3. DIMENSIONAL ANALYSIS

3.1 GENERAL:

Dimensional reasoning is extensively used in discussion of sediment transport problems (Raudkivi A.J., 1976). Mathematically, a physical phenomenon is described by a set of n independent, necessary and sufficient parameters, \( a_1, \ldots, a_n \). The n parameters, \( a_i \), are independent if none of them is expressible as a function of the remaining (n-1) parameters. Stated differently, an independent parameter (variable) is one whose value is imposed externally on the system and as a corollary, a dependent variable is one whose value is determined by the values specified for the independent parameters. Any property, \( A \), of the phenomenon must be definable by the \( a_i \) parameters as

\[
A = f_A(a_1, a_2, \ldots, a_n)
\]

(3.1)

The independent or characteristic parameters of the fluid are its density, \( \rho \), and its viscosity, \( \mu \) (Yalin, M.S., 1977). These two describe the clear fluid for studies of mechanics. The cohesionless granular matrix is described by its density, \( \rho_s \), a size characteristic, \( d \), and by the shape of the grain and grain size distribution. The theory of dimensions is not suited for description of geometry and shape, and these are, therefore, omitted. This means that a particular functional relationship will be valid only for a given shape and grading properties of the sediment. When the shape or grading changes, the function \( f_A \) changes.
3.2 DIMENSIONAL ANALYSIS RELATED TO THE PRESENT STUDY:

In a combined flow of fluid and sediment in an open channel, the flow is determined by its depth $h$, slope $S$, and gravity $g$, which provides the driving force. Hence all variables of unsteady flow with sediment of given shape characteristics and grading are:

$$q_s, U, h, \rho, \Delta\rho, g, \mu, d, \frac{\Delta h}{\Delta t}, S$$

Where

$q_s$ = sediment discharge (N/s) per unit width of the channel

$U$ = mean velocity of flow,

$h$ = depth of flow

$\rho$ = mass density of fluid

$\Delta\rho$ = $(\rho_s - \rho)$ in which $\rho_s$ is the mass density of sediment particles

$g$ = acceleration due to gravity

$\mu$ = dynamic viscosity of fluid

$d$ = diameter of the particles

$\frac{\Delta h}{\Delta t}$ = rate of change of depth

$S$ = Energy slope

In the present study, the sediment load, $q_s$, is the dependent variable, and hence the functional relationship amongst these variables can be written in the form

$$q_s = f(U, h, \rho, \Delta\rho, g, \mu, d, \frac{\Delta h}{\Delta t}, S) \quad \text{(3.2)}$$
Taking $U$, $h$, and $\rho$ as repeating variables and applying the Buckingham $\pi$-theorem, the following functional relationship can be obtained:

$$f_3\left(\frac{q_s}{\rho U^3}, \frac{\Delta \rho}{\rho}, \frac{gh}{U^2}, \frac{\Delta h}{h}, \frac{\rho}{\Delta \rho}, \frac{\mu}{\Delta \mu}, \frac{S}{U}, \frac{\rho}{\Delta \rho}ight) = 0 \quad \cdots \cdots \quad (3.3)$$

Combining the first four $\pi$-terms and rearranging gives:

$$\frac{q_s}{\rho g^{3/2} d^{3/2}} = f_3\left(\frac{U}{\Delta h/\Delta t}, \frac{S}{\Delta \rho}, \frac{h}{U \rho}, \frac{\Delta h}{\Delta \mu}, \frac{\rho}{\Delta \rho}, \frac{S}{U^2}, \frac{gh}{U^2}\right) \quad \cdots \cdots \quad (3.4)$$

Since $S$ has been included in $\frac{S}{\Delta \rho}$ and the gravity effect is known to be very well accommodated for through the dimensionless sediment load parameter, one can drop $S$ and $\frac{gh}{U^2}$ to obtain

$$\frac{q_s}{\rho g^{3/2} d^{3/2}} = f_3\left(\frac{U}{\Delta h/\Delta t}, \frac{S}{\rho}, \frac{h}{U \rho}, \frac{\Delta h}{\Delta \mu}, \frac{\rho}{\Delta \rho}\right) \quad \cdots \cdots \quad (3.5)$$

The parameter $\frac{U \rho}{\mu}$, which represents the flow Reynolds number is less significant, as in case of open channel flow the flow is governed by the Froude number of the flow. Hence $\frac{U \rho}{\mu}$ is further dropped out, and the final functional relationship is given as:

$$\frac{q_s}{\rho g^{3/2} d^{3/2}} = f_3\left(\frac{U}{\Delta h/\Delta t}, \frac{S}{\rho}, \frac{h}{\Delta \rho}, \frac{h}{\Delta d}\right) \quad \cdots \cdots \quad (3.6)$$

Therefore, in the present study, the relationship has been sought among the above-mentioned four dimensionless terms. The term on the left hand side of Eq.(3.6) represents the sediment load parameter and is designated by $\psi$. 

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