6. DATA ANALYSIS

6.1 GENERAL:

The raw data of water levels and sediment concentration have been analysed to determine discharge, average flow depth \( h \), average velocity of flow \( U \), sediment discharge per unit width \( q_s \), Reynolds no. \( R_e \), and Froude No. \( F_r \); for every experimental run. The water discharge \( q_w \) has been plotted against the sediment discharge \( q_s \). The variation is given in Fig. 6.1:i-xxviii below:

Fig. 6.1:i-iv : Existence of hysteresis between sediment and water discharges for hydrographs H-1 to H-4.
Fig. 6.1. (v-x) : Existence of hysteresis between sediment and water discharges for hydrographs H-5 to H-10.
Fig. 6.1.(xi-xvi) : Existence of hysteresis between sediment and water discharges for hydrographs H-11 to H-16.
Fig. 6.1 (xvii-xxii): Existence of hysteresis between sediment and water discharges for hydrographs H-17 to H-22.
Fig. 6.1.(xxiii-xxviii) : Existence of hysteresis between sediment and water discharges for hydrographs H-23 to H-28.

From Fig.(6.1) it is clear that there is a prominent clockwise hysteresis that has been observed in all the 28 hydrographs. Further analysis has been carried out to yield the values of the four dimensionless parameters viz.,
Using different sets of values for all the above-mentioned four dimensionless parameters, regression analysis has been carried out and equations developed among these four dimensionless parameters, for both rising and falling stages of the hydrographs.

6.2 REGRESSION ANALYSIS:

Let \( x_1 = \frac{q_i}{\rho_s g^{1/2}d^{3/2} \left( \Delta \rho_i \right)^{1/2} \rho} \), \( x_2 = \frac{U}{\Delta h_i / \Delta t} \), \( x_3 = \frac{h}{d} \) and \( x_4 = \frac{S}{\Delta \rho_i / \rho} \)

Then one can write functional relationship expressed by Eq.(3.6) as

\[ x_1 = f(x_2, x_3, x_4) \]  \hspace{1cm} (6.1)

Using the multiple linear regression equation of the form (Chow, 1964)

\[ x_1 = b_1 x_2 + b_2 x_3 + b_3 x_4 + b_4 \]  \hspace{1cm} (6.2)

Taking logarithm of both sides we get:

\[ \log x_1 = \log b_1 + B_2 \log x_2 + B_3 \log x_3 + B_4 \log x_4 \]  \hspace{1cm} (6.3)

Let \( X_1 = \log x_1 \), \( B_1 = \log b_1 \), \( X_2 = \log x_2 \), \( X_3 = \log x_3 \) and \( X_4 = \log x_4 \)

Then Eq.(6.3) may be rewritten as

\[ X_1 = B_1 + B_2 X_2 + B_3 X_3 + B_4 X_4 \]  \hspace{1cm} (6.4)

The constants can be determined using the following equations:

\[ B_2 \sum (X_2 - \bar{X}_2)^2 + B_3 \sum (X_2 - \bar{X}_2)(X_3 - \bar{X}_3) + B_4 \sum (X_2 - \bar{X}_2)(X_4 - \bar{X}_4) = \sum (X_1 - \bar{X}_1)(X_2 - \bar{X}_2) \]  \hspace{1cm} (6.5)

\[ B_2 \sum (X_2 - \bar{X}_2)(X_3 - \bar{X}_3) + B_3 \sum (X_3 - \bar{X}_3)^2 + B_4 \sum (X_3 - \bar{X}_3)(X_4 - \bar{X}_4) = \sum (X_1 - \bar{X}_1)(X_3 - \bar{X}_3) \]  \hspace{1cm} (6.6)
\[ B_2 \sum (X_2 - \overline{X}_2)(X_4 - \overline{X}_4) + B_3 \sum (X_3 - \overline{X}_3)(X_4 - \overline{X}_4) + B_4 \sum (X_4 - \overline{X}_4)^2 = \sum (X_1 - \overline{X}_1)(X_4 - \overline{X}_4) \]
\[ \text{(6.7)} \]

and
\[ B_4 = \overline{X}_4 - B_2 \overline{X}_2 - B_3 \overline{X}_3 - B_4 \overline{X}_4 \]
\[ \text{(6.8)} \]

where \( \overline{X}_1, \overline{X}_2, \overline{X}_3 \) and \( \overline{X}_4 \) are the arithmetic average values of \( X_1, X_2, X_3 \) and \( X_4 \) series.

6.2.1 RISING STAGE:

On carrying out regression analysis of the rising stage data for all the runs taken,
\[ \overline{X}_1 = 0.8882, \overline{X}_2 = 3.677, \overline{X}_3 = 2.568 \text{ and } \overline{X}_4 = -2.547 \]
\[ \sum (X_1 - \overline{X}_1) = -8.361 \times 10^{-6}, \sum (X_2 - \overline{X}_2) = 0.00175, \sum (X_3 - \overline{X}_3) = -0.00262 \]
\[ \sum (X_4 - \overline{X}_4) = -0.00428, \sum (X_1 - \overline{X}_1)^2 = 15.092, \sum (X_2 - \overline{X}_2)^2 = 3.24 \]
\[ \sum (X_3 - \overline{X}_3)^2 = 629, \sum (X_1 - \overline{X}_1)(X_2 - \overline{X}_2) = -4.2495 \]
\[ \sum (X_1 - \overline{X}_1)(X_3 - \overline{X}_3) = 5.455, \sum (X_1 - \overline{X}_1)(X_4 - \overline{X}_4) = -3.294 \]
\[ \sum (X_2 - \overline{X}_2)(X_3 - \overline{X}_3) = -5.027, \sum (X_2 - \overline{X}_2)(X_4 - \overline{X}_4) = 11.055 \text{ and } \sum (X_3 - \overline{X}_3)(X_4 - \overline{X}_4) = -4.4607 \]

Substituting these values in Eqs.(6.5) to(6.8), and solving for \( B_1, B_2, B_3 \) and \( B_4 \) one gets:
\[ B_1 = -5.2744, B_2 = 0.192, B_3 = 2.465, B_4 = 0.343 \]
\[ b_1 = \text{antilog } B_1, \ldots b_1 = 5.316 \times 10^6 \]

Hence for rising stage the functional relationship will take the form of the following equation
Now representing the sediment load dimensionless parameter \( q_s \) by \( \psi \). The above regression gives a multiple correlation coefficient \( R^2 \) of 0.933. Multiple correlation coefficient is defined as the ratio of standard deviation of computed \( \psi \) values to the standard deviation of observed \( \psi \) values. Coefficient of multiple determination \( D_1 = R^2 = 0.8702 \). Figure (6.2) shows the variation of computed and observed \( \psi \) values for the rising stage.

\[
\frac{q_s}{\rho, g^{1/2} d^{3/2} \sqrt{\Delta \rho/\rho}} = 5.316 \times 10^{-6} \times \left[ \frac{U}{\Delta h/\Delta t} \right]^{0.192} \times \left[ \frac{h}{d} \right]^{2.465} \times \left[ \frac{S}{\Delta \rho, \rho} \right]^{0.541}
\]

\[\text{(6.9)}\]
6.2.2 FALLING STAGE:

\[ \bar{X}_1 = 0.5027, \quad \bar{X}_2 = 3.859, \quad \bar{X}_3 = 2.555 \text{ and } \bar{X}_4 = -2.5754 \]

\[ \sum (X_1 - \bar{X}_1) = -6.6 \times 10^{-5} \quad \sum (X_2 - \bar{X}_2) = 5.95 \times 10^{-5} \quad \sum (X_3 - \bar{X}_3) = -4.0 \]

\[ \times 10^{-5} \sum (X_4 - \bar{X}_4) = 0.008122 \quad \sum (X_1 - \bar{X}_1)^2 = 98.671 \]

\[ \sum (X_2 - \bar{X}_2)^2 = 49.975 \quad \sum (X_3 - \bar{X}_3)^2 = 4.48 \]

\[ \sum (X_4 - \bar{X}_4)^2 = 27.778 \quad \sum (X_1 - \bar{X}_1)(X_2 - \bar{X}_2) = 10.904 \]

\[ \sum (X_1 - \bar{X}_1)(X_3 - \bar{X}_3) = 16.054 \quad \sum (X_1 - \bar{X}_1)(X_4 - \bar{X}_4) = -8.013 \]

\[ \sum (X_2 - \bar{X}_2)(X_3 - \bar{X}_3) = -3.22 \quad \sum (X_2 - \bar{X}_2)(X_4 - \bar{X}_4) = 15.11 \text{ and } \]

\[ \sum (X_3 - \bar{X}_3)(X_4 - \bar{X}_4) = -4.982 \]

Substituting these values in Eqs.(6.5) to(6.8), and solving for \( B_1, B_2, B_3 \) and \( B_4 \) one gets:

\[ B_1 = -11.07, \quad B_2 = 0.416, \quad B_3 = 4.13, \quad B_4 = 0.227 \]

\[ b_1 = \text{antilog } B_1, \quad . . b_1 = 8.51 \times 10^{-12} \]

Hence for falling stage the functional relationship will take the form of the following equation:

\[
\frac{q_x}{\rho g^{3/2} d^{3/2} \sqrt{\Delta \rho_s / \rho}} = 8.51 \times 10^{-12} \left[ \frac{U}{\Delta h / \Delta t} \right]^{0.416} \left[ \frac{h}{d} \right]^{4.13} \left[ \frac{S}{\Delta \rho_s / \rho} \right]^{0.237}
\]

\[ \text{-------------(6.10)} \]

giving a multiple correlation coefficient \( R_1 \) of \( 0.9872 \). Coefficient of multiple determination works out as \( D_1 = R_1^2 = 0.9745 \).

Figure (6.3) shows the variation of computed and observed \( \psi \) values for the falling stage.
Fig. 6.3. Computed versus observed values of dimensionless sediment load parameter $\psi$ for falling stage.

6.3 GRAPHICAL ANALYSIS:

Let the non-dimensional velocity term $\frac{U}{\Delta h/\Delta t}$, be represented by $\Gamma$, the dimensionless slope term $\frac{S}{\Delta \rho_1 / \rho}$, by $S_1$ and the dimensionless flow depth $\frac{h}{d}$, by $H_1$

$S_1 = 0.01$

Fig. 6.4. Relation between $\Gamma$ and $\psi$ for rising stage
Figure 6.4, a plot between \( \psi \) and \( \Gamma \) for different values of \( H_1 \), for rising stage of hydrographs H-1 to H-14, and other similar plots (not shown in the thesis) indicate that there is a systematic trend in values of \( \psi \) w.r.t. \( H_1 \) as well as \( \Gamma \).

Accordingly it was considered appropriate to write Eq.(3.6) as:

\[
\frac{K_{1R}K_{2R}}{\rho g^{3/2} d^{3/2}} \frac{q_1}{\Delta \rho / \rho} = f_6(\Gamma) \tag{6.11}
\]

where \( K_{1R} = f_7(H_1) \) \tag{6.12}

and \( K_{2R} = f_8(\Gamma) \) \tag{6.13}

The subscript \( R \) indicates that the corresponding value is for rising stage.

The experimental data were, therefore analysed in accordance with Eqs. (6.11, 6.12 and 6.13) to develop an empirical relation, Eq.(6.13). For example, for the rising stage data, lines of constant \( H_1 \) can be drawn on Fig. 6.4. All the data points can be replotted by multiplying \( \psi \) with suitable value of \( K_{1R} \) so that all the data merge along a single line. Obviously, the value of \( K_{1R} \) would depend on \( H_1 \), Fig. 6.5. This method is repeated for all the plots for different values of \( S_1 \) in order to obtain a single plot between \( K_{1R} \psi \) and \( \Gamma \) for all values of \( S_1 \).
Fig. 6.5. Relation between $K_{IR}$ and $H_1$ for rising stage

$K_{IR} = f(H_1)$ for Rising Stage

Fig. 6.6. Relation between $K_{IR} \psi$ and $\Gamma$, for different values of $S_1$, for rising stage

This plot (Fig. 6.6) has different lines for different values of $S_1$. These lines were also, similarly, merged into a single line, Fig. 6.7, by plotting $K_{IR} \ K_{2R} \ \psi$ against $\Gamma$, with $K_{2R}$ as function of $S_1$ as shown in Fig. 6.8.
Likewise, for the falling stage data too, the relationship for $K_{1F}K_{2F} \psi$ versus $\Gamma$ (Fig. 6.9), was obtained with $K_{1F}$ and $K_{2F}$ varying, respectively, with $H_{1}$ and $S_{i}$ as shown in Figs. 6.10 and 6.11.
Fig. 6.9. Relation between $K_{IF} K_{2F} \psi$ and $\Gamma$ for falling stage

$K_{IF} K_{2F} \psi = f(\Gamma)$ for falling Stage

Fig. 6.10. Relation between $K_{IF}$ and $H_1$ for falling stage

$K_{IF} = f(H_1)$ for Falling Stage
Fig. 6.11. Relation between $K_{2f}$ and $S_1$ for falling stage