CHAPTER I

INTRODUCTION
CHAPTER I

INTRODUCTION

1.1 Measurement of production efficiency posed a number of challenges to applied researchers in the fields of MANAGEMENT SCIENCE, APPLIED STATISTICS, INFORMATION THEORY and ECONOMICS. This problem is examined and attempted to solve by two distinct approaches, viz., (i) the parametric approach and (ii) the non-parametric approach.

Whenever a number of units participate in production of one or more outputs employing one or more inputs, they need not be equally efficient. These production units differ from each other in a number of ways such as, the techniques they employ, their ability to (i) reallocate inputs and/or outputs in response to changes in costs of inputs and/or prices of outputs, (ii) to reduce scale inefficiency by increasing or decreasing the scale of production.

Before setting goals to achieve productive efficiency the producer and the policy maker always wish to know the extent of inefficiency of production. Farrell* in

his path breaking article introduced three major efficiency concepts two at the firm level and one at industry level, viz., (i) TECHNICAL EFFICIENCY, (ii) PRICE EFFICIENCY and (iii) STRUCTURAL EFFICIENCY. While the first two efficiencies are at firm level, the later refers to industry. To measure technical and price efficiencies Farrell assumes that the FRONTIER PRODUCTION FUNCTION is piece-wise linear and homogeneous of degree one.*

Since the frontier production function is linear homogeneous, the REFERENCE TECHNOLOGY** used by FARRELL for efficiency measurement is the UNIT ISOQUANT.

* The function that associates with each input combination maximum possible output is called a frontier production function. If its ISOQUANTS are PIECE-WISE LINEAR we call the production frontier as PIECE-WISE LINEAR. Suppose inputs are increased proportionally by $\lambda$, if this leads to an increase in outputs by the same proportion $\lambda$, then the production frontier is homogeneous of degree one.

The locus of all input combinations which yields the same level of output is called an ISOQUANT.

** The ISOQUANT used by Farrell for efficiency measurement is constructed by observed inputs and outputs. This isoquant is called REFERENCE TECHNOLOGY because relative to this the efficiency of each production unit is computed.
Figure (1.1.1)

The reference technology for Farrell efficiency measurement is constructed by the production units A, B, C and D. Since the production frontier \( U = f(x_1, x_2) \) is linear homogeneous,

\[
l = f \left( \frac{x_1}{u}, \frac{x_2}{u} \right)
\]

which is nothing but UNIT ISOQUANT.

To measure technical efficiency of the point P, we compare it with point Q on the unit isoquant. To measure cost efficiency or price efficiency of P the inefficient point P is compared with the point R whose associated cost
equals the cost at the point B that belongs to the Farrell's unit isoquant. Third Farrell's efficiency is 'STRUCTURAL EFFICIENCY' which refers to the efficiency of the INDUSTRY on the whole. Farrell's efficiency measurement is deficient in a number of ways: (i) since the frontier is assumed to be linear homogeneous, the implicit returns to scale are CONSTANT, thus there is no way to measure SCALE INEFFICIENCY, (ii) it admits zero marginal products, but not negative marginal products, (iii) it assumes a full frontier.*

Farrell's efficiency estimation approach is NON-PARAMETRIC, therefore, no classical production function is postulated for estimating productive efficiency.

Kopp** extended Farrell's measures by assuming a reference technology that need not necessarily be linear homogeneous. Thus, Kopp's measures include the measure of scale inefficiency. However, his measures vary depending

* A frontier production function is said to be full if the outputs of various production units fall either on or below the frontier. However, when a stochastic frontier is considered for efficiently measurement, some of the observed outputs are allowed to fall above the frontier so that frontier efficiency estimators are ROBUST.

upon the choice of the production frontier. Kopps frontier is a full parametric frontier as such it can be estimated either by statistical procedures or by mathematical programming.

Of the two approaches to measure productive efficiency the PARMETRIC APPROACH postulates a parametric frontier such as (i) Cobb-Douglas (ii) translog (ii) Zellner-Revankar's variable returns to scale and so on.* Such a production frontier can be estimated by either statistical


procedures or mathematical programming such as linear
programming technique.*

* Aigner, D.J., C.A.K. Lovell and P. Schmidt, 'Formula-
tion and estimation of scholastic frontier production
function models', Journal of Economics, Vol.23 (1977),
pp.21-37.

Aigner, D.J., and Chu, S.F., 'On estimating the
Industry Production Function', American Economic

Green, W.H., 'A Gamma distributed stochastic frontier
141-163.

Green, W.H., 'Maximum Likelihood Estimation of Eco-
metric Frontier Functions', Journal of Econometrics,

Jondrow, J., Lovell, C.A.K., Materov, I.S., and
Schmidt, P., 'On the estimation of technical efficiency
in the stochastic frontier production function model',

Lovell, C.A.K. and P. Schmidt, 'A Comparison of alter-
native approaches to the measurement of productive
efficiency', Applications of Modern Productive theory :
Efficiency and Productivity, Kluwer Academic Publishers
Boston, M.A.,

Schmidt, P., 'On the Statistical estimation of param-
etric frontier production functions', Review of

Timmer, C.P. .Using probabilistic, frontier production
function to measure technical efficiency, Journal of
In the non-parametric approach of measuring productive efficiency a frontier is constructed by the observed inputs and outputs and the various efficiencies are estimated by mathematical programming such as linear programming in relation to the empirical production frontier. In non-parametric approach two methods are prominent, (i) the DATA ENVELOPMENT ANALYSIS introduced by Banker, Charnes and Cooper* and (ii) the method introduced by Shephard and Rolf Fare**.


Both the non-parametric approaches are axiomatic. The approach of Banker, Cooper and Charness is based on the following axioms, which characterise the production possibility set $T$:

**AXIOM-I** : $(x_j, u_j) \in T$, $j = 1, 2, \ldots, n \Rightarrow$

$$\sum_{j}^{n} \lambda_j (x_j, u_j) = (x', u', \lambda_j u_j) \in T, \lambda_j \geq 0, \sum_{j}^{n} \lambda_j = 1.$$

**AXIOM-II** : (a) $(x, u) \in T \Rightarrow (\overline{x}, u) \in T$, where $\overline{x} \geq x$.

(b) $(x, u) \in T \Rightarrow (x, \overline{u}) \in T$, where $\overline{u} \leq u$.

**AXIOM-III** : $T$ is the intersection of all production possibility sets which satisfy axioms I and II.

Thus, the production possibility set that satisfies axioms I, II and III is written as follows:

$$T = \left\{ (x, u) : x \geq \sum_{j=1}^{n} \lambda_j x_j, \ x \leq \sum_{j=1}^{n} \lambda_j u_j, \ \lambda_j \geq 0, \sum_{j}^{n} \lambda_j = 1 \right\}.$$

Figure (1.1.2)
As already mentioned the non-parametric approach of Färe et al., is also axiomatic. They suggest three orientations to measure technical efficiency viz., (i) the input orientation, (ii) the output orientation and (iii) the graph orientation. Each approach depends on a number of axioms.

AXIOMS OF INPUT ORIENTATION PROBLEMS:

Let \( L(u) \) be the input correspondence associated with the output vector \( u \), where \( L(u) \) is the collection of all input vectors which can produce at least \( u \).

\begin{align*}
\text{L.1.} & \quad 0 \notin L(u), \text{ for } u \succ 0, \text{ and } L(0) = \mathbb{R}^n_+ \quad (\ast) \\
\text{L.2.} & \quad ||u|| \to \infty \text{ as } \lambda \to \infty \Rightarrow \bigcap_{l=1}^{\infty} L(u_l) = \emptyset \\
\text{L.3.} & \quad x \in L(u) \Rightarrow \lambda x \in L(u), \text{ whenever } \lambda \succ 1. \\
\text{L.4.} & \quad L \text{ is a CLOSED CORRESPONDENCE} \\
\text{L.5.} & \quad L(\theta u) \subseteq L(u), \theta \succ 1.
\end{align*}

(\ast) \( u \succ 0 \) means at least one component of \( u \) is positive.
\( u \succ\succ 0 \) means all the components of \( u \) are zeroes is a possibility.
AXIOMS OF OUTPUT CORRESPONDENCE:

Let $P(x)$ be an output correspondence associated with the input vector $x$, where $P(x)$ is the collection of all output vectors which can be producable by the input vector $x$.

**P.1** $P(o) = \{ o \}$

**P.2** $P(x)$ is bounded, $x \in R^+$.  

**P.3** $P(x) \subseteq P(\lambda x)$, whenever $\lambda \geq 1$.

**P.4** $P$ is a CLOSED CORRESPONDENCE.

**P.5** $x \in P(x) \Rightarrow \exists u \in P(x), 0 \leq \theta \leq 1$.

There is DUALITY between $L(u)$ and $P(x)$. Thus, $x \in L(u) \iff u \in P(x)$.

AXIOMS OF GRAPH:

The graph of a production technology is defined by,

$$GR = \{(x, u) : \begin{array}{l} x \in L(u), \quad u \in R^n \\ \leftarrow \left\{ (x, u) : u \in P(x), \quad x \in R^+ \right\} \end{array}$$

**GR.1** $0 \in GR, \quad (0, u) \in GR \Rightarrow u = 0$.

**GR.2** $GR \cap \left\{ (x, u) : \begin{array}{l} x \leq \bar{x} \leftarrow \left\{ \right\} \end{array} \right\}$ is bounded for each $\bar{x} \in R^+$.

**GR.3** $(x, u) \in GR \Rightarrow (\lambda x, u) \in GR$ for $\lambda \geq 1$.

**GR.4** $GR$ is CLOSED SET.

**GR.5** $(x, u) \in GR \Rightarrow (x, \theta u) \in GR, 0 \leq \theta \leq 1$.

10
Figure (1.1.3) is an input correspondence that satisfies the axioms L.1 - L.5, figure (1.1.4) is an output correspondence that is consistent with the axioms P.1 - P.5. Figure (1.1.5) represents a graph that satisfies the axioms GR.1 - GR.5.

Input orientation efficiency measurement considers certain subsets of the input correspondence such as Isoqunt L(u), weak efficient L(u) and efficient L(u), as reference technology. Output orientation efficiency measurement recognises certain similar subsets of P(x) as reference sets. Graph efficiency measurement is based on subsets of graph as reference technology.

1.2 In the present study it is proposed to estimate (a) pure technical efficiency, (b) weak technical efficiency (c) overall technical efficiency, (d) congestion efficiency, (e) allocative efficiency, and (f) scale efficiency for total manufacturing sectors of various major states, viz., maharashtra, tamil nadu, uttar pradesh, gujarat, west bengal, madhya pradesh, andhra pradesh, karnataka, bihar and punjab.

The above efficiencies are computed by (i) input, (ii) output, and (iii) graph approaches. While input oriented measurement assumes cost minimization, the output orientation is consistent with revenue maximization. In the graph
oriented efficiency measurement the implicit assumption is PROFIT MAXIMIZATION.

Input based efficiency measurement enquires possible input reduction holding output constant. On the other hand, output oriented efficiency enquires possible output augmentation when input vector is held at constant level. The GRAPH approach to efficiency measurement enquires about possible radial input reduction and radial output augmentation, thereby allows both input and output vectors to vary.

1.3 THE DATA:

The data are provided by Annual Survey of Industries (ASI) for the year 1992. 85 per cent of the contribution to the country's total VALUE ADDED is due to the ten states selected for study.

The variables choosen for efficiency measurement are (a) VALUE ADDED, (b) TOTAL NUMBER OF EMPLOYEES, (c) FIXED CAPITAL, (d) TOTAL EMOLUMENTS.

(a) **VALUE ADDED**: Value added is the increment to the value of goods and services that is contributed by the factory and is obtained by deducting the value of total inputs and depreciation from value of output.

(b) **EMPLOYEES**: Employees include all workers and persons receiving wages and holding supervisory or managerial positions engaged in administrative office, storekeeping section and welfare section, sales department as also those engaged in purchase of raw materials etc., or production of fixed assets for the factory and watch and ward staff.

(c) **FIXED CAPITAL**: Fixed capital represents the depreciated value of fixed assets owned by the factory as on the closing day of the accounting year. Fixed assets are those which have a normal productive life of more than one year.

(d) **TOTAL EMOLUMENTS**: Total emoluments is the sum of wages paid to the workers and salaries paid to non-workers.
1.4 **CHAPTER SCHEME** :

The thesis is divided into five chapters as follows:

CHAPTER - I : INTRODUCTION

CHAPTER - II : REVIEW OF LITERATURE

CHAPTER - III: THEORY AND METHODOLOGY

CHAPTER - IV : EMPIRICAL INVESTIGATION

CHAPTER - V : SUMMARY AND CONCLUSIONS.