CHAPTER II
PROFIT EVALUATION OF A PRODUCTION STATION SUBJECT TO TEMPERATURE CONDITION

2.1 INTRODUCTION

Two unit cold stand-by systems with two operating modes—partial and total failure have been widely studied by several authors in the field of reliability. Proctor and Singh [104] had firstly incorporated the concept of three modes—normal, partial failure and total failure and obtained several characteristics of the system under study. Goel et al. [42], Gopalan et al. [28], working with three modes—normal, partial, and total failure have investigated two unit cold stand-by systems under different set of assumptions. In all the stand-by systems, stand-by units are only used to increase the reliability of the systems. The normal mode is defined as functioning of machine at full capacity. The partial failure mode may be defined as tolerable functioning of machine, say, at 70% of its full capacity. In total failure mode, the capacity of the machine is below 70% i.e. the machine does not operate at all and in catastrophic failure mode a machine or an equipment carries out some unwanted operation i.e. a machine operates when it should not operate. In some industrial systems machine operates on specific temperature and any fluctuation from it not only damages the machine but also the job which is processed on it. The affect of temperature condition of the machine is totally overlooked. Also, in the most of the papers of cold stand-by systems profit has been calculated by considering two characteristics namely availability and busy period of the repairman in (0, t). It is supposed there that system is continuously in use. However, there are many systems in which machines are used to process the jobs and in case of non-availability of the jobs, system remains idle. In idle period system suffers from loss while it is in operation. Recently Gopalan et al. [29, 35-37, 39] have analyzed models of production systems which never fail and processed of one machine is considered as the inputs for other machine.
Now-a-days, most of the modern sophisticated systems operate under stochastic environment which may have significant influence on the system performance. The certain specific environment conditions are required to operate such system which when satisfied may correspond to normal weather otherwise it may be supposed that weather is abnormal. Dhillon et al [14,15] have proposed Markov models for several repairable and non-repairable redundant systems in fluctuating environmental and human reliability factors. Dhillon et al [12-23], Cao [9] have used supplementary variable and alternative renewal techniques respectively to solve the environmental effect on one unit system. Gupta et al [69], Goel et al [50,51] have studied the effect of weather on complex and two unit system under three types of weathers—normal, abnormal 1 and abnormal 2 by taking all the failure, repair and weather change time distributions as negative exponential. However, the effect of temperature condition on a production system has not been studied so far. Most of the production systems need a specific environment and failing to which make the system inactive. For example in milkopack machine, a fix temperature is used for cutting a packet of milk and any fluctuation from that, make the machine down. Besides this, in many production stations, jobs are processed by operator whose physical condition affects the system in many ways. Firstly, Dhillon [13] incorporated the concept of physical state of the operator as good and poor which affects the performance of the system under study. Singh et al [120,122] revised the Dhillon's [13] model under the assumption that the operator physical condition can deteriorate when system is failed. i.e. it improved from poor to good but not vice-versa. However, very few works have been reported so far, for the study of the effect of temperature conditions on the production systems on which jobs are processed.

2.2 MODEL A

The purpose of the present model A is to study a production station of one machine at which jobs are arrived by poisson law and are processed by an operator whose condition is always perfect. Initially machine is in operation with diminishing load. Whenever job is operated on it, it works with load. So, its failure rate
increases. It is assumed here that if more than one jobs arrive at the production station before the completion of the processing of the job in hand then only one of them is allowed to wait. The jobs processing is highly affected by the temperature condition of the machine. The temperature of the machine is controlled by an operator whose physical condition is always good. Using regenerative point technique, following measures of system effectiveness are obtained:

(i) mean time to system failure;
(ii) expected idle time of the machine in \((0,t]\) and in steady state;
(iii) expected busy time of the machine in job processing in \((0,t]\) and in steady state;
(iv) expected busy time of the repairman in \((0,t]\) and in steady state;
(v) expected profit earned by the machine in \((0,t]\) and in steady state.

2.3 (a) SYSTEM DESCRIPTION AND ASSUMPTIONS

(1) System consists of one machine which is used to process the jobs. Initially machine is in operation with diminishing load.

(2) Jobs arrive on the production station with a poisson law. Processing time of the job is negative.

(3) During a job processing, if other jobs arrive then only one of them is allowed to wait for processing.

(4) Change of temperature from normal to abnormal and vice-versa follows negative exponential.

(5) The completion of job processing and failure of the machine follow negative exponential distribution while as repair time distribution are arbitrary.

(6) If a machine fails within the processing of the job, then all jobs are terminated at once and after repair machine goes to initial state \(S_0\) or \(S_2\) as the case may be.

(7) Machine fails either due to hardware fault (type I) or due to temperature fluctuation (type II).

(8) There is a single repair facility who repairs both type of failures. Repair is not affected.
by the operator.

(9) After repair machine works as new.

(b) NOTATIONS

\( g_i(t), G_i(t) \)
\( i = 1, 2 \)  
pdf and cdf of repair rate when machine fails in diminishing / non-diminishing load condition.

\( J_{wp} \)  
job waiting for processing.

\( \eta \)  
job processing rate on the machine.

\( a_1, a_2 \)  
the failure rate of the machine when it works with and without a diminishing load.

\( a_3, a_4 \)  
the failure rate of the machine due to temperature fault when it works with load and temperature condition is normal/abnormal.

\( \theta, \delta \)  
rates of change of the temperature condition from normal to abnormal and vice-versa.

\( T_N, T_A \)  
temperature condition is normal/abnormal.

(c) STATE SPACE OF THE MODEL

The system may be in any one of the following states:

\[ S_0 = \left( \begin{array}{c} N_F \\ T_N \end{array} \right) \]  
Up state of the system having one operative unit with diminishing load and temperature condition is normal.

\[ S_1 = \left( \begin{array}{c} N_B \\ T_N \end{array} \right) \]  
Up state of the system having one unit with load and temperature condition is normal.
S_2 = \left( \frac{N_F}{T_A} \right) \quad \text{Up state of the system having one operative unit with diminishing load and temperature condition is abnormal.}

S_3 = (F_{r_1}) \quad \text{Failed state of the system with one unit is under type I (hardware fault) repair.}

S_4 = (F_{r_2}) \quad \text{Failed state of the system with one unit is under type II (temperature fluctuation) repair.}

S_5 = \left( \frac{N_B}{T_A} \right) \quad \text{Up state of the system having one unit with load and temperature condition is abnormal.}

S_6 = \left( \frac{N_B}{T_N} \right) \quad \text{Up state of the system having one unit busy with job processing and other job waits for processing and temperature condition is normal.}

S_7 = \left( \frac{N_B}{T_A} \right) \quad \text{Up state of the system having one unit busy with job processing and other job waits for processing and temperature condition is abnormal.}

Transition between states are shown in Fig A.1.
Fig. A.1

State transition diagram

Up state
Failed state
Regenerative point
2.4 TRANSITION PROBABILITIES AND SOJOURN TIMES

Simple probabilistic arguments yields the following non-zero elements $p_{ij}$

$$Q_{01}(t) = \int_0^t \lambda e^{-(\lambda+a_1+\theta)t} dt = \frac{\lambda}{\lambda+a_1+\theta} [1-e^{-(\lambda+a_1+\theta)t}]$$

$$= \lambda [B_{11}]^{-1} [1-e^{-B_{11}t}]$$

$$Q_{02}(t) = \int_0^t \theta e^{-(\lambda+a_1+\theta)t} dt = \theta [B_{11}]^{-1} [1-e^{-B_{11}t}] = \theta Q_{01}(t)$$

$$Q_{03}(t) = a_1 Q_{01}(t)$$

$$Q_{10}(t) = \int_0^t \eta e^{-(\eta+\theta+\lambda+a_2+a_3)t} dt$$

$$= \frac{\eta}{\eta+\theta+\lambda+a_2+a_3} [1-e^{-(\eta+\theta+\lambda+a_2+a_3)t}]$$

$$= \eta [B_{12}]^{-1} [1-e^{-B_{12}t}]$$

$$Q_{13}(t) = \eta^{-1} a_2 Q_{10}(t) \quad ; \quad Q_{14}(t) = \eta^{-1} a_3 Q_{10}(t)$$

$$Q_{15}(t) = \eta^{-1} \theta Q_{10}(t) \quad ; \quad Q_{16}(t) = \eta^{-1} \lambda Q_{10}(t)$$

$$Q_{20}(t) = \int_0^t \delta e^{-(\delta+\lambda+a_1)t} dt = \frac{\delta}{\delta+\lambda+a_1} [1-e^{-(\delta+\lambda+a_1)t}]$$

$$= \delta [B_{13}]^{-1} [1-e^{-B_{13}t}]$$
\[ Q_{23}(t) = a_1 \delta^{-1} Q_{20}(t) \quad ; \quad Q_{25}(t) = \lambda \delta^{-1} Q_{20}(t) \]
\[ Q_{30}(t) = \int_0^t d \, G_1(t) \quad ; \quad Q_{40}(t) = \int_0^t d \, G_2(t) \]

\[ Q_{51}(t) = \int_0^t \delta \, e^{-\left(\delta + \eta + a_4 + \lambda + a_2\right) t} \, dt \]
\[ = \frac{\delta}{\delta + \eta + a_2 + a_4 + \lambda} \left[ 1 - e^{-\left(\delta + \eta + a_4 + a_2 + \lambda\right) t} \right] \]
\[ = \delta \left[ B_{14} \right]^{-1} \left[ 1 - e^{-B_{14} t} \right] \]

\[ Q_{52}(t) = \delta^{-1} \eta \, Q_{51}(t) \quad ; \quad Q_{53}(t) = \delta^{-1} a_2 \, Q_{51}(t) \]

\[ Q_{54}(t) = \delta^{-1} a_4 \, Q_{51}(t) \quad ; \quad Q_{57}(t) = \delta^{-1} \lambda \, Q_{51}(t) \]

\[ Q_{61}(t) = \int_0^t \eta \, e^{-\left(\eta + a_2 + \theta + a_3\right) t} \, dt \]
\[ = \frac{\eta}{\eta + a_2 + \theta + a_3} \left[ 1 - e^{-\left(\eta + a_2 + \theta + a_3\right) t} \right] \]
\[ = \eta \left[ B_{15} \right]^{-1} \left[ 1 - e^{-B_{15} t} \right] \]

\[ Q_{63}(t) = \eta^{-1} a_2 \, Q_{61}(t) \quad ; \quad Q_{64}(t) = \eta^{-1} a_3 \, Q_{61}(t) \quad ; \quad Q_{67}(t) = \eta^{-1} \theta \, Q_{61}(t) \]

\[ Q_{73}(t) = \int_0^t a_2 \, e^{-\left(\eta + a_2 + \delta + a_4\right) t} \, dt \]
\[ = \frac{a_2}{\eta + a_2 + \delta + a_4} \left[ 1 - e^{-\left(\eta + a_2 + \delta + a_4\right) t} \right] \]
\[ = a_2 \left[ B_{16} \right]^{-1} \left[ 1 - e^{-B_{16} t} \right] \]
\[ Q_{74}(t) = a_2^{-1}a_4 \ Q_73(t) \ ; \quad Q_{75}(t) = a_2^{-1} \eta \ Q_73(t) \]

\[ Q_{76}(t) = a_2^{-1} \delta \ Q_73(t) \]

Let \( t \to \infty \), expression (1-26) give non-zero transition probabilities \( (p_{ij}) \)

\[ p_{01} = \lambda \ [B_{11}]^{-1}, \quad p_{02} = \theta \ [B_{11}]^{-1}, \quad p_{03} = a_1 \ [B_{11}]^{-1} \]

\[ p_{10} = \eta \ [B_{12}]^{-1}, \quad p_{13} = a_2 \ [B_{12}]^{-1}, \quad p_{14} = a_3 \ [B_{12}]^{-1} \]

\[ p_{20} = \delta \ [B_{13}]^{-1}, \quad p_{23} = a_1 \ [B_{13}]^{-1}, \quad p_{25} = \lambda \ [B_{13}]^{-1} \]

\[ p_{30} = p_{40} = 1, \quad p_{51} = \delta \ [B_{14}]^{-1}, \quad p_{52} = \eta \ [B_{14}]^{-1} \]

\[ p_{53} = a_2 \ [B_{14}]^{-1}, \quad p_{54} = a_4 \ [B_{14}]^{-1}, \quad p_{57} = \lambda \ [B_{14}]^{-1} \]

\[ p_{61} = \eta \ [B_{15}]^{-1}, \quad p_{63} = a_2 \ [B_{15}]^{-1}, \quad p_{64} = a_3 \ [B_{15}]^{-1} \]

\[ p_{67} = \theta \ [B_{15}]^{-1}, \quad p_{73} = a_2 \ [B_{16}]^{-1}, \quad p_{74} = a_3 \ [B_{16}]^{-1} \]

\[ p_{75} = \eta \ [B_{16}]^{-1}, \quad p_{76} = \delta \ [B_{16}]^{-1} \]

\( B_{11} = (\lambda + a_1 + \theta), \quad B_{12} = (\eta + \theta + \lambda + a_1 + a_2), \quad B_{13} = (\delta + \lambda + a_1) \)

\( B_{12} = (\delta + \eta + a_2 + a_4), \quad B_{12} = (\eta + a_2 + \theta + a_3), \quad B_{16} = (\eta + a_2 + \delta + a_4) \)
To calculate mean sojourn time $\mu_0$ in state $s_0$, we observe that so long as the system is in state $s_0$, there is no transition in to $s_0$, $s_2$ and $s_3$, hence if $T_0$ denotes the sojourn times in $S_0$, then

$$\mu_0 = \int_0^\infty P[T_0 \geq t] dt = [B_{11}]^{-1}$$

similarly

$$\mu_1 = [B_{12}]^{-1}, \mu_2 = [B_{13}]^{-1}, \mu_5 = [B_{14}]^{-1}, \mu_6 = [B_{15}]^{-1}, \mu_7 = [B_{16}]^{-1},$$

$$\mu_i = \int_0^\infty G_{12}(t) dt \quad (i = 3, 4) \quad [50-56]$$

To calculate $m_{ij}$ we note that Laplace-Stieltjes transform of $Q_{ij}(t)$ is equal to the Laplace transform of $q_{ij}(t)$ i.e. $\tilde{Q}_{ij}(s) = q_{ij}(s)$ so that $\tilde{Q}_{ij}(0) = q_{ij}(0) = p_{ij}$. In terms of Laplace-Stieltjes of $Q_{ij}(t)$ we have

$$\tilde{Q}_{01}(s) = \int_0^\infty e^{-st} q_{01}(t) dt = \int_0^\infty e^{-(s+\lambda+a_1+\theta)t} dt = \frac{\lambda}{s+\lambda+a_1+\theta}$$

$$\tilde{Q}_{02}(s) = \delta[s+B_{11}]^{-1} ; \quad \tilde{Q}_{03}(s) = a_1[s+B_{11}]^{-1} ; \quad \tilde{Q}_{10}(s) = \eta[s+B_{12}]^{-1}$$

$$\tilde{Q}_{13}(s) = a_2[s+B_{12}]^{-1} ; \quad \tilde{Q}_{14}(s) = a_3[s+B_{12}]^{-1} ; \quad \tilde{Q}_{15}(s) = \theta[s+B_{12}]^{-1}$$

$$\tilde{Q}_{16}(s) = \lambda[s+B_{12}]^{-1} ; \quad \tilde{Q}_{20}(s) = \delta[s+B_{13}]^{-1} ; \quad \tilde{Q}_{23}(s) = a_4[s+B_{13}]^{-1}$$

$$\tilde{Q}_{25}(s) = \lambda[s+B_{13}]^{-1} ; \quad \tilde{Q}_{30}(s) = \int_0^\infty e^{-sT} \xi_1(t) dt = \xi_1(s) \quad \tilde{Q}_{40}(s) = g_2(s)$$

$$\tilde{Q}_{51}(s) = \delta[s+B_{14}]^{-1} ; \quad \tilde{Q}_{52}(s) = \eta[s+B_{14}]^{-1} ; \quad \tilde{Q}_{53}(s) = a_2[s+B_{14}]^{-1}$$

$$\tilde{Q}_{54}(s) = a_4[s+B_{14}]^{-1} ; \quad \tilde{Q}_{57}(s) = \lambda[s+B_{14}]^{-1} ; \quad \tilde{Q}_{61}(s) = \lambda[s+B_{15}]^{-1}$$

$$\tilde{Q}_{63}(s) = a_2[s+B_{15}]^{-1} ; \quad \tilde{Q}_{64}(s) = a_3[s+B_{15}]^{-1} ; \quad \tilde{Q}_{67}(s) = \theta[s+B_{15}]^{-1}$$

$$\tilde{Q}_{73}(s) = a_2[s+B_{16}]^{-1} ; \quad \tilde{Q}_{74}(s) = a_4[s+B_{16}]^{-1} ; \quad \tilde{Q}_{75}(s) = \eta[s+B_{16}]^{-1}$$
\[ \tilde{Q}_{76}(s) = \delta[s + B_{16}]^{-1} \]

In view of relation (57-82) we have

\[ m_{01} = -\tilde{Q}_{01}(0) = \lambda \int_{0}^{\infty} t e^{-(B_{11})t} \, dt = \lambda [B_{11}]^{-2} \]

\[ m_{02} = \theta [B_{11}]^{-2}, \quad m_{03} = \lambda [B_{11}]^{-2} \]

\[ m_{10} = -\tilde{Q}_{10}(0) = \eta \int_{0}^{\infty} t e^{-(B_{11})t} \, dt = \eta [B_{11}]^{-2} \]

\[ m_{13} = \sigma_{2} [B_{12}]^{-2}, \quad m_{14} = \alpha_{3} [B_{12}]^{-2}, \quad m_{15} = \theta [B_{11}]^{-2} \]

\[ m_{16} = \lambda [B_{12}]^{-2}, \quad m_{20} = -\tilde{Q}_{20}(s)\int_{0}^{\infty} t e^{-(B_{11})t} \, dt = \delta [B_{13}]^{-2} \]

\[ m_{23} = \alpha_{1} [B_{13}]^{-2}, \quad m_{25} = \lambda [B_{13}]^{-2} \]

\[ m_{30} = \int_{0}^{\infty} t \xi_{1}(t) \, dt, \quad m_{40} = \int_{0}^{\infty} t \xi_{2}(t) \, dt \]

\[ m_{51} = -\tilde{Q}_{51}(t) = \delta \int_{0}^{\infty} t e^{-(B_{14})t} \, dt = \delta [B_{14}]^{-2} \]

\[ m_{53} = \sigma_{2} [B_{14}]^{-2}, \quad m_{54} = \alpha_{4} [B_{14}]^{-2}, \quad m_{57} = \alpha_{2} [B_{14}]^{-2} \]

\[ m_{61} = -\tilde{Q}_{61}(0) = \eta \int_{0}^{\infty} t e^{-(B_{15})t} \, dt = \eta [B_{15}]^{-2} \]

\[ m_{63} = \sigma_{2} [B_{15}]^{-2}, \quad m_{64} = \alpha_{3} [B_{15}]^{-2}, \quad m_{67} = \theta [B_{15}]^{-2} \]

\[ m_{73} = -\tilde{Q}_{73}(0) = \sigma_{2} \int_{0}^{\infty} t e^{-(B_{16})t} \, dt = \alpha_{2} [B_{16}]^{-2} \]

\[ m_{74} = \alpha_{4} [B_{16}]^{-2}, \quad m_{75} = \eta [B_{16}]^{-2}, \quad m_{76} = \delta [B_{16}]^{-2} \]

\[ m_{20} = \lambda [B_{11}]^{-2} \]

It can be easily seen that

\[ m_{01} + m_{02} + m_{03} = \int_{0}^{\infty} t(\lambda + a_{1} + \theta) e^{-(\lambda + a_{1} + \theta)t} \, dt \]
\[
\int_0^\infty e^{-(\lambda + \alpha_1 + \theta)t} dt = \mu_0
\]

Similarly
\[
m_{10} + m_{13} + m_{14} + m_{15} + m_{16} = \mu_1
\]
\[
m_{20} + m_{23} + m_{25} = \mu_2
\]
\[
m_{30} = \mu_3 , m_{40} = \mu_4
\]
\[
m_{51} + m_{52} + m_{53} + m_{54} + m_{57} = \mu_5
\]
\[
m_{61} + m_{63} + m_{64} + m_{67} = \mu_6
\]
\[
m_{73} + m_{74} + m_{75} + m_{76} = \mu_7
\]

### 2.5 TIME TO SYSTEM FAILURE

Let \( T_i \) be the random variable depicting time to system failure when system starts from state \( S_i \in E(l=0,1,2,5-7) \) and

\[
\pi_i(t) = P \left[ T_i \leq t \right]
\]

To calculate the distribution function \( \pi_i(t) \) we regard the failed states \( S_3, S_4 \) as absorbing. To obtain \( \pi_0(t) \), we consider the possible transitions from state \( S_0 \). From \( S_0 \) the system transits any one of the states \( S_i (i = 1,2,3) \). Suppose that the system enters state \( S_i (i = 1,2) \) during \((u,u+du)\) and then starting from \( S_1 \) it fails before the expiry of the further time \((t-u)\), the probability of this contingency is

\[
\int_0^t \Pi_i(t-u) dQ_{0i}(u) = Q_{0i}(t) \quad \Pi_i(t) \quad (l=1,2)
\]

The other possibility is that starting from \( S_0 \) system directly transits to failed state \( S_3 \) in time \((u,u+du)\). Thus
\[ \pi_0(t) = Q_{01}(t) \pi_1(t) + Q_{02}(t) \pi_2(t) + Q_{03}(t) \]  

Similarly

\[ \pi_1(t) = Q_{10}(t) \pi_0(t) + Q_{15}(t) \pi_5(t) + Q_{16}(t) \pi_6(t) + Q_{13}(t) + Q_{14}(t) \]

\[ \pi_2(t) = Q_{20}(t) \pi_0(t) + Q_{25}(t) \pi_5(t) + Q_{23}(t) \]

\[ \pi_5(t) = Q_{51}(t) \pi_1(t) + Q_{52}(t) \pi_2(t) + Q_{57}(t) \pi_7(t) + Q_{53}(t) + Q_{54}(t) \]

\[ \pi_6(t) = Q_{61}(t) \pi_1(t) + Q_{67}(t) \pi_7(t) + Q_{63}(t) + Q_{64}(t) \]

\[ \pi_7(t) = Q_{75}(t) \pi_5(t) + Q_{76}(t) \pi_6(t) + Q_{73}(t) + Q_{74}(t) \]

Taking Laplace-Stieltjes transforms of equation (1-6) the solution for \( \pi_i(s) \) will be

\[ \{\pi_0, \pi_1, \pi_2, \pi_5, \pi_6, \pi_7\} = Q^{-1}(\tilde{Q}_{03}, \tilde{Q}_{13} + \tilde{Q}_{14}, \tilde{Q}_{23}, \tilde{Q}_{53} + \tilde{Q}_{54}, \tilde{Q}_{63} + \tilde{Q}_{64}, \tilde{Q}_{73} + \tilde{Q}_{74}) \]

\[ \tilde{Q} = \begin{bmatrix}
1 & -\tilde{Q}_{01} & -\tilde{Q}_{02} & 0 & 0 & 0 \\
-\tilde{Q}_{10} & 1 & 0 & -\tilde{Q}_{15} & -\tilde{Q}_{16} & 0 \\
-\tilde{Q}_{20} & 0 & 1 & -\tilde{Q}_{25} & 0 & 0 \\
0 & -\tilde{Q}_{51} & -\tilde{Q}_{52} & 1 & 0 & -\tilde{Q}_{57} \\
0 & -\tilde{Q}_{61} & 0 & 0 & 1 & -\tilde{Q}_{67} \\
0 & 0 & 0 & -\tilde{Q}_{75} & -\tilde{Q}_{76} & 1
\end{bmatrix} \]

Computing the relevant elements of \( \tilde{Q}^{-1} \) we have
\[ \hat{\pi}_0(s) = \frac{N_1(s)}{D_1(s)} \]  \hfill [9]

Where

\[ N_1(s) = \hat{\alpha}_{02}[(1-\hat{\alpha}_{25}\hat{\alpha}_{52})(1-\hat{\alpha}_{67}\hat{\alpha}_{76})-\hat{\alpha}_{57}\hat{\alpha}_{75}-\hat{\alpha}_{15}\hat{\alpha}_{51}(1-\hat{\alpha}_{67}\hat{\alpha}_{76})-\hat{\alpha}_{16}\hat{\alpha}_{51} \]
\[ -\hat{\alpha}_{67}\hat{\alpha}_{75}+\hat{\alpha}_{61}[(-\hat{\alpha}_{15}\hat{\alpha}_{57}\hat{\alpha}_{76}-\hat{\alpha}_{16}(1-\hat{\alpha}_{57}\hat{\alpha}_{75})+\hat{\alpha}_{16}\hat{\alpha}_{25}\hat{\alpha}_{52}]-(\hat{\alpha}_{13}+\hat{\alpha}_{14}) \]
\[ -\hat{\alpha}_{01}[(1-\hat{\alpha}_{67}\hat{\alpha}_{76})(1-\hat{\alpha}_{25}\hat{\alpha}_{52})(1-\hat{\alpha}_{57}\hat{\alpha}_{75})-\hat{\alpha}_{51}\hat{\alpha}_{02}\hat{\alpha}_{25}(1-\hat{\alpha}_{57}\hat{\alpha}_{76})-\hat{\alpha}_{02} \]
\[ \hat{\alpha}_{25}\hat{\alpha}_{57}\hat{\alpha}_{61}\hat{\alpha}_{76}]+\hat{\alpha}_{23}[\hat{\alpha}_{01}\hat{\alpha}_{52}\hat{\alpha}_{16}(1-\hat{\alpha}_{67}\hat{\alpha}_{76})+\hat{\alpha}_{16}\hat{\alpha}_{57}\hat{\alpha}_{75}\hat{\alpha}_{52}]+\hat{\alpha}_{02} \]
\[ ((1-\hat{\alpha}_{67}\hat{\alpha}_{76}-\hat{\alpha}_{57}\hat{\alpha}_{75})-\hat{\alpha}_{15}\hat{\alpha}_{51}(1-\hat{\alpha}_{67}\hat{\alpha}_{76})-\hat{\alpha}_{75}\hat{\alpha}_{16}\hat{\alpha}_{67}\hat{\alpha}_{51}-\hat{\alpha}_{16}\hat{\alpha}_{15} \]
\[ \hat{\alpha}_{57}\hat{\alpha}_{76}-\hat{\alpha}_{61}\hat{\alpha}_{16}(1-\hat{\alpha}_{57}\hat{\alpha}_{75})]-\hat{\alpha}_{53}+\hat{\alpha}_{54}][\hat{\alpha}_{01}[-\hat{\alpha}_{15}(1-\hat{\alpha}_{67}\hat{\alpha}_{76}) \]
\[ -\hat{\alpha}_{75}\hat{\alpha}_{16}\hat{\alpha}_{67}]+\hat{\alpha}_{02}[-\hat{\alpha}_{25}(1-\hat{\alpha}_{67}\hat{\alpha}_{76})+\hat{\alpha}_{61}\hat{\alpha}_{16}\hat{\alpha}_{25}]]+(\hat{\alpha}_{63}+\hat{\alpha}_{64})[-\hat{\alpha}_{01} \]
\[ \hat{\alpha}_{15}(1-\hat{\alpha}_{57}\hat{\alpha}_{76})-\hat{\alpha}_{16}(1-\hat{\alpha}_{57}\hat{\alpha}_{75})+\hat{\alpha}_{52}\hat{\alpha}_{16}\hat{\alpha}_{25}]+\hat{\alpha}_{02}[\hat{\alpha}_{25}\hat{\alpha}_{57}\hat{\alpha}_{76}+ \]
\[ \hat{\alpha}_{51}\hat{\alpha}_{16}\hat{\alpha}_{25}]]-(\hat{\alpha}_{73}+\hat{\alpha}_{74})][\hat{\alpha}_{01}[-\hat{\alpha}_{15}\hat{\alpha}_{57}+\hat{\alpha}_{16}\hat{\alpha}_{75}-\hat{\alpha}_{52}\hat{\alpha}_{67}\hat{\alpha}_{16}\hat{\alpha}_{25}]+ \]
\[ \hat{\alpha}_{02}[-\hat{\alpha}_{25}\hat{\alpha}_{57}-\hat{\alpha}_{51}\hat{\alpha}_{67}\hat{\alpha}_{16}\hat{\alpha}_{25}+\hat{\alpha}_{61}\hat{\alpha}_{57}\hat{\alpha}_{16}\hat{\alpha}_{25}] \]

\[ D_1(s) = [(1-\hat{\alpha}_{25}\hat{\alpha}_{52})(1-\hat{\alpha}_{67}\hat{\alpha}_{76})-\hat{\alpha}_{57}\hat{\alpha}_{75}-\hat{\alpha}_{15}\hat{\alpha}_{51}(1-\hat{\alpha}_{67}\hat{\alpha}_{76})-\hat{\alpha}_{16}\hat{\alpha}_{51} \]
\[ \hat{\alpha}_{67}\hat{\alpha}_{75}+\hat{\alpha}_{16}(-\hat{\alpha}_{15}\hat{\alpha}_{57}\hat{\alpha}_{76}-\hat{\alpha}_{16}(1-\hat{\alpha}_{57}\hat{\alpha}_{75})+\hat{\alpha}_{16}\hat{\alpha}_{25}\hat{\alpha}_{52}]]+\hat{\alpha}_{10} \]
\[ [-\hat{\alpha}_{01}[(1-\hat{\alpha}_{67}\hat{\alpha}_{76})(1-\hat{\alpha}_{25}\hat{\alpha}_{52})-\hat{\alpha}_{57}\hat{\alpha}_{75})-\hat{\alpha}_{51}\hat{\alpha}_{02}\hat{\alpha}_{25}(1-\hat{\alpha}_{67}\hat{\alpha}_{76}) \]
\[ -\hat{\alpha}_{02}\hat{\alpha}_{25}\hat{\alpha}_{57}\hat{\alpha}_{61}\hat{\alpha}_{76}-\hat{\alpha}_{20}\hat{\alpha}_{01}\hat{\alpha}_{52}\hat{\alpha}_{15}(1-\hat{\alpha}_{67}\hat{\alpha}_{76})+\hat{\alpha}_{16}\hat{\alpha}_{67}\hat{\alpha}_{75}\hat{\alpha}_{52} \]
\[ +\hat{\alpha}_{02}[(1-\hat{\alpha}_{67}\hat{\alpha}_{76}-\hat{\alpha}_{57}\hat{\alpha}_{75})-\hat{\alpha}_{15}\hat{\alpha}_{51}(1-\hat{\alpha}_{67}\hat{\alpha}_{76})-\hat{\alpha}_{75}\hat{\alpha}_{16}\hat{\alpha}_{67}\hat{\alpha}_{51}-\hat{\alpha}_{61}\hat{\alpha}_{15}\hat{\alpha}_{57}\hat{\alpha}_{76}-\hat{\alpha}_{61}\hat{\alpha}_{16}(1-\hat{\alpha}_{57}\hat{\alpha}_{75})]] \]

Using the equations (27-49) of section 2.4 and \( \lim_{s \to 0} \hat{\alpha}_{ij}(s) = p_{ij} \), we have

\[ N_1(0) = [(1-\hat{\alpha}_{52}\hat{\alpha}_{52})(1-\hat{\alpha}_{67}\hat{\alpha}_{76})-\hat{\alpha}_{57}\hat{\alpha}_{75}-\hat{\alpha}_{15}\hat{\alpha}_{51}(1-\hat{\alpha}_{67}\hat{\alpha}_{76})-\hat{\alpha}_{16}\hat{\alpha}_{51}\hat{\alpha}_{67}\hat{\alpha}_{75} + \hat{\alpha}_{61}[-\alpha_{15}\hat{\alpha}_{67}\hat{\alpha}_{76}-\hat{\alpha}_{16}(1-\hat{\alpha}_{57}\hat{\alpha}_{75})+\hat{\alpha}_{16}\hat{\alpha}_{25}\hat{\alpha}_{52}]]+\hat{\alpha}_{10}[-\hat{\alpha}_{01}[(1-\hat{\alpha}_{67}\hat{\alpha}_{76}) \]
\[ (1-\hat{\alpha}_{25}\hat{\alpha}_{52})-\hat{\alpha}_{57}\hat{\alpha}_{75})-\hat{\alpha}_{51}\hat{\alpha}_{02}\hat{\alpha}_{25}(1-\hat{\alpha}_{67}\hat{\alpha}_{76})-\hat{\alpha}_{02}[(1-\hat{\alpha}_{67}\hat{\alpha}_{76}-\hat{\alpha}_{57}\hat{\alpha}_{75})-\hat{\alpha}_{15}\hat{\alpha}_{51}(1-\hat{\alpha}_{67}\hat{\alpha}_{76})-\hat{\alpha}_{75}\hat{\alpha}_{16}\hat{\alpha}_{67}\hat{\alpha}_{51}-\hat{\alpha}_{61}\hat{\alpha}_{15}\hat{\alpha}_{57}\hat{\alpha}_{76}-\hat{\alpha}_{61}\hat{\alpha}_{16}(1-\hat{\alpha}_{57}\hat{\alpha}_{75})]] \]
Thus $\bar{f}_0(0) = 1$. This shows that $\bar{f}_0(0)$ is a proper cdf. Therefore, mean-time-to-system-failure when the system starts from state $S_0$ is

$$E(T) = -\frac{d\bar{\pi}_0(s)}{ds} \bigg|_{s=0} = \frac{D_1(0) - N_1(0)}{D_1(0)}$$

[10]

To obtain the numerator of (10) in the coefficients of relevant $m_{ij}$ in $D_1(0) = N_1(0)$, we have,

Coefficient of $m_{01} = [(1-p_{67}p_{76})(1-p_{25}p_{52} - p_{15}p_{51}) - p_{16}p_{61}(1-p_{25}p_{52} - p_{57}p_{75}) - p_{57}p_{75} - p_{15}p_{57}p_{61}p_{76}]$

Coefficient of $m_{02} = [(1-p_{67}p_{76})(1-p_{25}p_{52} - p_{15}p_{51}) - p_{16}p_{61}(1-p_{25}p_{52} - p_{57}p_{75}) - p_{57}p_{75} - p_{15}p_{57}p_{61}p_{76}]$

Coefficient of $m_{03} = [(1-p_{67}p_{76})(1-p_{25}p_{52} - p_{15}p_{51}) - p_{16}p_{61}(1-p_{25}p_{52} - p_{57}p_{75}) - p_{57}p_{75} - p_{15}p_{57}p_{61}p_{76}]$

Coefficient of $m_{10} = p_{01}((1-p_{67}p_{76})(1-p_{25}p_{52} - p_{57}p_{75}) + p_{02}p_{25}p_{51}(1-p_{67}p_{76}) + p_{02}p_{25}p_{57}p_{61}p_{76}$

Coefficient of $m_{13} = p_{01}((1-p_{67}p_{76})(1-p_{25}p_{52} - p_{57}p_{75}) + p_{02}p_{25}p_{51}(1-p_{67}p_{76}) + p_{02}p_{25}p_{57}p_{61}p_{76}$

Coefficient of $m_{14} = p_{01}((1-p_{67}p_{76})(1-p_{25}p_{52} - p_{57}p_{75}) + p_{02}p_{25}p_{51}(1-p_{67}p_{76}) + p_{02}p_{25}p_{57}p_{61}p_{76}$
Coefficient of $m_{15} = p_{01}\{(1-p_{67}p_{76})(1-p_{25}p_{52}) - p_{57}p_{75}\} + p_{02}p_{25}p_{51}(1-p_{67}p_{76}) + p_{02}p_{25}p_{57}p_{61}p_{76}$

Coefficient of $m_{16} = p_{01}\{(1-p_{67}p_{76})(1-p_{25}p_{52}) - p_{57}p_{75}\} + p_{02}p_{25}p_{51}(1-p_{67}p_{76}) + p_{02}p_{25}p_{57}p_{61}p_{76}$

Coefficient of $m_{20} = [p_{01}\{p_{52}p_{15}(1-p_{67}p_{76}) + p_{16}p_{67}p_{75}p_{52}\} + p_{02}\{(1-p_{67}p_{76} - p_{57}p_{75}) - p_{15}p_{51}(1-p_{67}p_{76}) - p_{16}p_{51}p_{67}p_{75} - p_{15}p_{57}p_{61}p_{76} - p_{16}p_{61}(1-p_{57}p_{75})\}]

= (1-p_{67}p_{76})(p_{01}p_{15}p_{52} + p_{02}(1-p_{15}p_{51})) + p_{16}p_{67}p_{75}
(p_{01}p_{52} - p_{02}p_{51}) - p_{02}p_{57}p_{75}(1-p_{16}p_{61}) - p_{02}p_{16}p_{61}
-p_{02}p_{15}p_{57}p_{61}p_{76}

= (1-p_{67}p_{76})(p_{01}p_{15}p_{52} + p_{02}(1-p_{15}p_{51})) + p_{16}p_{67}p_{75}
(p_{01}p_{52} - p_{02}p_{51}) - p_{02}(p_{57}p_{75}(1-p_{16}p_{61}) + p_{61}(p_{16} + p_{15}p_{57}p_{76}))

Coefficient of $m_{23} = (1-p_{67}p_{76})(p_{01}p_{15}p_{52} + p_{02}(1-p_{15}p_{51})) + p_{16}p_{67}p_{75}
(p_{01}p_{52} - p_{02}p_{51}) - p_{02}(p_{57}p_{75}(1-p_{16}p_{61}) + p_{61}(p_{16} + p_{15}p_{57}p_{76}))$

Coefficient of $m_{25} = [p_{52}(1-p_{67}p_{76}) - p_{16}p_{61}p_{52} - p_{01}p_{10}(1-p_{67}p_{76})p_{52}
-p_{16}p_{10}p_{52}] + (p_{13} + p_{14})p_{01}(1-p_{67}p_{76})p_{52} - (p_{13} + p_{14})p_{54}p_{02}
(1-p_{67}p_{76}) - (p_{13} + p_{14})p_{02}p_{57}p_{61}p_{76} - (p_{53} + p_{54})p_{02}(1-p_{67}p_{76})
+(p_{53} + p_{54})p_{62}p_{16}p_{61} + (p_{63} + p_{64})p_{01}p_{52}p_{16} - (p_{63} + p_{64})p_{02}p_{57}
-p_{76} - (p_{63} + p_{64})p_{02}p_{57}p_{16} + (p_{73} + p_{74})p_{01}p_{52}p_{67}p_{16} - (p_{73} + p_{74})p_{02}p_{57} - (p_{73} + p_{74})p_{02}p_{57}p_{67}p_{16} + (p_{73} + p_{74})p_{61}p_{57}p_{16}p_{02}]$
Coefficient of $m_{25} = \{p_{52}(1-p_{67}p_{76}) - p_{61}p_{16}p_{52} - p_{10}p_{10}(1-p_{67}p_{76})p_{52} + p_{10}p_{02}p_{51}(1-p_{67}p_{76}) + p_{10}p_{02}p_{57}p_{61}p_{76}\} - [p_{03} \{p_{52}(1-p_{67}p_{76}) - p_{61}p_{16}p_{52}\} + (p_{13} + p_{14})p_{01}(1-p_{67}p_{76}) - (p_{13} + p_{14})p_{02}p_{57}p_{61}p_{76} - (p_{3} + p_{54})p_{02}(1-p_{67}p_{76}) + (p_{3} + p_{54})p_{62}p_{16}p_{61} + (p_{3} + p_{64})p_{01}p_{52}p_{16} - (p_{3} + p_{64})p_{02}p_{57}p_{76} - (p_{3} + p_{64})p_{02}p_{51}p_{16} + (p_{73} + p_{74})p_{01}p_{52}p_{67}p_{16} - (p_{73} + p_{74})p_{02}p_{57} - (p_{73} + p_{74})p_{02}p_{51}p_{67}p_{16} + (p_{73} + p_{74})p_{61}p_{57}p_{16}p_{02}]$

$= \{p_{52}(1-p_{67}p_{76}) - p_{61}p_{16}p_{52}\}(p_{01} + p_{02}) + [p_{01}(1-p_{67}p_{76})p_{52} - p_{02}p_{51}(1-p_{67}p_{76}) - p_{02}p_{57}p_{61}p_{76}](-1 + p_{15} + p_{16}) + [p_{02}(1-p_{67}p_{76}) - p_{02}p_{61}p_{16}](-1 + p_{51} - p_{52} - p_{57}) + [p_{01}p_{52}p_{16} - p_{02}p_{57}p_{76} - p_{02}p_{51}p_{16}](-1 + p_{61} + p_{67}) + [p_{01}p_{52}p_{67}p_{16} - p_{02}p_{57} - p_{02}p_{51}p_{67}p_{16} + p_{61}p_{57}p_{16}p_{02}](-1 + p_{57} + p_{76})$

$= (1-p_{67}p_{76})[p_{01}p_{15}p_{52} + p_{02}(1-p_{15}p_{51})] + p_{16}p_{67}p_{75}(p_{01}p_{52} - p_{02}p_{51}) - p_{02}(p_{57}p_{75}(1-p_{16}p_{61}) + p_{61}(p_{16} + p_{15}p_{57}p_{76}))$

Coefficients of $m_{51} = \{p_{15}(1-p_{67}p_{76}) + p_{16}p_{67}p_{75} + p_{10}p_{02}p_{25}(1-p_{67}p_{76}) - p_{20}p_{02}p_{15}(1-p_{67}p_{76}) - p_{20}p_{02}p_{75}p_{16}p_{67}\} - [p_{03} \{p_{15}(1-p_{67}p_{76}) + p_{16}p_{67}p_{75}\} - (p_{13} + p_{14})p_{02}p_{25} \{1-p_{67}p_{76}\} + p_{23}p_{02}p_{15}(1-p_{67}p_{76}) + p_{23}p_{02}p_{75}p_{16} - p_{67}(p_{63} + p_{64})p_{02}p_{16}p_{25} - (p_{73} + p_{74})p_{67}p_{16}p_{25}p_{02}]$

$= \{p_{15}(1-p_{67}p_{76}) + p_{16}p_{67}p_{75}\}(p_{01} + p_{02}) + [p_{02}p_{25}(1-p_{67}p_{76})(1-p_{15} - p_{16}) + p_{02}p_{15}(1-p_{67}p_{76}) + p_{02}p_{75}p_{16}p_{67}](-1 + p_{25}) + [p_{02}p_{16}p_{25}(1-p_{61} - p_{62}) + (p_{67}p_{16}p_{25}p_{02})(1-p_{75} - p_{76})$
Coefficients of $m_{52}$

$$= \sum_{i,j} \alpha_{ij} \beta_{ij}$$

$$= \begin{bmatrix}
\alpha_{1,1} & \alpha_{1,2} & \ldots & \alpha_{1,n} \\
\alpha_{2,1} & \alpha_{2,2} & \ldots & \alpha_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
\alpha_{n,1} & \alpha_{n,2} & \ldots & \alpha_{n,n}
\end{bmatrix} \begin{bmatrix}
\beta_{1,1} \\
\beta_{1,2} \\
\vdots \\
\beta_{1,n} \\
\beta_{n,1} \\
\beta_{n,2}
\end{bmatrix}$$

Coefficients of $m_{53}$

$$= \sum_{i,j} \gamma_{ij} \delta_{ij}$$

$$= \begin{bmatrix}
\gamma_{1,1} & \gamma_{1,2} & \ldots & \gamma_{1,n} \\
\gamma_{2,1} & \gamma_{2,2} & \ldots & \gamma_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
\gamma_{n,1} & \gamma_{n,2} & \ldots & \gamma_{n,n}
\end{bmatrix} \begin{bmatrix}
\delta_{1,1} \\
\delta_{1,2} \\
\vdots \\
\delta_{1,n} \\
\delta_{n,1} \\
\delta_{n,2}
\end{bmatrix}$$

Coefficients of $m_{54}$

$$= \sum_{i,j} \epsilon_{ij} \zeta_{ij}$$

$$= \begin{bmatrix}
\epsilon_{1,1} & \epsilon_{1,2} & \ldots & \epsilon_{1,n} \\
\epsilon_{2,1} & \epsilon_{2,2} & \ldots & \epsilon_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
\epsilon_{n,1} & \epsilon_{n,2} & \ldots & \epsilon_{n,n}
\end{bmatrix} \begin{bmatrix}
\zeta_{1,1} \\
\zeta_{1,2} \\
\vdots \\
\zeta_{1,n} \\
\zeta_{n,1} \\
\zeta_{n,2}
\end{bmatrix}$$

Coefficients of $m_{57}$

$$= \sum_{i,j} \eta_{ij} \theta_{ij}$$

$$= \begin{bmatrix}
\eta_{1,1} & \eta_{1,2} & \ldots & \eta_{1,n} \\
\eta_{2,1} & \eta_{2,2} & \ldots & \eta_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
\eta_{n,1} & \eta_{n,2} & \ldots & \eta_{n,n}
\end{bmatrix} \begin{bmatrix}
\theta_{1,1} \\
\theta_{1,2} \\
\vdots \\
\theta_{1,n} \\
\theta_{n,1} \\
\theta_{n,2}
\end{bmatrix}$$
Coefficients of $m_{61}$

$$= P_0P_1P_5 + P_0P_2P_5 - P_0P_2P_6P_1P_6P_2 - P_0P_1P_5P_7P_7 + P_0P_1P_1P_6P_7P_7 - P_0P_2P_5P_7P_7$$

$$= (1-P_6P_7)(P_0P_1 + P_0P_2) + P_1(P_7P_7 - P_6P_7)$$

Coefficients of $m_{63}$

$$= [P_0P_1P_5P_7 + P_1P_6(1-P_5P_7) - P_6P_2P_2P_5](P_0 + P_2)$$

$$+ [P_0P_2P_5P_7](1-P_1P_6) + [P_0P_2P_1P_7P_7 + P_0P_1P_1P_7P_7P_7](-1+P_2) + [P_0P_2P_1P_7P_7P_7P_7P_7P_7](1-P_5P_5) + [P_0P_2P_1P_7P_7P_7P_7P_7P_7](1-P_5P_5 + P_5P_5)$$

$$= P_1P_5P_7P_7P_0 + P_1P_6P_0 - P_1P_0P_5P_7P_7 - P_0P_1P_6P_2P_2$$

$$+ P_0P_2P_5P_7 + P_5P_0P_2P_1P_6P_2$$

$$= P_1P_0P_1P_6P_5P_7P_7P_0 + P_5P_7P_7P_0P_0P_1P_5P_7P_7$$

$$+ P_0P_1P_6P_5P_7P_7P_0P_0P_1P_5P_7P_7$$

Coefficients of $m_{64}$

$$= -[P_0P_1P_5P_7P_7P_0 + P_1P_6(1-P_5P_7) + P_5P_0P_2P_1P_6P_2P_2]$$

$$+ P_0P_2P_5P_7P_7P_0P_0P_1P_5P_7P_7P_0P_0P_1P_5P_7P_7$$
\[
P_{16}P_{01}(1-p_{25}p_{52} - p_{57}p_{75}) + p_{57}p_{75}(p_{01}p_{15} + p_{02} p_{25}) + p_{02}p_{16} p_{25}p_{51}
\]

Coefficients of \( \text{m}_{67} \) = \[
(p_{76} - p_{25}p_{52} p_{76} - p_{15}p_{51}p_{76} + p_{16}p_{51}p_{75})(p_{01} + p_{02})
\]
\[
+ [p_{01}p_{76} - p_{01}p_{76} p_{25}p_{52} + p_{51}p_{02} p_{25}p_{76})(-1 + p_{15} + p_{16}) + [p_{01}p_{52} p_{25}p_{76} - p_{01}p_{16} p_{75}p_{52} + p_{02}p_{76} - p_{15}
\]
\[
p_{51}p_{76} p_{02} + p_{02}p_{75} p_{16}p_{51})(-1 + p_{25}) + [p_{01} p_{15} p_{76} - p_{75}p_{16} p_{01} + p_{02}p_{25} p_{76})(-1 + p_{51} + p_{52} + p_{57}) + [p_{01} p_{16}
\]
\[
- p_{01}p_{52} p_{16}p_{25} + p_{02}p_{51} p_{16}p_{25})(1-p_{75} - p_{76})
\]
\[
= p_{01}p_{16} - p_{01}p_{16} p_{25}p_{52} - p_{01}p_{16} p_{57}p_{75} + p_{57} p_{76} p_{01}
\]
\[
p_{15} + p_{57} p_{76} p_{02}p_{25} + p_{02}p_{51} p_{16}p_{25}
\]
\[
= p_{16} p_{01}(1-p_{25}p_{52} - p_{57}p_{75}) + p_{57} p_{75}(p_{01}p_{15} + p_{02} p_{25} + p_{02} p_{16} p_{25} p_{51}
\]

Coefficients of \( \text{m}_{73} \) = \[-[(p_{15}p_{57} p_{16}p_{67} + p_{52}p_{67} p_{16}p_{25})] + p_{02}(-p_{25}p_{57} - p_{51}p_{67} p_{16}p_{25} + p_{61}p_{57} p_{16}p_{25})
\]
\[
= p_{01} p_{15} p_{57} + p_{01} p_{16} p_{67} - p_{01} p_{52} p_{67} p_{16}p_{25} + p_{02} p_{25} p_{57} + p_{02} p_{51} p_{67} p_{16}p_{25} - p_{02} p_{61} p_{57} p_{16}p_{25}
\]
\[
= p_{01}(p_{15} p_{57} + p_{16} p_{67}(1-p_{25}p_{52})) + p_{02}(p_{25} p_{57} + p_{16}
\]
\[
p_{25}(p_{51} p_{67} - p_{61} p_{57})
\]

Coefficients of \( \text{m}_{74} \) = \[
p_{01}(p_{15} p_{57} + p_{16} p_{67}(1-p_{25}p_{52})) + p_{02}(p_{25} p_{57} + p_{16}
\]
\[
p_{25}(p_{51} p_{67} - p_{61} p_{57})
\]
Coefficients of \( m_{75} \)

\[
\begin{align*}
&= (p_{57} + p_{16}p_{51}p_{67} - p_{61}p_{16}p_{57})(p_{01} + p_{02}) + (p_{01}p_{57}) \\
&\quad - (1 + p_{15} + p_{16}) + (p_{01}p_{16}p_{67}p_{52} - p_{02}p_{57} - p_{02}p_{16}p_{67}p_{51} + p_{02}p_{61}p_{16}p_{57}) (1-p_{25}) + (p_{01}p_{16}p_{57}) (1-p_{51} - p_{52} - p_{57}) + (p_{01}p_{16}p_{57}) (-1 + p_{61} + p_{67}) \\
&= p_{15}p_{01}p_{57} - p_{01}p_{16}p_{67}p_{52}p_{25} + p_{02}p_{57}p_{25} + p_{02}p_{16}p_{67}p_{51}p_{25} - p_{02}p_{61}p_{16}p_{57}p_{25} + p_{01}p_{16}p_{67}p_{52}p_{25}
\end{align*}
\]

Coefficients of \( m_{76} \)

\[
\begin{align*}
&= [p_{67}(1-p_{25}p_{52}) - p_{15}p_{51}p_{67} + p_{61}p_{15}p_{57}](p_{01} + p_{02}) \\
&\quad + [p_{01}p_{67}(1-p_{25}p_{52}) - p_{51}p_{02}p_{25}p_{67} - p_{02}p_{25}p_{57}p_{61}] \\
&\quad (-1 + p_{15} + p_{16}) + [p_{01}p_{52}p_{15}p_{67} + p_{02}p_{67} - p_{02}p_{15}p_{51}p_{67} + p_{02}p_{61}p_{15}p_{57}] (1-p_{23}) + [p_{01}p_{15}p_{67} + p_{02}p_{25}p_{67}] (1-p_{51} - p_{52} + p_{57}) + [p_{01}p_{15}p_{57} + p_{02}p_{25}p_{57}] \\
&\quad (1-p_{61} - p_{67}) \\
&= p_{16}p_{01}p_{67} - p_{16}p_{01}p_{67}p_{25}p_{52} + p_{16}p_{51}p_{02}p_{25}p_{67} - p_{16}p_{02}p_{25}p_{57}p_{61} + p_{01}p_{15}p_{57} + p_{02}p_{25}p_{57}
\end{align*}
\]

\[
\begin{align*}
&= p_{01}(p_{15}p_{57} + p_{16}p_{67}(1-p_{25}p_{52})) + p_{02}(p_{25}p_{57} + p_{16}p_{25}(p_{51}p_{67} - p_{61}p_{57}))
\end{align*}
\]

and using the relation (27-49) of section 2.4, we have

\[
E(T) = \frac{N}{D_{1}}
\] [11]
\[ N_1 = \mu_0[(1-p_{67}p_{76})(1-p_{25}p_{52} - p_{15}p_{51}) - p_{16}p_{61}(1-p_{25}p_{52} - p_{57}p_{75}) - p_{57}p_{75} - p_{16}p_{67}p_{51}p_{75} - p_{61}p_{76}p_{15}p_{57}] + \mu_1[p_{01}(1-p_{67}p_{76})(1-p_{25}p_{52}) - p_{57}p_{75}] + p_{02}p_{25}p_{51}(1-p_{67}p_{76}) + p_{02}p_{25}p_{57}p_{61}p_{76}] + \mu_2[(1-p_{67}p_{76})]
\]
\[ \{p_{01}p_{15}p_{52} + p_{02}(1-p_{15}p_{51})\} + p_{16}p_{67}p_{75}(p_{01}p_{52} - p_{02}p_{51}) - p_{02}(p_{57}p_{75}) + p_{16}(p_{16}p_{61} + p_{15}p_{57}p_{76})]\]

\[ D_1 = [(1-p_{25}p_{52})(1-p_{67}p_{76}) - p_{57}p_{75}](1-p_{01}p_{10}) + [p_{15}p_{51}(1-p_{67}p_{76}) + p_{16}(1-p_{57}p_{75})(p_{02}p_{20} - 1) + p_{16}p_{61}p_{25}p_{52} + p_{10}(-p_{02}p_{25}p_{51}(1-p_{67}p_{76}) - p_{02}p_{25}p_{57}p_{61}p_{76}) - p_{20}(p_{01}\{p_{15}p_{52}(1-p_{67}p_{76}) + p_{16}p_{67}p_{75}p_{52}\} + p_{02}(1-p_{67}p_{76} - p_{57}p_{75})]
\]

2.6 IDLE TIME ANALYSIS OF THE MACHINE IN (0,1)

Let \( M_i(t) \) denote the probability that the system is up initially in regenerative state \( S_i \) at epoch \( t \) without passing through any other regenerative state. It might return to itself through one or more non-regenerative states, so that either it continues to remain in regenerative states \( S_i \) or in a non-regenerative state without visiting any regenerative state including itself. By probabilistic arguments, we have:

\[ M_0(t) = e^{-(\lambda + a_1 + \theta)t}, \quad M_2(t) = e^{-(\lambda + \delta + a_1)t} \]

We observe that the entry to any of the states \( S_0 - S_7 \) is a regenerative point. \( l_i(t) \) is defined as the probability that the machine is idle as epoch \( t \) given that initially it was in state \( S_i \in E \). To obtain it, we consider all possible contingencies. For example,

\( l_0(t) \) is the sum of the following three contingencies:

1. Probability that the system initially up in \( S_0 \) is up at epoch \( t \) without transiting to any regenerative state belongs to \( E \), which is \( M_0(t) \).

2. Probability that the system transits to \( S_i \in E (i=1,2,3) \) during \( (u,u+du) \) and then starting
from $S_i$, it is up at epoch $t$ which is

$$\int_{0}^{t} l_i(t-u) q_0(u) du = q_0(t) \sum l_i(t) \quad (i=1,2,3) \quad [3]$$

Thus

$$I_0(t) = M_0(t) + q_0_1(t) \bigg[ l_1(t) + q_0_2(t) l_2(t) + q_0_3(t) l_3(t) \bigg]$$

$$I_1(t) = q_1_0(t) l_0(t) + q_1_5(t) l_5(t) + q_1_3(t) l_3(t) + q_1_4(t) l_4(t)$$

$$+ q_1_6(t) l_6(t)$$

$$I_2(t) = M_2(t) + q_2_0(t) l_0(t) + q_2_3(t) l_3(t) + q_2_5(t) l_5(t)$$

$$I_3(t) = q_2_3(t) l_0(t)$$

$$I_4(t) = q_4_0(t) l_0(t)$$

$$I_5(t) = q_5_1(t) l_1(t) + q_5_2(t) l_2(t) + q_5_3(t) l_3(t)$$

$$+ q_5_4(t) l_4(t) + q_5_7(t) l_7(t)$$

$$I_6(t) = q_6_1(t) l_1(t) + q_6_3(t) l_3(t) + q_6_4(t) l_4(t) + q_6_7(t) l_7(t)$$

$$I_7(t) = q_7_3(t) l_3(t) + q_7_4(t) l_4(t) + q_7_5(t) l_5(t) + q_7_6(t) l_6(t) \quad [4-11]$$
Taking Laplace transform of (4-11) and computing the relevant elements of the inverse matrix, the Laplace transform of \( I_0(t) \) is seen to be

\[
I_0^*(s) = \frac{N_2(s)}{D_2(s)} \tag{12}
\]

Where

\[
N_2(s) = M_0 \left[(1 - q_3^* q_5^* - q_1^* q_3^*) (1 - q_6^* q_7^*) - q_5^* q_7^* - q_1^* q_6^* (1 - q_5^* q_2^* - q_5^* q_7^*) - q_1^* q_6^* q_1^* q_6^* q_1^* q_6^* \right] + q_2^* \left[ (1 - q_6^* q_7^* - q_5^* q_7^*) - q_1^* q_3^* (1 - q_6^* q_7^*) - q_1^* q_6^* q_7^* - q_6^* q_7^* \right] - q_1^* q_6^* q_1^* \left(1 - q_5^* q_7^* \right) \tag{13}
\]

\[
D_2(s) = (1 - q_0^* q_1^* - q_3^* q_0^*) \left[(1 - q_5^* q_2^*) (1 - q_6^* q_7^*) - q_5^* q_7^* \right] - (1 - q_0^* q_2^* - q_0^* q_3^* q_4^* - q_0^* q_3^* q_5^* - q_0^* q_3^* q_6^* - q_0^* q_3^* q_7^*) - (1 - q_0^* q_2^* - q_0^* q_3^* q_4^* - q_0^* q_3^* q_5^* - q_0^* q_3^* q_6^* - q_0^* q_3^* q_7^*) - (1 - q_0^* q_2^* - q_0^* q_3^* q_4^* - q_0^* q_3^* q_5^* - q_0^* q_3^* q_6^* - q_0^* q_3^* q_7^*) - (1 - q_0^* q_2^* - q_0^* q_3^* q_4^* - q_0^* q_3^* q_5^* - q_0^* q_3^* q_6^* - q_0^* q_3^* q_7^*) - (1 - q_0^* q_2^* - q_0^* q_3^* q_4^* - q_0^* q_3^* q_5^* - q_0^* q_3^* q_6^* - q_0^* q_3^* q_7^*) - (1 - q_0^* q_2^* - q_0^* q_3^* q_4^* - q_0^* q_3^* q_5^* - q_0^* q_3^* q_6^* - q_0^* q_3^* q_7^*) \tag{14}
\]
Using the relations $q_{ij}^{*}(0) = \int_{0}^{\infty} q_{ij}(t) = p_{ij}$ and (27-49) of section 2.4, we have

\[
D_2(0) = [(1-p_{25}p_{52})(1-p_{67}p_{76}) - p_{67}p_{75} - p_{15}p_{51}(1-p_{67}p_{76}) - p_{16}p_{61}(1-p_{67}p_{76}) + p_{16}p_{61}p_{25}p_{52} - p_{16}p_{61}p_{57}p_{75} + p_{15}p_{51}(1-p_{67}p_{76}) + p_{15}p_{51}p_{67}p_{75} + p_{16}p_{61}p_{25}p_{52} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) + p_{15}p_{51}(1-p_{67}p_{76}) - p_{15}p_{51}p_{67}p_{75}]
\]

\[
D_2(0) = 0 \ , \text{ as it should be} \ .
\]

Hence, starting from $S_0$, the steady state availability of the system is

\[
I_0(\infty) = \lim_{s \to 0} s I_0^*(s) = \frac{N_2(0)}{D_2(0)} \ .
\]  

[15] 

To find steady state availability, we first calculate
Substituting (16-17) in (15) and using (27-49) of section 2.4, we get

\[
M_0^* (0) = \int_0^\infty e^{-(\lambda + a_1 + \theta)t} dt = \mu_0
\]  

\[
M_2^* (0) = \int_0^\infty e^{-(\lambda + a_1 + \theta)t} dt = \mu_2
\]

To obtain the value of \(D_2^*(0)\), we collect the coefficients of relevant \(m_{ij}\) in \(D_2^*(0)\).

Coefficients of \(m_{01}\) = \((p_{10} + p_{13} + p_{14})\{(1 - p_{67}p_{76})(1 - p_{25}p_{52}) - p_{57}p_{75}\}
\text{ }+ p_{15}\{p_{52}(1 - p_{67}p_{76})(p_{20} + p_{23}) + (1 - p_{67}p_{76}(p_{53} + p_{54}) + p_{57})p_{67}p_{75} + p_{73} + p_{64}p_{76} + p_{74}\} + p_{16}\{p_{52}p_{67}p_{75}(p_{23} + p_{20}) + p_{67}p_{75}(p_{53} + p_{54}) + (1 - p_{25}p_{52})p_{63} + p_{67}p_{73}\} + (1 - p_{25}p_{52})p_{64} + p_{67}p_{74}) - p_{57}p_{75}\)
\text{ }(p_{63} + p_{64})
\]

\[
= (p_{10} + p_{13} + p_{14} + p_{15} + p_{16})\{(1 - p_{25}p_{52})(1 - p_{67}p_{76})
\text{ }- p_{57}p_{75}p_{67}p_{75} + p_{15}p_{51}(1 - p_{67}p_{76}) - p_{16}p_{61}(1 - p_{25}p_{52}
\text{ }- p_{57}p_{75}) - p_{15}p_{67}p_{76}p_{61}p_{57} - p_{16}p_{67}p_{75}p_{51}
\]
Coefficients of $m_{02} = \{ (1 - p_{25}p_{52})(1 - p_{67}p_{76}) - p_{57}p_{75} - p_{15}p_{51}(1 - p_{67}p_{76}) - p_{15}p_{76}p_{61}p_{57} - p_{16}p_{67}p_{75}p_{51} \} \\
= \{ (1 - p_{25}p_{52})(1 - p_{67}p_{76}) + p_{25}p_{57}p_{61}p_{76} \} + p_{20}(1 - p_{67}p_{76} - p_{57}p_{75} - p_{15}p_{51}(1 - p_{67}p_{76}) - p_{15}p_{76}p_{61}p_{57} - p_{16}p_{67}p_{75}p_{51})

Coefficients of $m_{03} = \{ (1 - p_{25}p_{52})(1 - p_{67}p_{76}) - p_{57}p_{75} - p_{15}p_{51}(1 - p_{67}p_{76}) - p_{15}p_{76}p_{61}p_{57} - p_{16}p_{67}p_{75}p_{51} \} \\
= \{ (1 - p_{25}p_{52})(1 - p_{67}p_{76}) - p_{57}p_{75} - p_{15}p_{51}(1 - p_{67}p_{76}) - p_{15}p_{76}p_{61}p_{57} - p_{16}p_{67}p_{75}p_{51} \} \\
= \{ (1 - p_{25}p_{52})(1 - p_{67}p_{76}) - p_{57}p_{75} - p_{15}p_{51}(1 - p_{67}p_{76}) - p_{15}p_{76}p_{61}p_{57} - p_{16}p_{67}p_{75}p_{51} \}$
Coefficients of $m_{10}$

\[ p_{01}((1 - p_{67}p_{76})(1 - p_{25}p_{52}) - p_{57}p_{75}) + p_{02}(p_{25}p_{51} (1 - p_{67}p_{76}) + p_{25}p_{57}p_{61}p_{76}) \]

\[ = p_{01}((1 - p_{67}p_{76})(1 - p_{25}p_{52}) - p_{57}p_{75}) + p_{02}p_{25}(p_{51} (1 - p_{67}p_{76}) + p_{57}p_{61}p_{76}) \]

Coefficients of $m_{13}$

\[ p_{01}((1 - p_{67}p_{76}) - p_{57}p_{75}) - p_{01}p_{52}(1 - p_{67}p_{76})p_{25} + p_{02}p_{25}p_{51}(1 - p_{67}p_{76}) + p_{25}p_{57}p_{61}p_{76} \]

\[ = p_{01}((1 - p_{67}p_{76})(1 - p_{25}p_{52}) - p_{57}p_{75}) + p_{02}p_{25}(p_{51} (1 - p_{67}p_{76}) + p_{57}p_{61}p_{76}) \]

Coefficients of $m_{14}$

\[ p_{75}p_{76}p_{61} + p_{51}(1 - p_{67}p_{76}) + (p_{20} + p_{23})\{p_{01}p_{52} (1 - p_{67}p_{76}) - p_{02}p_{51}(1 - p_{67}p_{76}) - p_{02}p_{57}p_{61}p_{76} \]

\[ + p_{01}((1 - p_{67}p_{76})(p_{53} + p_{54}) + p_{57}p_{76}(p_{63} + p_{64}) + p_{57}(p_{73} + p_{74})) + p_{03}(-p_{51}(1 - p_{67}p_{76}) - p_{61}p_{57}p_{76}) \]

\[ = -p_{25}p_{01}p_{52}(1 - p_{67}p_{76}) + p_{25}p_{02}p_{51}(1 - p_{67}p_{76}) + p_{01}(1 - p_{67}p_{76}) - p_{01}p_{57}p_{75} \]

……

Coefficients of $m_{15}$

\[ p_{01}((1 - p_{67}p_{76})(1 - p_{25}p_{52}) - p_{57}p_{75}) + p_{02}p_{25}(p_{51} (1 - p_{67}p_{76}) + p_{57}p_{61}p_{76}) \]

Coefficients of $m_{16}$

\[ (p_{20} + p_{23})\{p_{01}p_{52}p_{67}p_{75} - p_{02}p_{51}p_{67}p_{75} - p_{02}p_{61} (1 - p_{57}p_{75}) + p_{25}[-p_{57}p_{52} - p_{01}p_{52}(p_{63} + p_{64}) + p_{67}(p_{73} + p_{74})] + p_{02}p_{51}(p_{63} + p_{64}) + p_{67}(p_{73} + p_{74})) - p_{02}p_{61}(p_{54} + p_{53}) + p_{57}(p_{73} + p_{74}) + p_{03}p_{51}p_{52} \]

\[ + p_{61}(1 - p_{57}p_{75}) + p_{51}p_{67}p_{75} + p_{01}p_{67}p_{75}\{(p_{53} + p_{54}) + p_{01}(p_{63} + p_{64}) + p_{67}(p_{73} + p_{74}) - p_{01}p_{57}p_{75} \]

\[ (p_{63} + p_{64}) - p_{03}p_{51}p_{67}p_{75} - p_{03}p_{61}(1 - p_{57}p_{75}) \]
\[\begin{align*}
\text{Coefficients of } m_{20} &= \left( p_{20} + p_{23} + p_{25} \right) \left[ p_{01} p_{52} p_{67} p_{75} - p_{02} p_{51} p_{67} p_{75} \\
&\quad - p_{02} p_{61} \left( 1-p_{57} p_{75} \right) \right] - p_{01} p_{25} p_{52} \left( 1-p_{67} p_{76} \right) + p_{02} \\
&\quad - p_{25} p_{51} \left( 1-p_{67} p_{76} \right) - p_{02} p_{25} p_{61} + p_{02} p_{25} p_{61} p_{57} p_{76} \, + \\
&\quad + p_{61} \left( 1-p_{57} p_{75} \right) + p_{15} p_{67} p_{75} + p_{01} p_{67} p_{75} - p_{01} p_{67} \\
&\quad - p_{75} p_{51} - p_{01} p_{67} p_{75} p_{52} - p_{01} p_{67} p_{75} p_{57} + p_{01} \left( 1-p_{67} p_{76} \right) \, - p_{01} p_{61} - p_{01} p_{67} p_{75} - p_{01} p_{57} p_{75} + p_{01} p_{57} p_{61} p_{75} \, + \\
&\quad + p_{01} p_{57} p_{75} p_{61} - p_{01} p_{61} \left( 1-p_{57} p_{75} \right) + p_{01} p_{67} \left( 1-p_{57} p_{75} \right) + \\
&\quad + p_{02} p_{61} \left( 1-p_{57} p_{75} \right) - p_{51} p_{67} p_{75} + p_{01} p_{51} p_{67} p_{75} + p_{02} p_{51} p_{67} p_{75} \\
&\quad + p_{01} p_{25} p_{52} \left( 1-p_{67} p_{76} \right) + p_{02} p_{25} p_{51} \left( 1-p_{67} p_{76} \right) \, - p_{01} p_{67} p_{76} + p_{02} p_{25} p_{57} p_{61} p_{76} - p_{01} p_{57} p_{75} \\
&\quad + p_{01} p_{57} p_{75} p_{61} - p_{01} p_{61} \left( 1-p_{57} p_{75} \right) - p_{01} p_{57} p_{61} p_{76} + p_{02} \left( 1-p_{67} p_{76} - p_{57} p_{75} \right) \\
&\quad + p_{15} \left\{ p_{01} p_{52} \left( 1-p_{67} p_{76} \right) - p_{02} p_{51} \left( 1-p_{67} p_{76} \right) - p_{57} p_{61} p_{76} \right\} \, + \\
&\quad + p_{16} \left\{ p_{01} p_{52} p_{67} p_{75} - p_{02} p_{51} p_{67} p_{75} - p_{02} p_{61} \left( 1-p_{57} p_{75} \right) \right\} \, + \\
&\quad + p_{02} \left\{ \left( 1-p_{67} p_{76} - p_{57} p_{75} \right) \right\} \\
&\quad = \left[p_{01} p_{52} \left( 1-p_{67} p_{76} \right) + p_{16} p_{67} p_{75} \right] + p_{02} \left\{ \left( 1-p_{67} p_{76} - p_{57} p_{75} \right) \right\} \\
&\quad + p_{15} \left\{ p_{01} p_{52} \left( 1-p_{67} p_{76} \right) - p_{02} p_{51} \left( 1-p_{67} p_{76} \right) - p_{57} p_{61} p_{76} \right\} \, + \\
&\quad + p_{16} \left\{ p_{01} p_{52} p_{67} p_{75} - p_{02} p_{51} p_{67} p_{75} - p_{02} p_{61} \left( 1-p_{57} p_{75} \right) \right\} \, + \\
&\quad + p_{02} \left\{ \left( 1-p_{67} p_{76} - p_{57} p_{75} \right) \right\} \\
\text{Coefficients of } m_{23} &= p_{01} \left\{ p_{52} \left( 1-p_{67} p_{75} \right) p_{15} + p_{52} p_{16} p_{67} p_{75} \right\} + p_{02} \left\{ \left( 1-p_{67} p_{76} - p_{57} p_{75} \right) - p_{15} p_{51} \left( 1-p_{67} p_{76} \right) - p_{15} p_{57} p_{61} p_{76} - p_{16} p_{51} p_{67} p_{75} - p_{16} p_{61} \left( 1-p_{57} p_{75} \right) \right\}
\end{align*}\]
\[
\text{Coefficients of } m_{25} = \left( p_{01}p_{52}(p_{15}(1-p_{67}p_{76}) + p_{16}p_{67}p_{75}) + p_{02}(1-p_{57}p_{75} - p_{67}p_{76} - p_{15}p_{57}p_{61}p_{76} - p_{16}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75})) \right)
\]

\[
\text{Coefficients of } m_{30} = \left( p_{01}p_{52}(p_{15}(1-p_{67}p_{76}) + p_{16}p_{67}p_{75}) + p_{02}(1-p_{57}p_{75} - p_{67}p_{76} - p_{15}p_{57}p_{61}p_{76} - p_{16}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75})) \right)
\]

\[
= (1-p_{67}p_{76}) + p_{16}p_{67}p_{75} - p_{15}p_{57}p_{76} - p_{16}p_{51}p_{67}p_{75} - p_{16}p_{61}(1-p_{57}p_{75}) \]

Coefficients of $m_{40}$

$$= p_{01}[p_{14}(1-p_{67}p_{76} - p_{57}p_{75}) + p_{15}(p_{54}(1-p_{67}p_{76}) + p_{57}(p_{64}p_{76} + p_{74})) + p_{16}(p_{54}p_{67}p_{75} + (p_{64} + p_{67}p_{74}) - p_{57}p_{64}p_{75}) + p_{25}p_{52}(-p_{14}(1-p_{67}p_{76}) - p_{16}(p_{64} + p_{67}p_{74})))] + p_{02}p_{25}[(p_{54}(1-p_{67}p_{76}) + p_{57}(p_{64}p_{76} + p_{74})) - p_{14}(p_{51}(1-p_{67}p_{76}) + p_{57}p_{61}p_{76}) + p_{16}(p_{51}(p_{64} + p_{67}p_{74}) - p_{61}(p_{54} + p_{57}p_{74}))]
$$

Coefficients of $m_{51}$

$$= p_{10}[p_{02}p_{25}(1-p_{67}p_{76})] + p_{13}[p_{02}p_{25}(1-p_{67}p_{76})] + p_{14}[p_{02}p_{25}(1-p_{67}p_{76}) - p_{20}[p_{02}p_{15}(1-p_{67}p_{76}) + p_{02}p_{16}p_{67}p_{75}] + p_{25}[p_{02}p_{16}(p_{63} + p_{67}p_{73}) + p_{02}p_{16}(p_{64} + p_{67}p_{74})] + p_{15}(1-p_{67}p_{76}) + p_{16}p_{67}p_{75} - p_{03}(p_{15}(1-p_{67}p_{76}) + p_{16}p_{67}p_{75})

= (1-p_{15} - p_{16})[p_{02}p_{25}(1-p_{67}p_{76})] - (1-p_{25})[p_{02}p_{25}(1-p_{67}p_{76}) + p_{02}p_{16}p_{67}p_{75}] + p_{25}[p_{02}p_{16}((1-p_{61} - p_{67}) + p_{67}(1-p_{75} - p_{76}))] + p_{01}p_{15}(1-p_{67}p_{76}) + p_{01}p_{16}p_{67}p_{75} + p_{02}p_{15}(1-p_{67}p_{76}) + p_{02}p_{16}p_{67}p_{75}

= p_{01}(p_{15}(1-p_{67}p_{76}) + p_{16}p_{67}p_{75}) + p_{02}p_{25}(1-p_{67}p_{76} - p_{16}p_{61})$$
Coefficients of $m_{52}$

$$= (p_{01}p_{15} + p_{02}p_{25})(1 - p_{67}p_{76}) + p_{16}(p_{01}p_{67}p_{75} - p_{02}p_{25}p_{61})$$

$$= -(p_{01} + p_{13} + p_{15})[p_{01}p_{25}(1 - p_{67}p_{76})] + (p_{20} + p_{23})[p_{01}p_{15}(1 - p_{67}p_{76}) + p_{01}p_{16}p_{67}p_{75}] + p_{25}$$

$$[-p_{01}p_{16}(p_{63} + p_{64} + p_{67}(p_{73} + p_{74})) + (1 - p_{67}p_{76}) - p_{16}p_{61} - p_{03}(1 - p_{67}p_{76}) + p_{03}p_{16}p_{61}]$$

$$= (-1 + p_{15} + p_{16})[p_{01}p_{25}(1 - p_{67}p_{76})] + (1 - p_{25})$$

$$[p_{01}p_{15}(1 - p_{67}p_{76}) + p_{01}p_{16}p_{67}p_{75}] + p_{25}[-p_{01}p_{16}(1 - p_{67}p_{76}) + p_{01}(1 - p_{67}p_{76}) + p_{02}(1 - p_{67}p_{76})$$

$$+ p_{16}p_{61} - p_{01}p_{16}p_{61} - p_{02}p_{16}p_{61}]$$

$$= p_{01}(p_{15}(1 - p_{67}p_{76}) + p_{16}p_{67}p_{75}) + p_{02}p_{25}(1 - p_{67}p_{76} - p_{16}p_{61})$$

$$= (p_{01}p_{15} + p_{02}p_{25})(1 - p_{67}p_{76}) + p_{16}(p_{01}p_{67}p_{75} - p_{02}p_{25}p_{61})$$

Coefficients of $m_{53}$

$$= (p_{01}p_{15} + p_{02}p_{25})(1 - p_{67}p_{76}) + p_{16}(p_{01}p_{67}p_{75} - p_{02}p_{25}p_{61})$$

$$= (p_{01}p_{15} + p_{02}p_{25})(1 - p_{67}p_{76}) + p_{16}(p_{01}p_{67}p_{75} - p_{02}p_{25}p_{61})$$

Coefficients of $m_{53}$
Coefficients of $m_{57}$

\[
= p_{57} - p_{16}p_{6}p_{75} + p_{15}p_{6}p_{76} - (1 - p_{15} - p_{16})p_{p_{01}p_{75}}
- p_{02}p_{25}p_{6}p_{76} - (1 - p_{25})p_{02}p_{25} + p_{02}p_{15}p_{6}p_{76}
- p_{02}p_{16}p_{6}p_{76} + p_{01}p_{15}(p_{76} - p_{62}p_{76} - p_{67}p_{76} + 1 - p_{75}
- p_{76}) - p_{01}p_{16}p_{75}(1 - p_{61} - p_{66}) + p_{02}p_{25}(p_{76} - p_{62}p_{76}
- p_{67}p_{76} + 1 - p_{75} - p_{76}) - p_{02}p_{25}p_{16}p_{51}(1 - p_{75} - p_{76}) -
(1 - p_{01} - p_{02})p_{75} + p_{61}p_{15}p_{76} - p_{16}p_{75}p_{61}
\]

\[
= - p_{01}p_{15}p_{67}p_{76} + p_{01}p_{15} + p_{01}p_{16}p_{75}p_{67} - p_{02}p_{25}p_{67}p_{76}
+ p_{02}p_{25} - p_{02}p_{25}p_{16}p_{61}
\]

\[
= p_{01}(p_{15}(1 - p_{67}p_{76}) + p_{16}p_{67}p_{75}) + p_{02}p_{25}(1 - p_{67}p_{76}
- p_{16}p_{67})
\]

\[
= (p_{01}p_{15} + p_{02}p_{25})(1 - p_{67}p_{76}) + p_{16}(p_{01}p_{67}p_{75} - p_{02}p_{25}p_{61})
\]

Coefficients of $m_{61}$

\[
= p_{16}(1 - p_{25}p_{52} - p_{57}p_{75}) + p_{15}p_{57}p_{76} + (1 - p_{15} - p_{16})
(p_{02}p_{25}p_{57}p_{76}) - (1 - p_{25})(p_{02}p_{25}p_{57}p_{76} + p_{02}p_{16}(1 -
 p_{57}p_{75})) - p_{02}p_{25}p_{16}(1 - p_{51} - p_{52} - p_{57} + p_{57}(1 - p_{75}
- p_{76})) - (1 - p_{01} - p_{02})(p_{15}p_{57}p_{76} + p_{16}(1 - p_{57}p_{75} -
p_{25}p_{52}))
\]

\[
= p_{02}p_{25}p_{57}p_{76} + p_{02}p_{25}p_{16}p_{51} + p_{01}p_{15}p_{57}p_{76} + p_{01}p_{16}
(1 - p_{25}p_{52} - p_{57}p_{75})
\]

Coefficients of $m_{63}$

\[
= p_{01}p_{15}p_{57}p_{76} + p_{01}p_{16}(1 - p_{25}p_{52} - p_{57}p_{75}) + p_{02}p_{25}
(p_{57}p_{76} + p_{16}p_{51})
\]
Coefficients of $m_{64} = p_0 p_1 p_{15} p_{57} p_{76} + p_0 p_{16} (1 - p_{25} p_{52} - p_{57} p_{75}) + p_0 p_{25} (p_{57} p_{76} + p_{16} p_{51})$

Coefficients of $m_{67} = p_0 p_1 p_{15} p_{57} p_{76} + p_0 p_{16} (1 - p_{25} p_{52} - p_{57} p_{75}) + p_0 p_{25} (p_{57} p_{76} + p_{16} p_{51})$

Coefficients of $m_{73} = p_0 p_1 p_{15} p_{57} + p_0 p_{16} p_{67} (1 - p_{25} p_{52}) + p_0 p_{25} p_{57} (1 - p_{16} p_{61}) + p_0 p_{25} p_{16} p_{5} p_{67}$

\[= p_0 (p_{15} p_{57} + p_{16} p_{67} (1 - p_{25} p_{52})) + p_0 p_{25} (p_{57} (1 - p_{16} p_{61}) + p_{16} p_{5} p_{67})\]

Coefficients of $m_{74} = p_0 p_1 p_{15} p_{57} + p_0 p_{16} p_{67} (1 - p_{25} p_{52}) + p_0 p_{25} p_{57} (1 - p_{16} p_{61}) + p_0 p_{16} p_{5} p_{67}$

Coefficients of $m_{75} = p_{57} - p_{16} p_{5} p_{57} + p_{16} p_{5} p_{67} (1 - p_{15} - p_{16}) p_0 p_{57} + (1 - p_{25}) (p_0 p_{16} p_{6} p_{57} - p_0 p_{25} p_{57} - p_0 p_{16} p_{5} p_{67} + p_0 p_{16} p_{6} p_{57}) + p_0 p_{16} p_{67} (1 - p_{51} - p_{52} - p_{57}) - p_0 p_{16} p_{57} (1 - p_{61} - p_{67}) - (1 - p_{15} - p_{16}) p_0 (p_{57} + p_{51} p_{67} - p_{67} p_{16} p_{57})$

\[= p_0 (p_{15} p_{57} + p_{16} p_{67} (1 - p_{25} p_{52}) + p_0 p_{25} (p_{57} (1 - p_{16} p_{61}) + p_{16} p_{5} p_{67})\]

Coefficients of $m_{76} = p_{67} (1 - p_{25} p_{52} - p_{15} p_{51}) + p_{15} p_{57} p_{61} - (1 - p_{15} - p_{16}) (p_0 p_{67} (1 - p_{25} p_{52}) + p_0 p_{25} p_{51} p_{67} - p_0 p_{25} p_{57} p_{61}) - (1 - p_{25}) (p_0 p_{52} p_{15} p_{67} + p_0 p_{67} (1 - p_{15} p_{51}) + p_0 p_{15} p_{57} p_{61} - p_0 p_{15} p_{67} (1 - p_{51} - p_{52} - p_{57}) + p_0 p_{15} p_{57} (1 - p_{61} - p_{67}) - (1 - p_{15} - p_{16}) p_0 (p_{67} (1 - p_{25} p_{52}) - p_{15} p_{51} p_{67} + p_{51} p_{57} p_{67})$
Using (27-49) of section 2.4 and above computed results, equation (15) gives:

\[
D_2'(0) = \mu_0 \left( (1 - p_{67}p_{76})(1 - p_{25}p_{52}) - p_{57}p_{75} - p_{16}p_{61}(1 - p_{25}p_{52} - p_{57}p_{75}) - p_{15}p_{51} (1 - p_{67}p_{76}) - p_{16}p_{67}p_{75}p_{51} - p_{15}p_{57}p_{76}p_{61} \right) + \mu_1 p_{01} \left( (1 - p_{67}p_{76})(1 - p_{25}p_{52}) - p_{57}p_{75} - p_{16}p_{61}(1 - p_{25}p_{52} - p_{57}p_{75}) - p_{15}p_{51} (1 - p_{67}p_{76}) - p_{16}p_{67}p_{75}p_{51} - p_{15}p_{57}p_{76}p_{61} \right) + \mu_2 p_{01} p_{52} (p_{15}(1 - p_{67}p_{76}) + p_{16}p_{67}p_{75}) + p_{02} \left( (1 - p_{57}p_{75} - p_{67}p_{76}) - p_{15}p_{51}(1 - p_{67}p_{76}) - p_{15} - p_{57}p_{75} - p_{15}p_{53}(1 - p_{67}p_{76}) + p_{57}(p_{63}p_{76} + p_{73}) + p_{16}(p_{53}p_{67}p_{75} + (p_{63} + p_{67}p_{73}) - p_{57}p_{63}p_{75}) - p_{52}(1 - p_{67}p_{76})(p_{13}p_{25} - p_{15}p_{23}) - p_{16}p_{23}p_{67}p_{75} p_{16}p_{25}(p_{63} + p_{67}p_{73})) + p_{02} p_{23}(1 - p_{67}p_{76})(1 - p_{15}p_{51}) - p_{57}p_{75} - p_{75} - p_{15}p_{57}p_{67}p_{76} - p_{16}p_{57}p_{67}p_{75} - p_{16}p_{61}(1 - p_{57}p_{75}) \right) + p_{25}(p_{53}(1 - p_{67}p_{76}) + p_{57}(p_{63}p_{67} + p_{73}) + p_{13}p_{51}(1 - p_{67}p_{76}) + p_{13}p_{57}p_{61}p_{67} + p_{16}p_{51} (p_{63} + p_{67}p_{73}) - p_{16}p_{61}(p_{53} + p_{57}p_{73})) + p_{03} \left( (1 - p_{67}p_{76})(1 - p_{25}p_{52}) - p_{57}p_{75} - p_{15}p_{57}p_{76}p_{61} - p_{67}p_{56}p_{75} + p_{16}p_{67}p_{75}) + p_{61}(-p_{15}p_{57}p_{76} - p_{16}(1 - p_{57}p_{75}) + p_{16}p_{25}p_{52})) \right) + \mu_4 p_{01}(p_{14}(1 - p_{67}p_{76}) - p_{57}p_{75}) + p_{15}(p_{54}(1 - p_{67}p_{76}) + p_{57} (p_{64}p_{76} + p_{74})) + p_{16}(p_{54}p_{67}p_{75} + (p_{64} + p_{67}p_{64}) - p_{57}p_{64}p_{75}) + p_{25}p_{52}(-p_{14} (1 - p_{67}p_{76}) - p_{16}(p_{64} + p_{67}p_{74})) + p_{02} p_{25} \left( p_{25}(1 - p_{67}p_{76}) + p_{57}(p_{64}p_{67} + p_{74}) - p_{14}(1 - p_{67}p_{76}) - p_{16}(p_{64} + p_{67}p_{74})) \right) + \mu_5 \left( (p_{01}p_{15} + p_{02}p_{25})(1 - p_{67}p_{76}) + p_{16}(p_{01}p_{67}p_{75} - p_{02}p_{25} p_{61}) \right) + \mu_6(p_{01}p_{15}p_{57}p_{76} + p_{01}p_{16}(1 - p_{25}p_{52} - p_{57}p_{75}) + p_{25}(p_{57}p_{76} + p_{16}p_{51})) + \mu_7p_{01}(p_{15}p_{57} + p_{16}p_{76}(1 - p_{25}p_{52}) + p_{02}p_{25}(p_{57}(1 - p_{16}p_{61}) + p_{16}p_{51}p_{67}))
\]

Using (18) and (19) in (15) we get the expression for $I_0(\infty)$
2.7 BUSY PERIOD ANALYSIS

(a) BUSY PERIOD ANALYSIS OF MACHINE IN (0,t]

$W_i(t)$ denotes the probability that the machine which is initially busy in job processing in state $S_i \in E$ and continues to be busy at epoch $t$ without an sitting to any other state or returning to itself through one or more regenerative states. By probability arguments, we have

\[
W_1(t) = e^{-(\eta + a_2 + \lambda + \theta + a_3)t}
\]
\[
W_5(t) = e^{-(\eta + a_2 + \lambda + a_4)t}
\]
\[
W_6(t) = e^{-(\eta + a_2 + \theta + a_3)t}
\]
\[
W_7(t) = e^{-(\eta + a_2 + \delta + a_3)t}
\]

Let $B_1^1(t)$ be the probability that at any time the machine is busy with job processing given that the system entered regenerative state $S_i$ at $t = 0$. By probabilistic arguments similar to those in section (2.6), $B_1^1(t)$ are seen to satisfy the following recursive relations:

\[
B_0^1(t) = q_{01}(t) B_1^1(t) + q_{02}(t) B_2^1(t) + q_{03}(t) B_3^1(t)
\]
\[
B_1^1(t) = W_1(t) + q_{10}(t) B_1^1(t) + q_{15}(t) B_1^1(t) + q_{13}(t) B_3^1(t) + q_{14}(t) B_4^1(t)
\]
\[
+ q_{16}(t) B_6^1(t)
\]
\[
B_2^1(t) = q_{20}(t) B_1^1(t) + q_{23}(t) B_3^1(t) + q_{25}(t) B_5^1(t)
\]
\[ B_3^1(t) = q_{30}(t) B_0^1(t) \]

\[ B_4^1(t) = q_{40}(t) B_0^1(t) \]

\[ B_5^1(t) = W_5(t) + q_{51}(t) B_1^1(t) + q_{52}(t) B_2^1(t) + q_{53}(t) B_3^1(t) + q_{54}(t) B_4^1(t) + q_{57}(t) B_7^1(t) \]

\[ B_6^1(t) = W_6(t) + q_{61}(t) B_1^2(t) + q_{63}(t) B_3^2(t) + q_{64}(t) B_4^2(t) + q_{67}(t) B_7^2(t) \]

\[ B_7^1(t) = W_7(t) + q_{73}(t) B_3^3(t) + q_{74}(t) B_4^3(t) + q_{75}(t) B_5^3(t) + q_{76}(t) B_6^3(t) \]  

[5-12]

Taking Laplace transform of (5-12) and computing the value of \( B_0^{1e}(s) \), we have

\[ B_0^{1e}(s) = \frac{N_3(s)}{D_2(s)} \]  

[13]

\[ N_3(s) = W_1^* [q_{01}^* (1 - q_{67}^* q_{76}^*) (1 - q_{25}^* q_{52}^*) - q_{57}^* q_{75}^*) + q_{02} q_{25}^* (q_{51}^* (1 - q_{67}^* q_{76}^*) \]

\[ + q_{57}^* q_{61}^* q_{76}^*) + W_5^* [q_{01}^* (q_{16}^* (1 - q_{67}^* q_{76}^*) + q_{16}^* q_{57}^* q_{75}^*) + q_{02} q_{25}^* (1 - q_{67}^* q_{76}^*) - q_{16}^* q_{57}^* q_{25}^*)] + W_6^* [q_{01}^* (q_{16}^* q_{57}^* q_{75}^* + q_{16}^* (1 - q_{25}^* q_{52}^* - q_{57}^* q_{75}^*) \]

\[ + q_{02} q_{25}^* (q_{57}^* q_{76}^* + q_{16}^* q_{51}^*)] + W_7^* [q_{01}^* (q_{16}^* q_{57}^* + q_{16}^* q_{57}^* (1 - q_{25}^* q_{52}^*)) \]

\[ - q_{02} q_{25}^* (q_{57}^* (1 - q_{16}^* q_{61}^*) + q_{16}^* q_{51}^* q_{67}^*)] \]  

[14]
To find, in steady state, the fraction of time for which the system is under repair, we first calculate

\[ W_1^*(0) = \int_0^\infty W_1(t) dt = \mu_1 \]
\[ W_5^*(0) = \int_0^\infty W_5(t) dt = \mu_5 \]
\[ W_6^*(0) = \int_0^\infty W_6(t) dt = \mu_6 \]
\[ W_7^*(0) = \int_0^\infty W_7(t) dt = \mu_7 \]

Using (27-49) and (50-56) of section 2.4 we get

\[ N_3(0) = \mu_1\{P_{01}((1-p_{67}p_{76})(1-p_{25}p_{52})+p_{57}p_{75})+p_{02}p_{25}\{p_{51}(1-p_{67}p_{76})+p_{57}p_{61}p_{76}\}\}+\mu_5\{p_{01}p_{15}(1-p_{67}p_{76})+p_{16}p_{67}p_{75}\}+p_{02}p_{25}\{1-p_{16}p_{61}-p_{67}p_{76}\}\}
\[ + \mu_6\{p_{01}p_{15}p_{57}p_{76}+p_{16}(1-p_{25}p_{52}-p_{57}p_{75})\}+p_{02}p_{25}\{p_{57}p_{76}+p_{16}p_{51}\}\}
\[ + \mu_7\{p_{01}p_{15}p_{57}+p_{16}p_{67}(1-p_{25}p_{52})\}+p_{02}p_{25}(1-p_{16}p_{61}+p_{16}p_{51}p_{67})\]

Therefore, in long run, the fraction of time for which the machine is busy with job processing is given by

\[ B_0^1(\infty) = \lim_{t \to \infty} B_0^1(t) = \lim_{s \to 0} s B_0^1(s) = \frac{N_3(0)}{D_2'(0)} \]

Using the result (19) of section 2.6 and (19) of this section in (20), we get the expression for \( B_0^1(\infty) \).
(b) EXPECTED BUSY PERIOD OF THE REPAIR MAN IN REPAIR IN (0,t]

Let $B^2_i(t)$ be the probability that repairman is busy in the repair of the machine in the state $S_3$ and $S_5$ at time $t$, given that it was busy at $t = 0$ in state $S_i$. Using the similar arguments as in (5-12) of section (2.7), we have

$$B^2_0(t) = q_{01}(t) B^2_1(t) + q_{02}(t) B^2_2(t) + q_{03}(t) B^2_3(t)$$

$$B^2_1(t) = q_{10}(t) B^2_0(t) + q_{15}(t) B^2_2(t) + q_{13}(t) B^2_3(t) + q_{14}(t) B^2_4(t) + q_{16}(t) B^2_6(t)$$

$$B^2_2(t) = q_{20}(t) B^2_0(t) + q_{23}(t) B^2_3(t) + q_{25}(t) B^2_5(t)$$

$$B^2_3(t) = W_3(t) + q_{30}(t) B^2_0(t)$$

$$B^2_4(t) = W_4(t) + q_{40}(t) B^2_0(t)$$

$$B^2_5(t) = q_{51}(t) B^2_1(t) + q_{52}(t) B^2_2(t) + q_{53}(t) B^2_3(t) + q_{54}(t) B^2_4(t) + q_{57}(t) B^2_7(t)$$

$$B^2_6(t) = q_{61}(t) B^2_1(t) + q_{63}(t) B^2_3(t) + q_{64}(t) B^2_4(t) + q_{67}(t) B^2_7(t)$$

$$B^2_7(t) = q_{73}(t) B^2_3(t) + q_{74}(t) B^2_4(t) + q_{75}(t) B^2_5(t) + q_{76}(t) B^2_6(t)$$
Taking Laplace transform of (1-8) and computing the relevant elements of the inverse matrix, Laplace transform of the pointwise busy period is seen to be

\[ E_0^2(s) = \frac{N_4(s)}{D_2(s)} \]

Where \( D_2(s) \) is obtained earlier in the expression of \( l_0(s) \) and

\[
N_4(s) = W_3(t)\{q_0q_13(1-q_57q_76)-q_57q_75) + q_{15}(q_{53}(1-q_57q_76) + q_{57}(q_{63}q_76 + q_{73})
+ q_{16}(q_{53}q_67q_75 + q_63 + q_{67}q_73) - q_{57}q_{63}q_75 - q_{52}(1-q_67q_76)
\{q_{13} q_{25} - q_{15}q_{23}) - q_{16}q_{23}q_67q_75 + q_{16}q_{25}(q_63 + q_{67}q_73)\}
+ q_{02}[-q_{23}(1-q_67q_76)(1-q_{15}q_{51}) - q_{57}q_75 - q_{15}q_{57}q_61q_76 - q_{16}q_{51}q_{67}q_75
- q_{16}q_{61}(1-q_{57}q_75) + q_{25}[-q_{53}(1-q_67q_76) - q_{57}(q_{63}q_76 + q_{73}) - q_{53}q_{51}
(1-q_67q_76) - q_{13}q_{57}q_61q_76 + q_{16}q_{51}(-q_{13} - q_{67}q_73)\}
- q_{16}q_{61}[-(-q_{53} - q_{57}q_73)]\} - q_{33}[(1-q_67q_76)(1-q_{25}q_{52}) - q_{57}q_75) - q_{51}q_{15}
(1-q_67q_76) + q_{16}q_67q_75\} + q_{61}[-q_{15}q_{57}q_76 - q_{16}(1-q_{57}q_76)
+ q_{16}q_{25}q_{52})] + W_4[-q_{01}q_{14}(1-q_67q_76 - q_{57}q_75) + q_{15}(q_{54}(1-q_67q_76) + q_{57}(q_{64}q_76 + q_74)\} + q_{16}(q_{54}q_{57}q_75(q_{64} + q_67q_74) - q_{57}q_{64}q_75) + q_{25}q_{52}
\{-q_{14}(1-q_67q_76)q_{16}(q_{64} + q_67q_74)) + q_{02}q_{25}[-q_{54}(1-q_67q_76) - q_{57}(q_{64}q_76
+ q_74)) + q_{14}[-q_{51}(1-q_67q_76) - q_{57}q_61q_76) - q_{16}q_{51}(q_64 + q_67q_74)\}
- q_{61}(q_{54} + q_{57}q_74))]]\]

[12]

To find, the fraction of time for which the system is under service of the repairman, we first calculate
Using (13-14) and (27-49) of section 2.4 we get

\[ W_3^*(0) = \int_0^\infty G_1(t)\, dt = \mu_3 \] [13]

\[ W_4^*(0) = \int_0^\infty G_2(t)\, dt = \mu_4 \] [14]

Using (13-14) and (27-49) of section 2.4 we get

\[ N_4(0) = \mu_3\{p_{01}p_{13}(1-p_{67}p_{76})+p_{53}(1-p_{67}p_{76})+p_{57}(p_{63}p_{76}+p_{73})+p_{16}(p_{53}p_{67}p_{75}+(p_{63}+p_{67}p_{73})-p_{57}p_{63}p_{75})-p_{52}((1-p_{67}p_{76})(p_{13}p_{25}-p_{15}p_{23})-p_{16}p_{23}p_{67}p_{75}+p_{16}p_{25}(p_{63}+p_{67}p_{73})))+p_{02}((1-p_{67}p_{76})(1-p_{15}p_{51})-p_{57}p_{75}-p_{15}p_{57}p_{67}p_{76}-p_{16}p_{51}p_{67}p_{76}-p_{16}p_{61}(1-p_{57}p_{75})+p_{25}(p_{53}(1-p_{67}p_{76})+p_{57}(p_{63}p_{76}+p_{73})+p_{13}p_{57}p_{61}p_{76}+p_{16}p_{51}(p_{63}+p_{67}p_{73})-p_{16}p_{61}(p_{53}+p_{57}p_{73}))]+p_{03}((1-p_{67}p_{76})(1-p_{25}p_{52})-p_{57}p_{75})-p_{51}(1-p_{67}p_{76}+p_{16}p_{67}p_{75})+p_{61}(-p_{15}p_{57}p_{76}-p_{16}(1-p_{57}p_{75})+p_{16}p_{25}p_{52})]+\mu_4\{p_{01}(p_{14}(1-p_{67}p_{76}-p_{57}p_{75})+p_{15}(p_{54}(1-p_{67}p_{76})p_{57}(p_{64}p_{76}+p_{74})+p_{16}(p_{54}p_{67}p_{75}+(p_{64}+p_{67}p_{74})-p_{57}p_{64}p_{75})+p_{25}p_{52}(-p_{14}(1-p_{67}p_{76})-p_{16}(p_{64}+p_{67}p_{74})))+p_{02}p_{25}((-p_{54}(1-p_{67}p_{76})-p_{57}(p_{64}p_{76}+p_{74})+p_{14}(-p_{51}(1-p_{67}p_{76})-p_{57}p_{61}p_{76})-p_{16}(p_{51}(p_{64}+p_{67}p_{74})-p_{61}(p_{54}+p_{57}p_{74})))\} \] [15]

Therefore, in long run, the fraction of time for which the system is under the repair of the repairman is given by

\[ B_0^2(\infty) = \lim_{t \to \infty} B_0^2(t) = \lim_{s \to 0} s B_0^2(s) = \frac{N_4(0)}{D_4^*(0)} \] [16]

Using results (27-49) of section 2.4 and (15) in (16), we get the expression for \( B_0^2(\infty) \).
2.8 COST ANALYSIS

The cost analysis of the system can be carried out by considering the expected busy and idle time of the machine and expected repair time of the repair facility in (0, t]. We have:

Expected repair time of the machine in (0, t] is
\[ \mu_R(t) = \int_0^t B_0^2(t) \, dt \]  \[1\]

Expected idle time of the machine in (0, t] is,
\[ \mu_1(t) = \int_0^t I_0(t) \, dt \]  \[2\]

so that
\[ \mu_1^*(s) = \frac{I_0^*(s)}{s} \]

Expected busy time of the machine in (0, t] is,
\[ \mu_m^*(t) = \int_0^t B_0^1(t) \, dt \]  \[3\]

so that
\[ \mu_m^*(s) = \frac{B_0^1(s)}{s} \]

\[ G(t) = \text{expected revenue earned by the production station in (0, t]} - \text{expected repair cost of the service facility} \]
\[ = C_1 \mu_m(t) - C_2 \mu_1(t) - C_3 \mu_R(t) \]  \[4\]
The expected cost earned by the system in steady state is

\[ G = \lim_{t \to \infty} \frac{G(t)}{t} = \lim_{s \to 0} s^2 G^*(s) = C_1 \mu_m - C_2 \mu_1 - C_3 \mu_R \]  

[5]

\( C_1 \) is the revenue per unit up time to be incurred and \( C_2 \) and \( C_3 \) are the loss per unit time to be incurred when no job is available for processing on the machine (idle time) and cost per unit time is given to the repairman respectively.

2.9 MODEL B

This model deals with the profit analysis of a production station that has one machine on which jobs are processed one by one. The analysis is carried out under the supposition that during the procession of a job, if a job arrives, then it is terminated at once.

2.10 (a) SYSTEM DESCRIPTION AND ASSUMPTIONS

1. Initially, one machine is operative with a diminishing load and temperature condition is normal.
2. The job arrives at the production station following the Poisson law. The processing time of the job is negative exponential.
3. The jobs are processed one by one on the machine.
4. The machine fails because of temperature fluctuation (type two) or hardware fault (type one).
5. There is a single repair facility that repairs both types of failures.
6. After repair the machine works as new.
The failure and repair time distribution are negatively distributed while repair time distributions are arbitrary.
Assumptions other than the above are similar to model A.

(b) NOTATION

All notations are similar to model A.

(c) STATE SPACE OF THE MODEL

The production station under study can be found in any one of the states at time t, shown in figure B.1.

2.11 TRANSITION PROBABILITES AND SOJOURN TIMES

Simple probabilistic arguments yield the following equations:

\[ p_{01} = \lambda [A_{11}]^{-1}, \quad p_{02} = \theta [A_{11}]^{-1}, \quad p_{03} = a_1 [A_{11}]^{-1}, \quad p_{04} = a_2 [A_{11}]^{-1} \]

\[ p_{10} = \theta [A_{12}]^{-1}, \quad p_{13} = a_2 [A_{12}]^{-1}, \quad p_{14} = a_3 [A_{12}]^{-1}, \quad p_{15} = \theta [A_{12}]^{-1} \]

\[ p_{20} = \delta [A_{13}]^{-1}, \quad p_{23} = a_1 [A_{13}]^{-1}, \quad p_{25} = \lambda [A_{13}]^{-1}, \quad p_{30} = 1 = p_{40} \]

\[ p_{51} = \delta [A_{14}]^{-1}, \quad p_{52} = \eta [A_{14}]^{-1}, \quad p_{53} = a_2 [A_{14}]^{-1}, \quad p_{54} = a_4 [A_{14}]^{-1} \]

\[ A_{11} = (\theta + \lambda + a_1), \quad A_{12} = (\theta + \eta + a_2 + a_3), \quad A_{13} = (\delta + \lambda + a_1) \]
Up state
Down state
Regenerative point
Fig. 6: State Transition Diagram.
Similarly

$$\mu_0 = [A_{11}]^{-1}, \quad \mu_1 = [A_{12}]^{-1}, \quad \mu_2 = [A_{13}]^{-1}, \quad \mu_5 = [A_{14}]^{-1}$$

$$\mu_i = \int_0^\infty G_{i-2}(t) \, dt \quad (i=3,4)$$

### 2.12 TIME TO SYSTEM FAILURE

By employing the arguments for the regenerative process we obtain the following recursive relation for \( \pi_i(t) \):

$$\pi_0(t) = \sum_{j=1,2} (Q_{0j}(t) \pi_j(t) + Q_{03}(t))$$

$$\pi_1(t) = \sum_{j=0,5} (Q_{1j}(t) \pi_j(t) + \sum_{j=3,4} Q_{1j}(t))$$

$$\pi_2(t) = \sum_{j=0,5} (Q_{2j}(t) \pi_j(t) + Q_{23}(t))$$

$$\pi_5(t) = \sum_{j=1,2} (Q_{5j}(t) \pi_j(t) + \sum_{j=3,4} Q_{5j}(t))$$

By taking the Laplace transforms of equations (1-4) and after solving for \( \hat{\pi}_0(s) \), it is found that

$$\text{MTSF} = E(T) = -\frac{d}{ds} \hat{\pi}_0(s)|_{s=0} = \frac{N_1}{D_1}$$
Where

\[ N_1 = \mu_0(1 - p_{25}p_{52} - p_{15}p_{51}) + \mu_1[p_{01}(1 - p_{25}p_{52}) + p_{02}p_{25}p_{51}] + \mu_2[p_{01}p_{15}p_{52} + p_{02}(1 - p_{15}p_{51})] + \mu_5[p_{01}p_{15} + p_{02}p_{25}] \]  

[6]

and

\[ D_1 = (1 - p_{25}p_{52})(1 - p_{10}p_{01}) - p_{02}p_{20}(1 - p_{15}p_{51}) - p_{15}p_{51} - p_{10}p_{02}p_{51}p_{25} - p_{01}p_{20}p_{15}p_{52} \]  

[7]

2.13 IDLE TIME ANALYSIS OF THE MACHINE IN (0,t]

Let \( I_0(t) \) be the probability that machine is idle at an instant \( t \), given that system was in state \( S_0 \) at \( t = 0 \). Hence, we get the following:

\[ I_0(t) = e^{-(\lambda + a_1 + \delta)t} + \sum_{j=1} I_j(t) \]

\[ I_1(t) = \sum_{j=0,3,4,5} (q_{0j}(t) c \ I_j(t)) \]

\[ I_2(t) = e^{-(\lambda + a_1 + \delta)t} + \sum_{j=0,3,5} (q_{2j}(t) c \ I_j(t)) \]

\[ I_i(t) = (q_{i0}(t) c \ I_0(t)) \ (i = 3,4) \]

\[ I_5(t) = \sum_{j=1} (q_{5j}(t) c \ I_j(t)) \]  

[1-6]
By taking the Laplace transforms these relations and solving for \( I^\ast(s) \), the steady state idle time \( I_0 \) of the system is

\[
I_0 = \frac{N_2}{D_2}
\]  

where

\[
N_2 = \mu_0[1 - p_{15}p_{25} - p_{25}p_{52}] + \mu_2[p_{01}p_{15}p_{25} + p_{02}(1 - p_{15}p_{51})]
\]

\[
D_2 = \mu_0[1 - p_{15}p_{25} - p_{25}p_{52}] + \mu_1[p_{01}(1 - p_{25}p_{52}) + p_{02}p_{25}p_{51}] + \mu_2[p_{01}p_{15}p_{25} + p_{02}(1 - p_{15}p_{51})] + \mu_5(p_{01}p_{15} + p_{02}p_{25})
\]

2.14 BUSY PERIOD ANALYSIS OF THE MACHINE IN \((0,t]\)

By probabilistic arguments similar to those in 2.7 \( B_1^i(t) \) are seem to satisfy the following relations

\[
B_0^1(t) = \sum_{j=1} (q_0^i(t) \quad B_j^1(t))
\]

\[
B_1^1(t) = e^{-(a_2 + a_3 + \eta \theta)t} + \sum_{j=0,3,4,5} (q_1^i(t) \quad B_j^1(t))
\]

\[
B_2^1(t) = \sum_{j=0,3,5} (q_2^i(t) \quad B_j^1(t))
\]

\[
B_3^1(t) = (q_0^i(t) \quad B_0^1(t)) (i = 3,4)
\]
By taking Laplace transforms of equations (1-6) and solving for $B_0^+(s)$, the fraction of time for which the machine is busy in job processing is given by

$$B_0^1(t) = e^{-(a_2 + a_4 + \eta + \delta)t} + \sum_{j=0,3,4,5} (q_j(t) c) B_j^1(t)$$

[1-6]

By taking Laplace transforms of equations (1-6) and solving for $B_0^+(s)$, the fraction of time for which the machine is busy in job processing is given by

$$B_0^1 = \lim_{t \to \infty} B_0^1(t) = \lim_{s \to 0} s M_0^+(s) = \frac{N_3}{D_2}$$

[7]

Where

$$N_3 = (a_1 + a_2 + \eta + \theta)^{-1}[p_{01}(1 - p_{25}p_{52}) + p_{51}p_{02}p_{25}] + (a_1 + a_2 + \eta + \theta)^{-1}
[ p_{04}p_{15} + p_{02}p_{25} ]$$

2.15 EXPECTED BUSY PERIOD OF THE REPAIRMAN IN REPAIR IN (0,t]

Probabilistic arguments similar to those in 2.7, $B_1^2(t)$ are seen to satisfy the following relations:

$$B_0^2(t) = \sum_{j=1}^3 (q_0(t) c) B_j^2(t)$$

$$B_1^2(t) = \sum_{j=0,3,4,5} (q_1(t) c) B_j^2(t)$$

$$B_2^2(t) = \sum_{j=0,3,5} (q_2(t) c) B_j^2(t)$$
By taking Laplace transforms of equation (1-6) and solving for $B_0^*(s)$, the fraction of time for which the system is under the service of the repairman is given by

$$B^2_0(t) = \bar{G}_{i-2}(t) + (q_{i0}(t) \bar{G} B^2_0(t)) \quad (i = 3,4)$$

$$B^2_0(t) = \sum_{j=1}^{4} (q_{5j}(t) \bar{G} B^2_j(t))$$

[1-6]

By taking Laplace transforms of equation (1-6) and solving for $B_0^*(s)$, the fraction of time for which the system is under the service of the repairman is given by

$$B^2_0 = \lim_{t \to \infty} B^2_0(t) = \lim_{s \to 0} s B^*_0(s) = \frac{N_4}{D_2} \quad [7]$$

Where

$$N_4 = \mu_3 [p_{01}(p_{13}(1-p_{25}p_{52}) - p_{15}(p_{53} + p_{23}p_{52}) + p_{02}(p_{23} + p_{25}p_{53}) + p_{03}(1-p_{25}p_{52}))
+ p_{51}[p_{02}(p_{13}p_{25} - p_{15}p_{23}) - p_{03}p_{13})] + \mu_4[p_{01}(p_{14}(1-p_{25}p_{52}) + p_{15}p_{54}) + p_{02}p_{25}
(p_{54} + p_{14}p_{52})]$$

2.16 COST ANALYSIS

The cost benefit analysis of the system can be carried out by considering the expected busy period and idle time of the service station in $(0,t]$. Therefore

$$G(t) = \text{Expected revenue earned by the service station during } (0,t] - \text{expected repair cost of the service facility}$$
$$= C_1\mu_m(t) - C_2\mu_1(t) - C_3\mu_R(t) \quad [1]$$

The expected cost earned by the system in a steady state is

$$G = \lim_{t \to \infty} \frac{G(t)}{t} = \lim_{s \to 0} s^2 G^*(s)$$
Where $C_1, C_2$ are the revenue and loss per unit up time to be incurred when the service station is busy and not busy respectively and $C_3$ is the cost per unit of time which is given to repairman.

2.17 GRAPHICAL REPRESENTATION

The effect of the job arrival rate $\lambda$, on the expected profit for varying values of $n_1(a_1, a_2), (a_3, a_4)$ and $(\theta, \delta)$ are studied in Fig. B.2-5, respectively, for the fixed values of the other parameters in the following case:

$$g_i(t) = r_i e^{-r_i t} \quad (i = 1, 2)$$
EFFECT OF JOB ARRIVAL RATE ON EXPECTED PROFIT FOR VARYING VALUES OF $\eta$

**Fig B.2**

EFFECT OF JOB ARRIVAL RATE ON EXPECTED PROFIT FOR VARYING VALUES OF $a_1$ and $a_2$

**Fig B.3**
EFFECT OF JOB ARRIVAL RATE ON EXPECTED PROFIT FOR VARYING VALUES OF $a_3$ and $a_1$.

**Fig B.4**

EFFECT OF JOB ARRIVAL RATE ON EXPECTED PROFIT FOR VARYING VALUES OF $\theta$ and $\delta$.

**Fig B.5**