac{\theta}{2} \right)
\]
\[
\psi_2 = \sqrt{Fr} \sin \left( \frac{\theta}{2} \right)
\]
\[
\begin{align*}
\eta_1 &= \psi_1 \cosh(\psi_1) \sin(\psi_2) - \psi_2 \sinh(\psi_1) \cos(\psi_2) \\
\eta_2 &= \psi_1 \cosh(\psi_1) \sin(\psi_2) + \psi_2 \sinh(\psi_1) \cos(\psi_2) \\
\eta_3 &= \psi_1 \sinh(\psi_1) \cos(\psi_2) - \psi_2 \cosh(\psi_1) \sin(\psi_2) \\
\eta_4 &= \psi_1 \sinh(\psi_1) \cos(\psi_2) + \psi_2 \cosh(\psi_1) \sin(\psi_2) \\
\eta_5 &= \psi_1 \sin(\psi_1) \cos(\psi_2) + \psi_2 \sinh(\psi_1) \cosh(\psi_2) \\
\eta_6 &= \cosh^2(\psi_1) - \cos^2(\psi_2) \\
\eta_7 &= \sinh(\psi_1) \sin(\psi_2) \\
\eta_8 &= \cosh(\psi_1) \cos(\psi_2) \\
\eta_9 &= \frac{\eta_3}{(\psi_1^2 + \psi_2^2)\eta_5 - 2\psi_1\psi_2\eta_6} \\
\eta_{10} &= \frac{\eta_4}{(\psi_1^2 + \psi_2^2)\eta_5 - 2\psi_1\psi_2\eta_6} \\
\eta_{11} &= \frac{(\psi_1^2 + \psi_2^2)[\alpha\sigma\eta_8 + \eta_3] - \alpha\sigma\eta_4}{(\psi_1^2 + \psi_2^2)\eta_5 - 2\psi_1\psi_2\eta_6} \\
\eta_{12} &= \frac{(\psi_1^2 + \psi_2^2)\eta_5 - 2\psi_1\psi_2\eta_6}{(\psi_1^2 + \psi_2^2)\eta_7 - \eta_2} \\
\eta_{13} &= \frac{(\psi_1^2 + \psi_2^2)\eta_5 - 2\psi_1\psi_2\eta_6}{(\psi_1^2 + \psi_2^2)\eta_7 - \eta_2} \\
\eta_{14} &= \frac{(\psi_1^2 + \psi_2^2)[\alpha\sigma\eta_7 + \eta_1] - \alpha\sigma\eta_2}{(\psi_1^2 + \psi_2^2)\eta_5 - 2\psi_1\psi_2\eta_6} \\
\eta_{15} &= \psi_1 \cosh(\psi_1) \cos(\psi_2) + \psi_2 \sinh(\psi_1) \sin(\psi_2) \\
\eta_{16} &= \psi_1 \sinh(\psi_1) \sin(\psi_2) - \psi_2 \cosh(\psi_1) \cos(\psi_2) \\
\eta_{17} &= 2\psi_1 \left(\psi_1^2 - 3\psi_2^2 - F\right) \\
\eta_{18} &= 2\psi_2 \left(3\psi_1^2 - \psi_2^2 - F\right) \\
\eta_{19} &= 1 + \eta_4\eta_{12} - \eta_2\eta_9 \\
\eta_{20} &= \eta_{15}\eta_{17} - \eta_{16}\eta_{18} \\
\eta_{21} &= \eta_{15}\eta_{18} + \eta_{16}\eta_{17} \\
\eta_{22} &= \eta_2\eta_{11} - \eta_4\eta_{14}
\end{align*}
\]
\[
\eta_{23} = \eta_4\eta_{13} - \eta_2\eta_{10}
\]
\[
\eta_{24} = \frac{\eta_3\eta_{12} - \eta_1\eta_9}{\eta_3\eta_{14} - \eta_1\eta_{11} + \bar{\alpha}\sigma}
\]
\[
\eta_{25} = \frac{1 + \eta_3\eta_{13} - \eta_1\eta_{10}}{\eta_3\eta_{14} - \eta_1\eta_{11} + \bar{\alpha}\sigma}
\]

and
\[
n_f = \frac{4Q_1\sigma^2(\psi_1^2 + \psi_2^2) + 21\sigma [\eta_{20}(\eta_{14}\eta_{25} - \eta_{13}) + \eta_{21}(\eta_{10} - \eta_{11}\eta_{25}) - 2F(\eta_{23} + \eta_{22}\eta_{25})]}{\eta_{20}(\eta_{14}\eta_{24} - \eta_{12}) + \eta_{21}(\eta_9 - \eta_{11}\eta_{24}) - 2F(\eta_{19} + \eta_{22}\eta_{24})}
\]  \hspace{1cm} (6.2.29)

\[
A_1 = \frac{1}{2(\psi_1^2 + \psi_2^2)} [(1 - \eta_2\eta_9 + \eta_4\eta_{12}) n_f + 2(\eta_4\eta_{13} - \eta_2\eta_{10}) \bar{\alpha}\sigma Q_1 + 2(\eta_2\eta_{11} - \eta_4\eta_{14}) u_B]
\]
\[
A_2 = -\frac{1}{4}\eta_{12} n_f - \frac{1}{2}\eta_{13}\bar{\alpha}\sigma Q_1 + \frac{1}{2}\eta_{14} u_B
\]
\[
A_3 = \frac{1}{4}\eta_9 n_f + \frac{1}{2}\eta_{10}\bar{\alpha}\sigma Q_1 - \frac{1}{2}\eta_{11} u_B
\]

The shear stress for the primary and secondary velocities are given by
\[
\tau_x = 2(\psi_1 A_2 + \psi_2 A_3) \sinh(\psi_1) \cos(\psi_2) + 2(\psi_2 A_2 - \psi_1 A_3) \cosh(\psi_1) \sin(\psi_2)
\]
\[
\tau_z = -\frac{1}{m F} [(\eta_{17} A_2 + \eta_{18} A_3) \sinh(\psi_1) \cos(\psi_2) + (\eta_{17} A_3 - \eta_{18} A_2) \cosh(\psi_1) \sin(\psi_2)]
\]

### 6.3 Results and Discussion

This study investigates the combined effects of Hartmann Number \(M\), porous parameter \(\bar{\alpha}\sigma\), slip parameter \(\bar{\alpha}\) and Hall parameter \(m\) on MHD flow through a channel bounded by porous media. In this investigation the boundary condition at the fluid-porous medium interface suggested by Beavers and Joseph [1967] has been applied.

In the present discussion, numerical evaluations have been done for the following values of the parameters: the Hartmann number \(M = 2, 3, 4, 5\) (Bhaskara Reddy and Bathaiah [1982]), porous parameter \(\bar{\alpha}\sigma = 10^1, 10^2, 10^3\), Darcy velocity \(Q_1 = 0\) to \(0.001\) (Shivakumar et al. [1986]), the slip parameter \(\bar{\alpha} = 0.1, 0.3, 0.5\) (Makinde et al. [2006]) and Hall parameter \(m = 0.5, 1.0, 1.5, 2.0\).
The non-dimensional velocity component \( u \) in the primary flow direction is plotted in Figures (6.2) and (6.3), for various values of the parameter \( M \) and fixed value of \( m \). Figure 6.2 and 6.3 show that by increasing the Hartmann number \( M \), the primary velocity is decreased. Thus, it is seen that the presence of magnetic field slows down the primary flow of the fluid, confirming the fact that the magnetic field brings in rigidity in conducting fluids. It is observed that the maximum peak occurs in the middle of the channel when \( M = 2.0 \), which flattens as \( M \) is increased.

From figures 6.2 and 6.3, it is also clear that when the porous parameter \((\bar{\alpha}\sigma)\) is increased from \( 10^1 \) to \( 10^2 \), the primary velocity decreases which indicates the fact that the effect of porosity is to retard the flow. However when the porous parameter is further increased beyond \( 10^4 \), there is no significant difference in the primary velocity, since in this case the walls behave like impermeable walls.

Figures 6.4 and 6.5 show the effect of Hall parameter\((m)\) in primary flow direction for fixed values of the Hartmann number\((M)\). It is observed that the primary velocity increases as \( m \) increases. It is also observed from these figures that the porous parameter \((\bar{\alpha}\sigma)\) reduces, the primary velocity.

The non-dimensional velocity component \( w \) in the secondary flow direction are plotted in Figures 6.6 and 6.7 for different values of \( M \) by fixing \( m = 0.5 \) and \( m = 2.0 \) respectively. It is found that the secondary flow velocity decreases in magnitude with increasing Hartmann number\((M)\). It is also seen that the secondary flow is decreased through the channel by increasing the porous parameter\((\bar{\alpha}\sigma)\).

Figures 6.8 and 6.9 bring out the effects of Hall parameter\((m)\) on the secondary flow velocity for \( M = 2.0 \) and \( M = 4.0 \) respectively. It is observed that the secondary flow velocity increases in magnitude with the increase in Hall parameter \( m \).

Figures 6.10 and 6.11 present the development of the primary velocity for various values of slip parameter \( \bar{\alpha} \) at fixed values of \( M \) and \( m \). These figures show that by increasing \( \bar{\alpha} \), the primary velocity \( u \) is decreased. From figure 6.10, the primary velocity decreases as the slip
Figure 6.2: Effect of $M$ on primary velocity $u$ Vs. channel width ($m = 0.5$)

Figure 6.3: Effect of $M$ on primary velocity $u$ Vs. channel width ($m = 2.0$)
Figure 6.4: Effect of $m$ on primary velocity $u$ Vs. channel width ($M = 2.0$)

Figure 6.5: Effect of $m$ on primary velocity $u$ Vs. channel width ($M = 4.0$)
Figure 6.6: Effect of $M$ on secondary velocity $w$ Vs. channel width ($m = 0.5$)

Figure 6.7: Effect of $M$ on secondary velocity $w$ Vs. channel width ($m = 2.0$)
Figure 6.8: Effect of $m$ on secondary velocity $w$ Vs. channel width ($M = 2.0$)

Figure 6.9: Effect of $m$ on secondary velocity $w$ Vs. channel width ($M = 4.0$)
Figure 6.10: Effect of $\alpha$ on primary velocity $u$ Vs. channel width ($M = 2.0$ and $m = 1.0$)

Figure 6.11: Effect of $\alpha$ on primary velocity $u$ Vs. channel width ($M = 4.0$ and $m = 2.0$)
Figure 6.12: Effect of $\alpha$ on secondary velocity $u$ Vs. channel width ($M = 2.0$ and $m = 1.0$)

Figure 6.13: Effect of $\bar{\alpha}$ on secondary velocity $u$ Vs. channel width ($M = 4.0$ and $m = 2.0$)
Table 6.1: Shear Stress ($\tau_x$) for primary velocity at different values of $M$ and $m$ ($\overline{\alpha} = 0.1$ and $\sigma = 10^2$)

<table>
<thead>
<tr>
<th>$M \downarrow m \rightarrow$</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>-0.0100</td>
<td>-0.0038</td>
</tr>
<tr>
<td>3.0</td>
<td>-0.0082</td>
<td>-0.0022</td>
</tr>
<tr>
<td>4.0</td>
<td>-0.0037</td>
<td>-0.0018</td>
</tr>
</tbody>
</table>

Table 6.2: Shear Stress ($\tau_z$) for secondary velocity at different values of $M$ and $m$ ($\overline{\alpha} = 0.1$ and $\sigma = 10^2$)

<table>
<thead>
<tr>
<th>$M \downarrow m \rightarrow$</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>-2.3218</td>
<td>-3.4776</td>
</tr>
<tr>
<td>3.0</td>
<td>-1.8228</td>
<td>-2.6775</td>
</tr>
<tr>
<td>4.0</td>
<td>-1.4764</td>
<td>-2.0317</td>
</tr>
</tbody>
</table>

Parameter increases in the central core region of the channel for $M = 2.0$ and $m = 1.0$. It is seen from figure 6.11 that negative primary velocity occurs near the channel walls when $M = 4.0$ and $m = 2.0$ indicating the possibility of flow reversal near the walls.

Figures 6.12 and 6.13 show the effect of slip parameter on the secondary flow velocity at fixed values of $M$ and $m$. It is observed from these figures that the secondary flow velocity decreases in magnitude as the slip parameter is increased.

Table 6.3: Shear Stress ($\tau_x$) for primary velocity at different values of $M$ and $m$ ($\overline{\alpha} = 0.3$ and $\sigma = 10^2$)

<table>
<thead>
<tr>
<th>$M \downarrow m \rightarrow$</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>-0.0300</td>
<td>-0.0244</td>
</tr>
<tr>
<td>3.0</td>
<td>-0.0142</td>
<td>-0.0099</td>
</tr>
<tr>
<td>4.0</td>
<td>-0.0112</td>
<td>-0.0059</td>
</tr>
</tbody>
</table>
Table 6.4: Shear Stress ($\tau_z$) for secondary velocity at different values of $M$ and $m$ ($\bar{\alpha} = 0.3$ and $\sigma = 10^2$)

<table>
<thead>
<tr>
<th>$M \downarrow m \rightarrow$</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>-2.4038</td>
<td>-3.5157</td>
</tr>
<tr>
<td>3.0</td>
<td>-1.8383</td>
<td>-2.7048</td>
</tr>
<tr>
<td>4.0</td>
<td>-1.5029</td>
<td>-2.0694</td>
</tr>
</tbody>
</table>

Finally, the values of shear stress at the lower wall ($y = -1$) for the primary ($\tau_x$) and secondary ($\tau_z$) flows are given in Table 6.1 and 6.2 at $\bar{\alpha} = 0.1$. The value of shear stress at the lower wall for the primary and secondary flows decreases numerically as $M$ increase. From table 6.2, it is also observed that the shear stress for secondary flow numerically increasing as $m$ increases. From tables 6.3 and 6.4, the same trend is observed in the case of $\bar{\alpha} = 0.3$. Comparing these tables, it is seen that the values of shear stress for both primary and secondary flows increase numerically when the slip parameter $\bar{\alpha}$ increases from 0.1 to 0.3. It is also observed that the values of shear stress at the upper wall are equal and opposite to those at the lower wall.

6.4 Conclusion

The effects $\bar{\alpha}\sigma$, $M$ and $m$ on the steady flow of conducting viscous incompressible fluid in a uniform channel covered by porous media using BJ sip condition were investigated. Our results reveal that $\bar{\alpha}\sigma_0$ and $M$ produce a retarding effect on both the primary and secondary velocities where as the other parameter i.e., $m$ has an accelerating effect on primary and secondary velocities. The effect of slip parameter $\bar{\alpha}$ is to decrease both the primary and secondary velocities. The shear stress for the primary and secondary flows increases numerically when the Hall parameter increases. From these we conclude that with a proper choice of $\bar{\alpha}\sigma_0$, $\bar{\alpha}$, $M$ and $m$, it is possible to achieve the control over the steady flow through a uniform channel bounded by porous media.