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LIST OF SYMBOLS

$\lfloor x \rfloor$ - Largest integer less than or equal to $x$.

$\lceil x \rceil$ - Smallest integer less than or equal to $x$.

$p$ - Order of a graph $G$.

$q$ - Size of a graph $G$.

d(u, v) - Distance between vertices $u$ and $v$ in a graph $G$.

$\text{deg}(v)$ - Degree of a vertex $v$ in a graph $G$.

$G$ - Complement of a graph $G$.

$G - v$ - Deletion of a vertex $v$ in a graph $G$.

$G - e$ - Deletion of an edge $e$ in a graph $G$.

$G + e$ - Addition of an edge $e$ in a graph $G$.

$G \cup H$ - Union of graphs $G$ and $H$.

$G + H$ - Join of graphs $G$ and $H$.

$G \square H$ - Cartesian product graphs $G$ and $H$.

$L(G)$ - Line graph.

$N(v)$ - Open neighbourhood of vertex $v$.

$N[v]$ - Closed neighbourhood of vertex $v$.

$\text{pn}[u, S]$ - Private neighbourhood set of vertex $u$ with respect to set $S$.

$(S)$ - Subgraph induced by a vertex subset $S$. 
α_0(G) -  Vertex covering number of graph G.
α_1(G) -  Edge covering number of graph G.
β_0(G) -  Indepandence number of graph G.
β_1(G) -  Edge independence number of graph G.
ω(G) -  Clique number of graph G.
χ(G) -  Chromatic number of graph G.
δ(G) -  Minimum degree of a vertex v in graph G.
∆(G) -  Maximum degree of a vertex v in graph G.
k(\text{G}) -  Vertex connectivity of graph G.
λ(\text{G}) -  Edge connectivity of graph G.
ir(\text{G}) -  Irredundance number of graph G.
IR(\text{G}) -  Upper irredundance number of graph G.
γ(\text{G}) -  Domination number of graph G.
Γ(\text{G}) -  Upper domination number of graph G.
γ_\text{g}(\text{G}) -  Global domination number of graph G.
γ_t(\text{G}) -  Total domination number of graph G.
τ(\text{G}) -  Independant domination number of graph G.
τ_e(\text{G}) -  Clique transversal number of graph G.
LIST OF ABBREVIATIONS

\( \tau_c - \text{set} \) - Minimum clique transversal set.

\( \gamma - \text{set} \) - Minimum dominating set.

\( \gamma_g - \text{set} \) - Minimum global dominating set.

\( \gamma_t - \text{set} \) - Minimum total dominating set.