CHAPTER-2

LITERATURE REVIEW
The work done on the topic of Direct Contact Heat Transfer between two immiscible liquids with change of phase is of recent origin. During the last decade water desalting schemes have given impetus to the study of direct contact heat transfer. As described by Spiegler (203), this mode of heat transfer forms the basis of a well established process, for the desalination of sea water by direct contact freezing method.

Applications of direct contact heat transfer can generally be classified as:

i) Liquid - Liquid

ii) Liquid: Vapour (or Gas)

iii) Liquid - Vapour

(a) Liquid - liquid to vapour (evaporation)

(b) Liquid - vapour to liquid (condensation)

The second type is commonly found in cooling towers and inert gas cryogenic subcooling systems. However, these techniques rely to a large extent on mass transfer phenomenon to accomplish the desired results. Liquid-Vapour direct contact heat exchange is of minimal interest so far as desalination and utilization of geothermal brine for
energy production are concerned. Liquid-liquid direct contact heat transfer has also received considerable attention.

The larger volumes of fluids required in liquid-liquid heat transfer can be reduced by making the dispersed phase evaporate (or condense) within the continuous liquid medium. Latent, rather than sensible, heat transfer takes place in these three-phase exchangers, thus permitting a more effective heat-transfer process. The reported work on direct contact heat transfer with phase change (29,48,91,147) is relatively little in comparison to the literature available on heat transfer to solid spheres and mass transfer to single drops and bubbles. Although the basic concepts and theories associated with mass transfer rates are usually applicable to heat transfer rates, but are not directly applicable to the present study because condensation of a bubble or vaporization of a drop results in interfacial turbulence due to the continuous change in density during the heat-transfer process. However, literature on allied topics is useful in order to understand the complex mechanism of heat transfer and dynamics of a two-phase bubble.

In view of the complexity of the nature of the present work, an attempt has been made during this literature survey to cover all important aspects relevant to
the main field of study. Fluid flow and interfacial phenomena will be reviewed first, then the allied topics in heat and mass transfer will be considered and lastly the work on the main topic will be discussed. The literature concerning less obvious side effects will be briefly reviewed in the end, to show what precautions were necessary in the experimental work.

2.1.0 Fluid Flow and Interfacial Phenomena

The importance of fluid flow to this study lies in its effect on the nature of the inside and outside resistances to heat flow.

2.1.1 Theoretical Concepts

Early work on flow around submerged bodies, drops and bubbles was mostly theoretical. It was based on ideal behaviour and consequently the results obtained were of little practical use, although they predicted the velocity and pressure distributions for a limited range of flow.

The Navier-Stokes equations were next developed to predict the flow behaviour, but a general solution of these equations has not yet been obtained. However, Navier-Stokes equations have been solved for potential flow,
creeping flow and certain other special cases. At low Reynolds numbers when the inertial forces are not signifi­cant the flow patterns in the approach and wake zones are similar. For large Reynolds numbers the situation becomes entirely different. The inertia forces are then of much more importance than the viscous forces, at least at a sufficient distance from the walls of the body i.e., with the exception of the layer of fluid adjacent to the obstacle. However, if the influence of viscosity is completely neglected erroneous results are obtained.

Prandtl (160) made an improvement on this situation by dividing the flow into two regions:

(a) Surrounding the surface of the body there is a thin layer of fluid in which the velocity gradient generally becomes very large, so that even with very small values of the velocity the shear stress assumes values which cannot be neglected. This depth of the fluid is known as boundary layer, and it is conventionally defined to extend from the interface out to a point where the velocity is 99 percent of the free stream velocity (184).

(b) In the region outside the boundary layer, the velocity gradient does not become as large, and the influence of viscosity is negligible. In this region the free stream velocity prevails and ideal fluid behaviour
can be assumed.

Such a division of the field of flow, brings about a considerable simplification in the mathematical theory of the motion of a fluid of low viscosity.

2.1.2 Drag

When a body moves through a fluid at rest, it experiences a force in a direction opposite to that of its motion. This force is called the 'drag'.

Total drag is composed of three components (161).

(a) Friction Drag

This results from the friction forces tangential to the surface of the body.

(b) Pressure Drag

A change in the geometry of the streamline shapes causes a change in the pressure field and consequently leads to pressure drag.

(c) Deformation Drag

This is a part of the friction drag for very
low Reynolds numbers. The work done by deformation drag is ultimately dissipated as heat in the total field.

Unlike deformation drag, the work of the friction drag in its more restricted sense and that of pressure drag are dissipated into heat more specifically in the wake of the body. The size of each drag force contribution depends on the form and the nature of the body as well as on the Reynolds number.

The drag coefficient is defined as

\[ C_D = \frac{F_D / A_p}{\frac{1}{2} \rho u^2} \] ... (2.1)

where, \( F_D \) = total force exerted on the body.

The drag coefficient \( C_D \) depends on the shape of the body and on the Reynolds number.

As reviewed by Hughes and Gilliland (97), acceleration of drop also affects the drag coefficient. The bodies being accelerated have higher drag coefficients than those moving at a constant velocity.

2.1.3 Flow Around a Solid Body

Before discussing the flow behaviour around drops and bubbles it would be better to have a good
understanding of the flow patterns around a solid body. Considering flow around a sphere let us visualize the changes in flow pattern with increasing Reynolds numbers.

At low Reynolds numbers the flow is symmetrical and satisfies Stoke's solution for creeping flow. The velocity first increases on upstream surface and then decreases on the downstream surface of the sphere. Along the downstream side as the velocity decreases the initial pressure is recovered.

With increasing free stream velocity the inertial effects become important and one has boundary layer flow. At this stage losses become significant and the pressure recovery at the rear stagnation point is no longer complete. This trend continues until separation of the forward flow occurs at about Reynolds number equal to 17, when a very small toroidal vortex is formed near the rear stagnation point. The vortex gains strength as the Reynolds number increases, and the point of separation advances towards the equator. At a Reynolds number equal to about 450, the angle of separation is equal to 104° (59). At this stage the wake becomes unstable and with increasing Reynolds number oscillates about the axis of motion spilling the contents downstream (57).
2.1.4 **Flow Around Bubbles and Drops**

If we now replace the rigid sphere with a fluid sphere, the following significant changes may occur:

(a) The fluid within the sphere can move under shear causing a finite velocity at the interface.

(b) The fluid sphere may distort and oscillate under the influence of the hydraulic and dynamic pressure gradients.

The transfer of shear energy will depend upon the surface tension and the viscosity of the drop phase fluid. If the drop is small or the viscosity of the drop phase fluid is high, shear resistance in the drop will be great and the fluid drops will behave more like rigid drops. On the other hand with large drop size and a low viscosity of the drop phase liquid, circulation may develop in the drops.

2.1.5 **Circulation in Drops**

This is by far the most important and stimulating mechanism and numerous workers have treated its momentum and heat and mass transfer aspects theoretically (10, 20, 21, 23, 54, 79, 80, 82, 93, 118, 119, 126, 180, 181, 182, 202, 39) and experimentally (24, 26, 29, 41, 48,
A drop (or bubble) moving in a viscous fluid tends to circulate internally due to shear stress applied at its interface by the viscous fluid. As a result of this circulation, the transfer rate is increased by a factor approximately equal to $1.9 \left( \frac{\pi}{3} \right)^{1/6}$ compared with the rigid drop. Circulation increases with the drop size and the viscosity ratio of the two fluids.

During circulation new surface is created at the front of the drop, the necessary energy being supplied by the loss of surface at the back of the drop. For a fluid with an appreciable viscosity some of this energy gets dissipated resulting in damping of circulation. Skin friction, however, provides energy to overcome this viscous damping. So if skin friction is sufficiently great, a fair amount of circulation can be expected. Hence it can be concluded (97).

(a) Bubbles of gas in liquids are nearly always circulating.

(b) Liquid drops in gases are rarely circulating.

(c) Liquid drops in liquids are usually in the transition region.
Fully developed circulation can be achieved only when skin friction is sufficient to supply all the energy needed to overcome the surface forces. The energy required to form the surface is \( 2 \pi R \sigma \Omega u_g \) ergs/sec (61).

Thus fluids with low interfacial tension will circulate more easily than will those with high interfacial tensions.

For a given system, the resistance to circulation in the drop depends on its diameter, the smaller the drop, the larger will be the surface pressure gradient between its front and rear, and the smaller will be the tangential friction stress (37). A force balance at any point of the interface gives (74, 183):

\[
\frac{\partial \sigma}{\partial x} = \mu_c \frac{du_c}{dy} - \mu_d \frac{du_d}{dy} \quad \ldots \quad (2.2)
\]

where \( x \) and \( y \) are the parallel and normal vector distances from the drop interface, respectively. Thus circulation within the drop will be retarded appreciably, or even arrested, if the interfacial tension gradient and external viscous drag are of the same order of magnitude.

According to the classical theories of Hadamard (79), Rybczynski (181) and Boussinesq (20, 21), circulation sets in whenever a drop moves in a viscous fluid. Experim-
ents (65), however, indicate that circulation takes place readily, though not exclusively (60), in the presence of a suitable driving force (temperature or concentration gradients) and only above a certain level of the Reynolds number, i.e., whenever the drag force exceeds a certain minimum. The Reynolds number at which circulation begins within the drop decreases with increasing viscosity of the continuous phase (63). Garner and Skelland (54) suggested a correlation for the critical Reynolds number for determining the onset of circulation, but it is rather inaccurate since circulation is highly dependent on fluid purity and on the presence of a surface-active impurities (60, 74, 131). The lower the interfacial tension, the lower the transitional Reynolds number (55), and the higher the overall transfer coefficient (12, 188). For a drop with high surface tension, minute amounts of impurities suffice to produce an appreciable surface-tension gradient at the interface, thus greatly retarding circulation. Bond (18, 19) included surface tension in his criterion for the onset of circulation. This criterion, however, was derived from experimental data in the streamline region and is rather inadequate and unreliable in the practical ranges of Reynolds numbers (61), especially since circulation is strongly affected by immeasurably small changes in surface tension due to surface active agents (37, 131). For a given system, the effect of a surface-tension gradient
increases with decreasing drop diameter, and below a certain size, circulation is reduced or arrested, with transfer coefficients approaching the rigid drop values.

According to Savic (182), an immobile cap forms at the bottom of the drop, and circulation is confined to its front part since velocities in the vicinity of the rear stagnation point are relatively small (80). Molecules moving backwards and returned forwards by diffusion. Thorsen and Tergesen (213) claim that, in contrast to transfer from solid particles at critical Reynolds number greater than 20, most of the transfer from drops takes part at the rear of the drop, as also reported by Garner (58). They suggested that hydrodynamic behaviour, rather than circulation, is responsible for the higher transfer rates inadrops compared with rigid drops and solid particles.

A quantitative semiempirical expression for the degree of drop circulation as a function of the viscous forces, drop diameter, densities and the compressional modulus of the surface film (surface-tension gradient), as well as the empirical fraction of liquid circulating has been suggested by Davies (57) and is given by:
\[ \% \text{circulation} = \frac{100 \cdot f_3 \cdot \sqrt{C_s}}{(1 + 1.5 \cdot \frac{\mu_d}{\mu_c}) \cdot R^2 \cdot (\rho_d - \rho_c)} \]

where the percent circulation is defined by the fraction of the total liquid in the drop which is circulating.

According to Elzinga and Hanchero (47), internal circulation causes a shift in the point of boundary layer separation on the drop surface, which in turn decreases the drop drag coefficient. Internal circulation also causes a marked increase in crop distortion.

Redfield and Houghton (171) studied the mass transfer and drag coefficient at different values of Reynolds number for single bubbles of carbon dioxide rising in pure water and various other aqueous solutions and arrived at the following conclusions:

(a) At Reynolds number less than 0.2 the drag coefficients are described by the circulating sphere models of Handamard-Ryoezynski.

(b) At Reynolds number of 0.2-1.0, the bubbles are still spherical but the drag coefficient is larger than that for circulating spheres; indicating that boundary layer and separation effects are beginning to influence
the behaviour.

(c) For Reynolds number between 1 and 10, small
deviations from the spherical shape occur, but the drag
coefficient appears to be a function of Reynolds number
only.

(d) For Reynolds number greater than 10 turbulence
appears to develop in the wake of the bubble, the drag
coefficient passes through a minimum and becomes a func-
tion of the physical properties of the solution as well
as of the bubble size.

(e) When the Reynolds number lies between 100 and
5000 the shape of the bubbles slowly changes from oblate
spheroidal to a mushroom shape, the wake becoming increa-
singly turbulent. Furthermore, the path of the bubbles at
high Reynolds numbers changes from a straight vertical
trajectory through a spiral path and side to side zig-zag
path back to a stable vertical trajectory for the mushroom
shaped bubbles with spherical caps.

An interesting point regarding the aging of
circulating drops was noted by Deindofer and Humphery
(41). Their experiments showed that the transfer coeffi-
cients of 1-2 sec. old drops were in agreement with those
obtained from theories of Higbie (93) or Boussinesq (20,
21), whereas the coefficients of 5 sec. old drops were close to those predicted by the rigid drop theories. This phenomenon is probably closely related to the observed decrease of transfer coefficients with increasing column height (22,46,151) and undoubtedly introduces a certain error in the determination of end effects by the variable column height technique.

Another aspect, usually overlooked, is the effect of natural convection inside the drop on the transfer coefficient. Transfer coefficients differing by about 25%, were found (231) when drops were heated or cooled in the continuous phase.

2.1.6 Bubble and Drop Dynamics

The motion of a bubble or a drop is characterized by its terminal velocity, defined as the velocity at which drag forces and gravitational forces exactly balance each other. The dynamics of bubbles and drops in terms of shapes and velocities has been extensively investigated (4, 5, 70, 152, 232, 70, 71, 223, 15, 107, 39, 72, 212, 150). Many empirical and semi-empirical equations are available for calculating terminal velocities for liquid drops (96,130,114,85).

Datta, Napier and Newitt (36) studied the
properties and behaviour of gas bubbles rising in a liquid column. Based on their experimental results the authors identified the following factors which may change the shape of the terminal velocity versus bubble radius plot:

(a) Temperature
(b) Wall effect
(c) Turbulence
(d) Measurement of bubble size
(e) Velocity measurement

Garner and Hammerton (51) measured the velocities of bubbles in water and gave a correlation in the form of a plot between drag coefficient and Reynolds number. The Reynolds number was based on the equivalent spherical diameter of the bubble. Upto Reynolds number of 600 the curve is quite close to that for solid spheres, but beyond this the drag coefficient increases rapidly with Reynolds number until a Reynolds number of 5000 after which the drag coefficient becomes more or less constant.

In general, bubbles of radius less than 0.1mm behave like solid spherical particles and obey Stokes law. Even larger size bubbles behave much like solid spheres in contaminated liquids. Levich (125) derived the expression for the velocity of a gas bubble moving in a pure
liquid and is given by:

\[ u = \frac{1}{3} \zeta^2 \rho C_0 \frac{g R_0^2}{\mu} \]  

(2.4)

The velocity predicted by equation (2.4) was found to be higher than that obtained from experimental data, may be due to contamination of the gas liquid interface.

The flow field around a bubble of radius in the range of 0.15 to 0.4 mm may be approximated by potential flow, at least up to the point of boundary layer separation for large Reynolds numbers (> 100). The drag exerted on a spherical bubble in this range was also analysed by Levich (125) who proposed an expression for the terminal velocity given by:

\[ u_{\infty} = \frac{1}{9} \zeta^2 \rho C_0 \frac{g R_0^2}{\mu} \]  

(2.5)

Equation (2.5) overpredicts the bubble rise velocity for bubbles larger than about 0.4 mm.

The nature of motion of the bubbles change sharply at Reynolds numbers close to 700, which corresponds to bubble diameter of 2-3 mm. Bubbles begin to be noticeably deformed acquiring the shape of flattened ellipsoids with minor axis in the direction of motion of the bubbles. In addition, the bubbles start oscillating.
and move in a spiral instead of a straight path. Peebles and Garber (152) have correlated terminal velocity data in the equivalent radius range from 0.7 to 3 mm, and the correlation is given as:

\[ u_\infty = 1.35 \frac{L}{R_0} \frac{\rho}{\rho_f} \frac{\nu}{\nu_f} \frac{1}{\sqrt{2}} \]  \hspace{1cm} (2.5)

Haberman and Morton (78) have shown that the terminal velocity, for the bubbles of equivalent radius greater than 3 mm, which assume the shape of a spherical cap, can be expressed as:

\[ u_\infty = 1.02 \left( \frac{\rho}{\rho_f} \right)^{1/2} \]  \hspace{1cm} (2.7)

Larger bubbles of about 40 to 60 mm radius having an approximate volume of $5 \times 10^{-5}$ m$^3$ were observed to deform and tend to fragment into smaller bubbles.

A useful starting point for describing the behaviour of bubbles and drops in liquids is the regime diagram developed by Grace and coworkers (70,71,33) and for each regime equations have been proposed for the prediction of terminal velocity as summarized below:

(a) Spherical Regime

For drops and bubbles with Reynolds number less than 0.2, the terminal velocity in the absence of surface-
active impurities is given \((79, 181)\) by

\[
U_\infty = \frac{g D^2 (\rho_c - \rho_d)}{6 \mu_c} \zeta \frac{\mu_d/\mu_c + 1}{3 \mu_d/\mu_c + 2 - 7} \quad \ldots (2.8)
\]

Equation \((2.8)\) reduces to the well known Stokes expression \((207)\) when \(\mu_d/\mu_c \to \infty\). There are a number of equations in the literature \((33, 87)\) which give expressions intermediate between equation \((2.8)\) and Stokes expression for cases of intermediate purity where surface-active impurities reduce, but do not completely eliminate internal circulation. However, these expressions are of limited utility for practical cases because the nature and concentration of impurities is seldom known.

At higher values of the drop or bubble Reynolds number, numerical solutions have been carried out for specific cases, e.g., for air bubbles in water with \(\mu_d/\mu_c = 1, 3\) and 10. An expression proposed by Rivikind and Ryskin \((173)\),

\[
C_D = \frac{4}{3} \left( \frac{\rho_c - \rho_d}{\rho_c} \right) \frac{C_D}{U_\infty} = \frac{1}{(\mu_d/\mu_c + 1)} \zeta \frac{\mu_d}{\mu_c} \left( \frac{24}{Re} + \frac{4}{Re^{1/3}} \right) + \frac{14.9}{Re^{0.78 - 7}} \quad \ldots (2.9)
\]

allows the terminal velocity to be calculated for spherical drops and bubbles in pure systems.
(b) Ellipsoidal Regime

For bubbles and drops in liquids where some surface-active contamination is present, number of empirical equations for predicting terminal velocities have been proposed. The most successful equation was proposed by Grace et al., (71) as an extension to that of Johnson and Braida (107), itself an extension of the correlation of Hu and Kintner (96). The correlation is:

\[ \text{Re} = (J - 0.857) \text{ Mo}^{-0.149} \]  
\[ \text{Mo} = 0.94 \text{ N}^{0.757} \quad (2 \leq \text{N} \leq 59.3) \]
\[ J = 3.42 \text{ N}^{0.441} \quad (59.3 < \text{N}) \]
\[ N = \left( \frac{4}{3} \right) \text{ Eo} \text{ No}^{-0.149} \left( \mu_c / 0.0009 \right)^{-0.14} \]

The continuous phase liquid viscosity, \( \mu_c \), must have SI units.

(c) Spherical-Cap Regime

This Regime is encountered by large bubbles and drops. For a spherical-cap, the velocity has been derived (223,39) by assuming that the pressure distribution near the leading edge is that for potential flow past a complete sphere and balancing pressures on the inside and outside
of the fluid particle in that vicinity. The resulting expression is

$$u_{bo} = \frac{2}{3} \left( \frac{\rho_c - \rho_d}{\rho_c} \right) \varepsilon R \sqrt{2} \quad \ldots \ (2.11)$$

where $R$ is the radius of curvature of the leading edges. Extension of this equation to the case where the front edge is oblate spheroidal leads (223) to

$$u_{bo} = \frac{1}{2} \sin^{-1} \left( \frac{e - e\sqrt{1 - e^2}}{e^3} \right) - \frac{1}{2} \sqrt{\left( \frac{\rho_c - \rho_d}{\rho_c} \right) \varepsilon b} \quad \ldots \ (2.12)$$

where $e = \sqrt{\frac{(a^2 - b^2)}{a^2}}$ is the eccentricity and $a$ and $b$ are the horizontal and vertical semi-axes, respectively.

Recently Abou-El-Hassan (1) has developed a generalized analytical correlation for bubble rise in gas-liquid systems on the basis of new dimensionless groups: flor ($F'$) and velocity numbers ($V''$). As claimed by the author, the correlation is not sensitive to regime limitations or bubble shape, and is valid for wide range of conditions (Re No. range from 0.1 to $10^4$). The analytical form of the correlation is:

$$V'' = 0.75 \ (\log F')^2 \quad \ldots \ (2.13)$$
where

\[ V' = \frac{u D^{2/3} \rho_c^{2/3}}{\mu_c^{4/3} \sigma_c^{4/3}} \]

\[ F' = g D^{8/3} L^{1/2} \left( \frac{\rho_c - \rho_d}{\mu_c^{4/3} \sigma_c^{4/3}} \right)^{2/3} \]

2.1.7 Drop Distortion and Oscillation

Drop distortion and oscillation will change the shape of a moving drop and hence will change the drag coefficient and terminal velocity. Distortion will also affect the surface area and thus the rate of heat transfer. It is also a cause of internal circulation. Distortion and oscillation tend to increase the drag and reduce the terminal velocity over that for a rigid sphere of equal volume. According to Calderbank and Korchinski (29) oscillation causes some boundary layer degradation and thus increases the continuous phase transfer coefficient.

Many efforts have been made to correlate the shape and amount of distortion as a function of drop or bubble size and other measurable system properties (97, 61, 38, 204).

Johnson and Braida (107) found that drop oscillation depends on
(a) Physical properties of the system

(b) Drop diameter

(c) Drop velocity.

Harmathy (86) correlated drag coefficients with Eötvös number within \( \pm 20 \) percent, for values of dispersed phase Reynolds number greater than 500 and taking into account the oscillation and distortion effects.

So far no method is available for predicting the exact nature or the amount of distortion of a fluid drop of any given size.

2.1.8 Dynamics of a Two-Phase Bubble

Dynamics of a condensing bubble or that of a vaporizing drop is relatively more complex than either that of a bubble or a drop of constant radius. The vertical motion of a condensing bubble is induced by five forces: liquid inertia within the bubble, buoyancy, form drag, viscous drag and surface tension. As the bubble leaves the nozzle, liquid inertia forces within the bubble act in the direction of rise and are prominently exhibited during the partial collapse of the bubble, accompanying the rapid deceleration. These forces are even more pronounced when the emerging bubble already contains some condensate. Buoyancy forces act to acceler
ate the bubble upwards, but decrease as the rising bubble collapses to a liquid state. Form (or impact) drag acts to flatten the bubble, while viscous drag tends to elongate it into an oblate spheroid. Both forces are opposed by surface tension, which acts to restore the bubble's spherical shape. Viscous drag forces are dependent on the extent of induced circulation.

Since, most of the volume of a two-phase bubble is occupied by the vapour phase even at about one percent vaporization, it is expected to behave like a bubble and the flow of the continuous medium around the bubble closely approximates potential flow conditions (21,24,179). Visual comparison between pictures taken of bubble collapse of single component (pentane-pentane) and two-component (pentane-water) systems (100) revealed that the surface of the pentane bubble in pentane liquid was highly irregular when compared to the smoother interface of the pentane bubble in water (192). The former is typical of gas-liquid interfaces, while the latter behaviour is more akin to liquid drops. Isenberg and Sideman (100), therefore, treated the condensing bubble as a rigid sphere with a laminar flow field around it. It was also argued that the presence of residual liquid inside the condensing two-phase bubble reduced or sometimes arrested internal circulation and, thus, the movement of
the interface.

Anticipating difficulties in finding a solution for laminar flow field around a contracting sphere an empirical approach involving velocity factor, $K_v$, which depends upon the external fluid properties, was introduced to express velocity in terms of modified potential flow solution. Griffith (74), was first to recommend the incorporation of a velocity factor into the equation for steady state heat transfer in potential flow in order to modify it for rapidly circulating drops.

The velocity factor, $K_v$, was derived through relations with known solutions for steady state heat transfer to a sphere. Rearranging Froessling's semiempirical relation (53) for the Nusselt number for steady state heat transfer to a rigid sphere under laminar flow condition yields

$$\text{Nu}_C = \frac{2}{(\pi)^{1/2}} \left(0.25 \frac{Pr}{Re^{1/3}} \frac{Pe}{C} \right)^{1/2} \quad \ldots \quad (2.14)$$

whereas the Nusselt number for steady state heat transfer to a sphere in potential flow as evaluated by Boussinesq (21), is

$$\text{Nu}_C = \frac{2}{(\pi)^{1/2}} \left(\frac{Pe}{C} \right)^{1/2} \quad \ldots \quad (2.15)$$

Comparison between equations (2.14) and (2.15) gives
which represents the factor by which the potential flow solution is "transformed" to that of laminar flow around a sphere of constant radius. In their condensation studies, Sideman and Hirsch (192) reported two distinct velocity regions, particularly in the small bubbles. After injections, the bubble attained a maximum velocity corresponding to that of a gas bubble. After remaining at this plateau for a while (approximately 0.1 second) the bubble was sharply decelerated, until it approached the terminal velocity of a liquid drop.

Isenberg and Sideman (99,100) have observed a constant rise velocity (200-225 mm/sec) for pentane and isopentane bubble condensing in water and a gradual decrease in rise velocity for pentane bubble collapsing in aqueous glycerol.

Selecki and Gradon (187) have numerically solved the equation of motion for a drop evaporating in an immiscible liquid medium. They included a term in the equation to account for the apparent weight of the residual liquid. The velocity was obtained as a function of time after making several simplifications. The velocity was found to be constant after about 0.5 seconds. The theoretical results were experimentally
verified using an optical system.

Simpson, Beggs and Nazir (198) studied evaporation of butane drops in a vertical column of water or brine. Height versus time data were obtained for various temperature differences, T, from the cine-film records. The data were differentiated to find the local velocities. Instantaneous velocity versus diameter ratio plot indicated a constant velocity of about 250 mm/sec., up to a critical diameter ratio. The velocity increased as the square root of the diameter ratio. The critical diameter ratio of 2 and 2.7 were reported for brine and deionized water respectively. There was lack of agreement between their data and that of Sideman and Taitel (197). Partly, the discrepancy was attributed to wall effects.

Higeta, Mori and Komotori (94) have studied the condensation of pentane in glycerol and have observed that the drag coefficients for condensing pentane bubbles fall in the range between the lines for solid and inviscid fluid spheres; while the experimental results by Tochitani et.al. (217) on vaporizing bubbles in the same system agree with Stokes theory rather than with the theory by Hadamard (79) and Rybczynski (181).

Mokhtarzadeh and E.Shirbine (140) used one of these methods to record the movement of bubbles. They
used still photography to obtain data for rising small and medium size air bubbles, cap shape bubbles of butane, vaporizing drops and condensing bubbles of butane in distilled water. It was reported that the motion of vaporizing drops could be rectilinear in a continuous liquid, if there was no turbulence in the medium surrounding the slow forming droplets.

Raina and Wanchoo (163, 167) have reported some theoretical studies on motion of two-phase bubbles. Recently, Wanchoo and Raina (224) proposed an analytical expression for the instantaneous velocity of a drop, vaporizing in an immiscible less viscous continuous phase medium. The expression can be written as:

\[ U_{\infty} = 2.4598 \left( \frac{\mu_c}{\rho_c D_0} \right) \left( \frac{hr L_0}{D_c^{0.929}} \right)^{0.4927} \]

...(2.16a)

This expression predicts, very well, the experimental data of Sideman and Taitel (197).

### 2.2.0 HEAT TRANSFER

Intimate contact between two immiscible liquids can be established by dispersing one of the liquids as droplets in the other liquid. Quantitative methods for predicting rates of heat transfer between the exposed surface of liquid droplets and the continuous liquid
medium are indispensable for direct contact heat transfer equipment design. The estimation of overall heat transfer coefficients require a knowledge of the dispersed and the continuous phase coefficients. With the advent of new techniques (91, 124, 157) it has been possible to have better understanding of heat transfer mechanism for single phase drops. There is, however, no technique available for determining the temperature profiles either inside or outside a vaporizing drop or condensing bubble moving in an immiscible continuous phase liquid medium. The situation is further complicated by an inadequate knowledge of the actual liquid-liquid heat transfer area. Most of the results on two-phase drops are, therefore, reported as overall transfer coefficients based on either initial drop or bubble diameter or the overall instantaneous area. An excellent review of pre-1964 work on direct contact heat transfer between two immiscible liquids is given by Sideman (196). More recently, the topic has been reviewed by Sideman (195) (dealing primarily with direct contact condensation), Sudhoff et.al. (208), and Jacobs (104). Jacob's review includes the topics related to direct contact evaporation, condensation and boiling as well as simple sensible heat transfer between the fluid streams. In this section a brief account of various heat transfer models associated with single phase drop is given followed by a review of heat transfer on two-phase bubbles.
2.2.1 Heat Transfer to Drops and Bubbles moving in a Constant Temperature Field

In the study of heat transfer to drops (or bubbles), the internal and external transfer coefficients are usually evaluated separately. The model is usually postulated as one of the four basic types, with the exact mechanism determined by the agreement of the experimental and calculated values. The four basic models are:

(a) Rigid drop;
(b) Drop with internal circulation;
(c) Oscillating drop;
(d) Completely mixed drop.

Since any of these postulated mechanisms may exist during the process of evaporation of a drop or condensation of a vapour bubble, a short discussion of these is pertinent. A detailed review is given elsewhere (189).

The rigid drop model is usually assumed for small drops less than 2 mm in diameter, whereas the circulating drop model is assumed for large drops. The transition, however, is not well defined, being highly sensitive to surfactants, which may render the interface immobile even in large drops. Large drops of relatively
low viscosity are usually characterized by the completely mixed drop model though large drops of highly viscous fluids tend to comply with the rigid drop model.

Interfacial mobility due to internal circulation increases the transfer rate by reducing the velocity gradient at the interface (thinning of the boundary layer), increasing the drop velocity (213) and retarding the boundary layer separation to higher Reynolds numbers and large boundary layer separation angles. Larger transfer rates are thus realised for the circulating drop as compared with a rigid drop. Still larger transfer rates are obtained with oscillating drops, probably due to frequent stretching of the interface and repeated thinning of the internal and external boundary layers. Oscillations start at Reynolds number of about 200, usually in systems having a low viscosity dispersed phase. The oscillation of the wake and its frequent shedding also account for the higher transfer rates.

The outside film coefficients are usually expressed in terms of the Nusselt number, whereas the inside coefficients are indirectly reported in terms of the transfer efficiency, $E_m$, representing the fractional approach to the maximum possible heat transfer. By definition:
\[ E_m = \frac{T_o - T_i}{T_c - T_i} \]  \( \ldots (2.17) \)

where \( T_o \) and \( T_i \) are the outlet and inlet mixing-cup average temperatures of the drop, and \( T_c \) is the constant temperature of the continuous medium. Assuming the specific heat capacities of the fluids as constant, \( E_m \) is the ratio of the heat actually absorbed by the drop to the maximum transferable heat over infinite contact time. The relationship between the internal transfer efficiency and the internal heat transfer coefficient \( h_d \), is given by (25, 82)

\[ h_d = \frac{\rho_d C_{pd} D_o \ln(1-E_m)}{6 t} \]  \( \ldots (2.18) \)

where \( t \) is the time of contact with the surrounding field.

2.2.2 Rigid Drop Model

This model is simply the transient case of radial conduction. If one defines the internal transfer coefficient in terms of \( E_m \), then for the case of zero resistance of the external film, transfer efficiency is given by (68, 149)

\[ E_m = 1 - \frac{5}{\hat{\lambda}^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp \left( -\frac{\hat{\lambda}^2 n^2 \alpha_d t}{R^2} \right) \]  \( \ldots (2.19) \)
Vermeulen (222), based on his experimental work, suggested a better solution to this problem

$$E_m = \left[ 1 - \exp \left( \frac{x^2 \alpha_d t}{R^2} \right) \right]^{1/2} \ldots \ (2.20)$$

This equation was later modified by Johnson and Hamielec (109) to give

$$L_m = 1 - 6 \sum_{n=1}^{\infty} A_n \exp \left( - \frac{x^2 \alpha_d t R^n}{R^2} \right) \ldots \ (2.21)$$

The factor $R^n$ is a function of the ratio between the rate of heat transfer by pure conduction and the actual internal rate of heat transfer; the value of $R^n$ being unity for pure conduction. The constants $A_n$ and eigenvalues $\lambda_n$ are available in the literature (189).

As reviewed by Sideman (189) many workers (184, 146, 117, 52, 53, 120, 27) have determined the continuous phase heat transfer coefficient assuming some form of velocity profile in the boundary layer. The boundary layer solutions apply only to the front part of the drop, up to the point of separation (168, 169, 115). Fortunately at low Reynolds numbers very little transfer takes place in the wake region (7). Hence fairly good estimates of heat transfer coefficients can be obtained by the general equation for the forward half of the drop, i.e.,
Kramers (117) derived the following equation for spheres at low Reynolds numbers, but reported that for very small values of Reynolds numbers this equation gives very high values of Nusselt number

\[
\text{Nu}_c = b \, R_{ec}^{1/2} \, Pr_c^{1/3} \ 
\] ...

(2.22)

At increasing Reynolds numbers the thickness of the boundary layer is reduced and turbulence grows in the wake region, making it more important for heat transfer. The thinning of the boundary layer also increases the transfer rate to the outside fluid (23). Drew and Ryan (45) reported equal rates of heat transfer for front and rear portions of the sphere at high Reynolds numbers. The increasing wake region transfer shifts the Reynolds number exponent of 0.5 to higher values. Steinberger and Treybal (225) suggested 0.62 for the Reynolds number exponent, whereas an exponent of 1.0 has been suggested by Harriott (88).

A detailed and critical review, including additional experimental data on heat transfer to solid spheres, was published by Rowe, Claxton and Lewis (176).
Most of the data on heat transfer to solid spheres have been correlated in the following form

\[ N_u C = a' + b' \frac{Re'_c}{Pr'_c}^{m'} \quad \ldots \quad (2.24) \]

For solid spheres in water, in the range of \( 30 < Re_c < 2000 \), Rowe et al. (176) recommended \( a'=2, \ b'=0.79, \ m'=1/2 \) and \( n'=1/3 \). Their analysis, further, indicates that the constants \( a' \) and \( b' \) may be varied considerably provided the exponents are chosen appropriately, thus explaining the apparent discrepancies between the numerous suggested correlations.

2.2.3 Drops with Internal Circulation

Transfer coefficients in circulating drops are dependent on the degree of induced circulation, which in turn depends on the Reynolds number and the viscosities of the two fluids.

Krong and Brink (118) solved the Fourier-Poisson equation and derived equations for the temperature field and heat transfer inside a drop with internal circulation as described by Hadamard (79). Under the assumption that diffusion is negligible along the streamlines and that the isotherms at any particular moment coincide with the streamlines, and neglecting outside resistance they obtained the following equation:
Krong and Brink's equation gives heat transfer coefficients about 2.5 times higher than the rigid drop model. The correct numerical values of the constants \( A_n \) and the eigenvalues \( \lambda_n \) in equation (2.25) have been calculated by Elzinga and Banchero (48).

Dankwerts (35) has indicated that the solution of Krong and Brink is not limited by the original assumption of low Reynolds number, as is evident from experiments (29) with Reynolds numbers up to 2000 under oscillation free conditions. A simpler empirical equation, accurately fitting equation (2.25) was proposed by Calderbank and Korchinski (29).
time equivalent to the Krong and Brink average circulation time, and arrived at an expression for eddy diffusivity via the Einstein equation

$$\chi_e = \frac{\text{Pe}_d}{2048(1 + \frac{\mu_d}{\mu_c})} \left(6 \frac{W^2}{D^2} - 8W + 3\right)$$

where

$$W = \frac{4r}{D}$$

$r$ being the radial distance from the centre of circulation toroid. Using the first eigenvalue only they showed that the transfer efficiency, can be expressed as:

$$E_m \sim 1 - \exp \left(-5.62 \times 10^{-3} \left(\frac{\text{Pe}_d}{\mu_c} \frac{\chi_d t}{R^2}\right)^{-\frac{1}{7}}\right)$$

... (2.28)

and internal Nusselt number takes the form:

$$\text{Nu}_d = 0.00375 \text{Pe}_d \left(1 + \frac{\mu_d}{\mu_c}\right)^{-1}$$

... (2.29)

Their experimental data agreed within ± 20 percent of the theoretical values. An extension of this model to include surface resistance is given by Welleck and Skelland (226).
The external film coefficients are conveniently obtained for pure liquid-liquid systems (with Pr > 1), by assuming the interfacial velocity to be of the same order of magnitude as that of the main stream. Additional simplifying approximations are also made by utilising the fact that under these conditions the thermal boundary layer is relatively thin.

Levich (125) treated the case of Reynolds number smaller than unity but with very large Peclet number. The following external stream function was used:

$$U_c = \frac{U_0 \mu_c}{2(\mu_c + \mu_d)} \frac{R}{(r-R)} \sin^2 \theta$$

... (2.30)

with $U_0$ as given by:

$$U_0 = \frac{2(\frac{P_d - P_c}{3 \mu_c})}{g R^2} \left( \frac{\mu_c + \mu_d}{2 \mu_c + 3 \mu_d} \right)$$

For a constant bubble-wall temperature, the Nusselt number was calculated to be:

$$Nu_c = 0.65 \text{Pe}^{0.5} \left( \frac{\mu_c}{\mu_c + \mu_d} \right)^{0.5}$$

... (2.31)

Similar equations have been proposed by others (74,132).

At very high Reynolds numbers the approximation of potential flow around the droplet has been made by
many workers (115, 179). The validity of this assumption has also been proved by a number of experiments (41, 92, 225) for many practical applications.

Boussinesq (20) first applied the potential flow theory in solving steady state forced-convection heat transfer problems; Ruckenstein (179) used it in determining heat transfer coefficients at the interface of growing vapor bubbles. With certain simplifying assumptions, the proposed equation was:

\[ \text{Nu}_c = 1.13 (\text{Pe}_c)^{0.5} \]  \hspace{1cm} (2.32)

Handlos and Baron (82) also agreed with the above equation for the external film heat transfer coefficient in a liquid-liquid system at high Reynolds numbers. The applicability of Equation (2.32) in liquid-liquid systems is obviously limited, since viscous shear at the interface cannot be neglected. However, by using perturbations consisting of small deviations from the potential flow field, the existence of a viscous boundary layer may be taken into account in terms of \( K_v \) as:

\[ \text{Nu}_c = 1.13 (K_v \text{Pe}_c)^{0.5} \]  \hspace{1cm} (2.33)

where \( K_v \) is defined here as the ratio of the true interfacial velocity to that obtained from potential theory. Expressions for \( K_v \) available in the literature (74, 132,
are of the form

\[ K_v = f \left( \frac{\mu_d}{\mu_c}, \frac{\rho_d \mu_d}{\rho_c \mu_c}, \text{Re}_c \right) \quad \ldots (2.34) \]

The maximum possible continuous-phase heat transfer coefficient obtainable for non-oscillating drops was suggested by Elzinga and Banchero (48). Their equation is based on the maximum heat transfer to a solid sphere calculated in the vicinity of the forward stagnation point. Applying it to drops with internal circulation, the obtained:

\[ \text{Nu}_c = 1.96 (\text{Pe}_c)^{0.5} \quad \ldots (2.35) \]

Equations for the external heat transfer coefficients for nonspherical drops with rapidly moving interfaces in axisymmetric flow have been derived by Lochiel and Calderbank (132). They found that flattening of the drops does not influence the transfer coefficient appreciably. This explains the fair agreement of experimental data with the commonly used spherical correlations, which utilize the diameter of a sphere of equivalent volume as the characteristic length. This similarity in transfer characteristics, inspite of the increased surface area of the flattened drop, is obviously due to the simultaneous reduction in angular velocity in the boundary layer.
Terjesen and coworkers (24,213) explained the high heat transfer coefficients of circulating liquid drops in contrast to those for non-circulating drops, as due to their high velocities. Their experiments proved that the effect of interfacial agitation is small compared with that of velocity. The higher coefficients are attributed to hydrodynamic disturbances in the unstable boundary layer and the vortex formed behind the moving drop. Thus they proved that for liquid drops the relative importance of the front and rear areas of heat transfer has been reversed from that for solid spheres.

2.2.4 Oscillating Drop Model

A correlation accounting for the effect of oscillations on the external mass transfer coefficient is given by Garner and Tayeban (66).

\[
\text{Nu}_C = 50 + 8.5 \times 10^{-3} \text{Re}_C \text{Pr}_C^{0.7} \quad \ldots (2.36)
\]

Skelland and Welleck (200) have reported a correlation for the internal mass transfer coefficient in terms of Weber number, a dimensionless group accounting for the unsteady nature of the transfer mechanism.

Rose and Kinter (174) have proposed a theoretical model based on the concepts of interface stretching
and internal mixing of the droplet between oscillations. Their motion picture studies on single drops indicate that in case of oscillating drops the circulation was either deformed or damped and once oscillations were damped, the circulation pattern was restored.

2.2.5 Completely Mixed Drop Model

This is actually the simplest model and may be applied whenever the internal film resistance is zero and the external film governs the transfer rate. Here, the temperature of the drop, $T_d$, is uniform and may change with time. Using the familiar overall heat transfer coefficient, $\overline{U}$, yields for the case where $T_c= constant$.

$$Q = V \int_{d}^{p} C_p d \frac{dT_d}{dt} = \overline{U} (T_c - T_d) \quad \ldots \ (2.37)$$

or

$$E_m = 1 - \exp \left( \frac{-3 \overline{U} t}{R \overline{C_p}} \right) \quad \ldots \ (2.38)$$

2.2.6 Heat Transfer to Vaporizing Drop

Klipstein (115), who studied the heat transfer mechanism of evaporating drops in a continuous liquid phase, took a sequence of still photographs of the drops of ethyl-chloride rising in a water column at constant temperature. He suggested that the internal phase is well mixed at all times and thus offers no appreciable resis-
tance to heat transfer. Hence, assuming the outside resistance to be controlling, he proposed the following relationship.

\[ \text{Nu}_c = 2 + 0.094 \text{Re}_c^{0.93} \text{Pr}_c^{0.73} \quad \cdots (2.39) \]

Klipstein also proposed an equation for average rate of heat transfer:

\[ Q_{av} = 2.84 D_0^2 \Delta T \quad \cdots (2.40) \]

The overall heat transfer coefficient was found to be independent of the initial drop diameter. Surfactants were found to decrease the heat transfer rate by reducing internal circulation and interfacial rippling, and increase the heat transfer coefficient by enhancing the oscillation tendencies. Quantitatively, the presence of surfactants resulted in 30 per cent increase in the heat transfer rates for small drops (\( D_0 = 2.39 \text{ mm} \)), no change for medium size drops (\( D_0 = 3.01 \text{ mm} \)), and 20 per cent reduction for large drops (\( D_0 = 3.78 \text{ mm} \)).

Simultaneous studies of the mechanism of evaporation of discrete particles, in direct contact heat transfer between immiscible liquids were made by Sideman and Taitel (197). They assumed potential flow, axial symmetry and steady state for their spherical model. This yielded after simplification for constant bubble radius:
\[
\sin \theta \frac{\partial T}{\partial \theta} - 2y \cos \theta \frac{\partial T}{\partial y} = \frac{2 \rho \alpha_c}{3 \frac{U_\infty}{\lambda}} \frac{\partial^2 T}{\partial y^2}
\]

where \( y \) is the dimensionless radial coordinate. The analysis considered the internal resistance to be negligible and assumed the vaporous portion of the bubble wall to constitute an adiabatic surface. They obtained an average Nusselt number based upon the vapor opening angle \( 2\beta \) and heat flux per unit area of the entire drop:

\[
\text{Nu}_c = \left( \frac{3 \cos \beta - \cos^2 \beta + 2}{\lambda} \right)^{0.5} \frac{\rho e^{0.5}}{\lambda}
\]

For \( \beta = 0 \), this yields a Nusselt number of \( 1.13 \rho e^{0.5} \), as predicted by the potential flow theory.

Experimentally, they obtained film coefficients two orders of magnitude higher than those for non-evaporating drops. The time and level of total drop evaporation were found to be inversely proportional to the temperature driving force. The average velocity was about \( 50\% \) higher than that for non-evaporating drops due to increased buoyancy. When total evaporation was attained, the bubble assumed the shape of a spherical bubble cap with a velocity of 280 mm/sec., as predicted by both Levich (125) and Davis and Taylor (39). Also, the depen-
dence of the heat transfer coefficients on the temperature driving forces in the 4-15°C range, was found to be very weak, but increased below this range. It was pointed out, however, that the sharp increase of the film coefficient at low driving force was probably due to the increased influence of the hydrostatic head. Smaller drops were found to yield higher coefficients, while also conforming more toward the spherical shape assumed in the theoretical analysis.

The heat transfer coefficient is, by definition, the instantaneous heat flux per unit of transfer area divided by the temperature difference. Here, however, the heat flux is changing with time and height, due to changes in the transfer area and the hydrostatic head. The latter will drastically increase the temperature difference at small temperature driving forces. Although it has been logically assumed that the actual heat transfer surface is the liquid-liquid area of the interface, photographic analysis affords only partial proof. When 1% of the mass of the liquid was evaporated, most of the volume of the drop is vaporous. After 10% evaporation, the liquid layer is no longer discernible. The data were based, therefore, on two types of area that could be physically measured. A coefficient based on the initial area, and a coefficient based on the instantaneous area of the entire drop were defined. The expression derived theoretically
for the Nusselt number was based on the latter. Care must be exercised in interpreting the data.

While the above analysis assumed the internal resistance of the bubble to be negligible, Sideman and Hirsch (193) included this effect in their model, which was presumed to be an oblate spheroid of constant eccentricity. The effective heat transfer area was assumed to be the liquid-liquid interface and the length of the major axis was time dependent. This length was determined as being proportional to the cube root of the vapor weight fraction. For the pentane-water system theoretical expression for outside and inside heat transfer coefficients were proposed which should be valid only beyond three percent evaporation.

\[ h_c = 0.21 a_1^{1/2} x^{-3/6} \quad \ldots \ (2.43) \]
\[ h_d = \frac{2}{3} \frac{x}{1-x} a_1^{1/2} \quad \ldots \ (2.44) \]

They also claimed that Taitel's data (210) was found to be in good agreement with the proposed correlations. The authors concluded that the external resistance controls over 70 per cent of the evaporation process in terms of the evaporation ratio, while in terms of the evaporation time, the two resistances are of equal magnitude.
After reviewing Sideman and Hirsch's paper (193) Prakash (159) found that the derivation of equation (2.44) was wrong, and that in fact it should be,

\[ h_d = \frac{2}{3} \frac{x}{1-x} \quad \ldots \ (2.45) \]

An attempt to correlate the heat transfer mechanism with initial drop size was reported by Sideman, Hirsch and Gat (190). It was noted that the determination of film coefficient, depends on a knowledge of the effective heat transfer area. This area is, however, virtually impossible to determine experimentally above an evaporation ratio of 3 per cent.

The overall average resistance was defined as:

\[ R' = \frac{t_v \Delta T}{G_{max}} \quad \ldots \ (2.46) \]

where \( t_v \) is the time required for total evaporation, until no further change in volume is observed. For an initial diameter \( D_0 \), the experimental correlation was assumed to be:

\[ R' = C_1 D_0 C_2 \quad \ldots \ (2.47) \]

This assumption was reasonable in view of the available data. The average heat flow rate was defined as:

\[ Q = \frac{Q_{max}}{t_v} = \frac{\Delta T}{R'} \quad \ldots \ (2.48) \]
Two alternatives were proposed to express the average overall heat transfer coefficient. For the first case, the coefficient $\overline{U}$ is related to the initial drop diameter by:

$$\overline{Q} = \overline{U} \times 4 \pi R_0^2 \Delta T \quad \ldots \quad (2.49)$$

which then leads to:

$$\overline{U} = \frac{1}{\pi C_1 R_0 (C_2+2)} = C_3 R_0^{-\alpha} \quad \ldots \quad (2.50)$$

The equation suggested for pentane-seawater system was,

$$\overline{U} = 5.07 \times 10^4 D_0^{-0.64} \quad \ldots \quad (2.51)$$

where,

$\overline{U}$ is in K cal / (hr) (m$^2$) (°C)

and

$D_0$ is in mm.

For the second case, defining the associated coefficient as $\overline{U}$ and the instantaneous area as $A$, then:

$$\overline{Q} = \overline{U} \times A \Delta T \quad \ldots \quad (2.52)$$

Here, it was contended that $A$ is proportional to the initial drop area since the ratio varies only moderately during the evaporation process. With this proportionality constant, $W$, the following was obtained:
Corresponding equation suggested for pentane-seawater system was:

$$ U = \frac{1}{\kappa C_1 \sqrt{D_0 C_2 + 2}} = C_3 D_0^{-0.64} \quad \ldots \ (2.53) $$

The exponent of -0.64 on $D_0$ as compared with an exponent of -0.5 suggested by Sideman and Taitel (197), was explained as due to the fact that Sideman and Taitel neglected the inside film resistance which is important in the beginning of the evaporation process. The correct exponent of -0.64 accounts for outside as well as inside film resistance for heat transfer.

It is interesting to note that the authors deduced that the surface tension forces are an important aspect of this process.

Prakash and Pinder (159) have studied the vaporization of furan, isopentane and cyclopentane in water using motion picture photography. The correlation which predicted the overall heat transfer coefficient for all the three systems was

$$ Nu = 0.0505 (Pe')^{0.417} \left( \frac{\rho_c}{\rho_d} \right)^{1.25} \quad \ldots \ (2.55) $$

where the Nusselt number and the modified Peclet number
were based on the dispersed phase liquid properties. This correlation was developed from the experimental data and was found to be valid only up to ten percent evaporation.

These authors have used dilatometric method (158) for the determination of total evaporation time. However, this technique did not give satisfactory results as the second order response made it hard to use. They have used a correction factor to take into account the error due to second order response of the dilatometer.

The average rate of heat transfer for all the three systems was correlated as:

$$\bar{Q} = C' D_0^2 \Delta T$$

where, 'C'' is a constant which is different for each system.

Adams and Pinder (2) combined the dilatometric method with cine-photography to obtain average heat transfer coefficient for isopentane and cyclopentane drops vaporizing in distilled water or aqueous glycerol. Their results indicated a significant difference in the initial drop size obtained by these two methods. An empirical correlation was reported for the average heat transfer coefficient as:
\[ \text{Nu} = 7550 \Pr^{0.75} \left( \frac{\mu_c}{\mu_c + \mu_d} \right)^{-7.3} (\text{Bo})^{0.33} \]

where Bo is the liquid Bond number.

Except for minor differences the experimental set-up used by Simpson, Beggs and Nazir (198) was similar to the one used by Sideman and Taitel (197). Both used same nucleation technique and cine-photography. Simpson, Beggs and Nazir (198) found the overall heat transfer coefficient increasing monotonically with the diameter ratio \((D/D_0)\) in contrast to a moderate decrease after \(D/D_0=2\), as reported by Sideman and Isenberg (194). These authors further observed that the vaporizing butane drop oscillated during its rise through the liquid column in addition to changes in shape from spherical through ellipsoidal, to a cap shape. These oscillations sloshed the residual liquid inside the two-phase bubble at such a high frequency as to form a thin film on the inside surface of the bubble. Based on these observations a mathematical model of the evaporation process was developed. An expression developed for the rate of change of bubble diameter was given by:

\[ \frac{d(D/D_0)}{dt} = C_2^{(m-1)} \frac{k_d C_p_d \Delta T}{\lambda_d} \left( \frac{g^{\frac{1}{6}}}{\rho_d^{\frac{1}{3}} D_0^{3/2}} \right) (\frac{D}{D_0})^{\frac{1}{6}} \]

\[ \ldots \ (2.58) \]
This equation was considered to be applicable to the region of the bubble growth curves where sloshing dominated. The constant $C^1_2$ was estimated from their experimental data and an expression for the overall heat transfer coefficient valid up to about 90 per cent vaporization was given by:

$$
\tau_0 = \frac{2.57(D/D_0)^{2/6}}{1 + 0.205(\frac{D}{D_0})^{5/12}} \ldots \ (2.59)
$$

representing their data to within $\pm 18\%$.

Tochitani, Mori and Komotari (218) studied the heat transfer characteristics of pentane and furan drop vaporizing in high viscosity aqueous glycerol. Their study was confined to the region where rectilinear motion and geometrically simple configuration of the vaporizing drop was possible. The salient feature of their work was that they measured the actual liquid-liquid heat transfer area up to as high as 50 percent vaporization. The liquid-liquid transfer area was found to take an almost constant value (in the range below 10 per cent vaporization) slightly higher than the initial surface area of the liquid drop. Beyond 10 per cent vaporization the liquid-liquid transfer area decreased gradually below the constant value. An analytical expression for the outside heat transfer coefficient was der-
ived, assuming Stoke's flow. Their expression for the average Nusselt number for the outside flow has the form:

$$\text{Nu}_c = 0.463 \text{Pe}_c^{4/3} \left( \frac{\pi - \beta + \frac{\sin 2\beta}{2}}{2} \right)^{2/3} \quad \ldots (2.60)$$

Mokhtarzadeh and El. Shirbini (139) have made theoretical analysis of vaporizing drops in an immiscible liquid medium. Equations of mass, momentum and energy were solved simultaneously using a numerical method. They used the overall heat transfer coefficient expression of Sideman et. al. (194) and Simpson et.al. (199) to predict the variation in the bubble radius, height, velocity and temperature with time. The total vaporization time and the corresponding distance travelled by the vaporizing drop were also reported.

Raina and Grover (164) analyzed the heat transfer to a vaporizing drop based on the configuration model of Mori (143). These authors assumed the transfer area to lie between the zenethal angles $\gamma$ and $\beta$ and Stoke's theory to be valid for the outside flow. Based on these assumptions, an analytical expression for Nusselt number was derived for non-sloshing drops ($D_0 < 2 \text{ mm}$) given by:

$$\text{Nu}_c = 0.4628 (\text{Pe}_c)^{4/3} \left( \sqrt{\frac{\pi - \beta + \sin 2\beta}{2}} \right)^{2/3} \quad \ldots (2.61)$$
The angle \( \gamma \) was supposed to be dependent on spreading coefficient characterizing the extent of spreading of dispersed liquid on the vapour surface. Later, these authors incorporated the sloshing effect (for \( D_0 > 2 \text{ mm} \)) in their model and modified their expression for non-sloshing drops (165) to yield:

\[
Nu_c = 0.4628 \left( \frac{Pec}{\beta} \right)^{1/3} \left( \sin \frac{\beta}{2} \right)^{2/3} \]

...(2.62)

Battya, Raghavan and Seetharamu (12,13) have reported some parametric studies on evaporation of a drop in an immiscible liquid. From their analysis they concluded that the bubble growth rate, Nusselt number and the time and height required for complete evaporation depended mainly on the Peclet and Jakob numbers. A regression analysis was also carried out using the experimental data of Sideman and Taitel (197) and the correlation thus obtained is given by:

\[
Nu_c = 0.64 Pec^{0.5} Ja_c^{-0.35} \]

...(2.63)

Recently Tadrist et al. (209) have studied the vaporization of refrigerant R 113 and pentane in water. These authors have used numerical analysis methods to solve the equations of motion and energy. They used the heat transfer coefficient expression of Battya et al. (13) and for drag coefficient that of Haberman and Morton.
In their analysis the authors have used effective heat transfer area defined as:

$$A_e = 2\pi R^2 \left(1 + \cos j \frac{\beta}{\rho}\right) \quad \ldots \quad (2.64)$$

where, $j$ is the unknown parameter introduced to account for the effective surface compared to its plane interface value ($j=1$). After numerous experiments with pentane-water system, they concluded that the values obtained for the parameter $j$ are such that $0.75 < j < 0.85$.

2.2.7 Heat Transfer from Condensing Bubble

According to gas kinetics concepts condensation occurs, when the number of vapour molecules striking the condensed liquid surface exceeds the number of molecules leaving it, and a temperature 'jump' exists between the saturation temperature of the vapor and the surface of the condensed liquid (134). The condensation coefficient '$a$' represents the ratio of the number of condensing molecules to number striking the surface.

Utilizing the Hertz-Knudsen and Clausius-Clapeyron equations, the interfacial heat transfer coefficient can be written as

$$h_i = 'a' \left(\frac{\bar{M}}{2} \rho \bar{R} T\right)^{1/2} \left(\frac{\lambda^2}{T V'}\right) \quad \ldots \quad (2.65)$$
where \( \bar{M} \) is the molecular weight, \( R \) is the gas constant, \( T \) is the absolute vapour temperature, \( \lambda \) is latent heat and \( V' \) is the specific volume of the vapour at \( T \).

The value of the interfacial heat transfer coefficient clearly depends on the value of 'a', for which conflicting values, ranging from 0.036 to 1.0 have been reported (90, 211, 28).  

Nabavian and Bromley (148) assumed that condensation and evaporation coefficients were equal. Their experiments in condensation yielded value of 'a' ranging from 0.35 to 1.0, but they speculated that values near the upper limit are more plausible. Moreover, as shown by Hasson, Luss and Navon (90), the influence of 'a', on the heat transfer rate at normal pressures is negligible, even when the lower values were used. Since the value of 'a' for non-polar liquids such as pentane, is known to be close to unity, this would indicate that the heat transfer rate is independent of the condensation coefficient.

Bubble collapse in an infinite medium is, in general, governed by both the momentum and energy equations for the medium. Virtually all studies of collapse rates have been restricted to radially symmetric motion. The momentum equation, for this regime,
as derived by Rayleigh (170) is:

\[ \rho c L \frac{d^2R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right)^2 \frac{\sigma}{R} = \Delta p \]

where surface tension effects are significant only at the end of collapse, i.e. at small bubble diameters.

The energy equation for this regime is the non-steady heat diffusion equation with radial symmetry:

\[ \frac{\partial T}{\partial t} + \frac{dr}{dt} \frac{\partial T}{\partial r} = \frac{\lambda}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) \]

where, from the continuity equation

\[ \frac{dr}{dt} = \frac{R^2}{r^2} \frac{dR}{dt} \]

The bubble collapse rate appears in the energy equation as well in one of the boundary conditions. The general solution of radially symmetric collapse rates, controlled by both the Rayleigh and the heat diffusion equations, has yet to be obtained (a simplified solution is given by Florschuetz and Chao (49)). However, there are two domains where one of the equations becomes trivial. One domain is that of an isothermal process, where the rate of heat transfer at the bubble wall is very large, so that it is considered a step function in time. The heat diffusion equation is then insignificant, and the collapse rate is controlled by the Rayleigh equ-
ation. The other domain is that of low temperature gra-
dients, where ΔP is negligible and the Rayleigh equa-
tion becomes trivial. This leads to a heat transfer
controlled collapse. In the first domain collapse rates
are high and increasing with time, while in the second,
the collapse rates are comparatively slow and decrease
as the collapse proceeds. Bankoff and Mikesell (9), for
example, reported data for highly subcooled boiling
conforming to the Rayleigh equation.

Numerous theoretical studies have been conduc-
ted on heat transfer controlled growth or collapse.
Most have been concerned with the asymptotic period,
which timewise, is by far the period of greatest inte-
rest. Forster and Zuber (51) combined the Rayleigh
equation with Green's function for a spherical source
and related Δp to ΔT by the Clausius-clapeyron
equation. For their calculations they used low tempera-
ture gradients and the liquid inertia terms became
negligible. Forster (50) later explained his 'Source'
theory in an attempt to demonstrate a more general
applicability. However, while feasible for radially
symmetric problems this technique becomes untractable
for the axisymmetric case with moving boundaries, such
as that encountered in the present work.
By assuming a thin thermal boundary layer, Plesset and Zwick (156) solved the heat diffusion equation by a method of successive approximations. Their solution for the asymptotic period of growth compares favourably with that obtained by Forster and Zuber, differing by about 13 per cent. Within the limit of experimental accuracy, both theories were verified by Dergarabedian (42).

The relative importance of liquid inertia and heat transfer effects on the collapse rate was evaluated numerically by Florschuetz and Chao (49). A dimensionless parameter was developed that effectively specifies the governing mode. The parameter is:

$$B_{\text{eff}} = \mathcal{E}^2 \left( \frac{\rho_c C_p \Delta T}{\overline{C_{\text{av}}} \lambda} \right)^2 \frac{k}{\kappa_0} \left( \frac{\rho_c}{\Delta p} \right)^{1/2}$$

where $\mathcal{E}$ is a coefficient accounting for the non-linearity of the vapour pressure-temperature relationship, and $\overline{C_{\text{av}}}$ is the average bubble density during the collapse. A heat transfer controlled mechanism was specified for $B_{\text{eff}} \ll 0.05$, and a liquid inertia controlled mechanism was specified for $B_{\text{eff}} \gg 10$. For example, the data used by Zwick and Plesset (233), in their theoretical study of bubble collapse, yields $B_{\text{eff}}$ of 13.3. It is, therefore, not surprising that their results adhere
closely to the Rayleigh curve. Florschuetz and Chao obtained excellent verification of their predicted modes of collapse. Translation motion was restricted by allowing the test chamber to drop two meters on each run, thus giving a weightless condition lasting 0.6 seconds.

Using the same test chamber Wittke (229,230) investigated collapse of translating steam bubbles in water. Assuming potential flow, the temperature field was solved numerically, and yielded good correlation between theory and experiment. Following the analytical bubble growth solution of Ruckenstein and Davis, a potential field was also assumed by Dümic (43) for a condensing one-component vapour bubble, with translatory motion. A quasi-linear solution, based on linear dependence of the motion of the bubble on its radius and a quasi-steady-state solution, neglecting the effect of radius on the form of the temperature field, were reported.

Bankoff and Mason (8) measured turbulent heat transfer coefficients at the surface of single bubbles formed by injecting steam into a subcooled water stream at atmospheric pressure. Depending on the steam flow rate, water temperature, and velocity, three distinct types of bubble behaviour were observed. At water temperatures between 90 and 130°F the bubbles took on an ellipsoidal shape, and either collapsed almost instantly at the
lower range, or formed a pinched bubble and collapsed at the higher range. Increasing the water temperature above 130°F caused the bubbles to take on a flattened ellipsoidal shape, grow to prolate spheroids, and collapse. In contrast to the smooth surface of the first group, the shape was irregular. At water temperatures of 160 to 180°F bubbles exhibited large oscillations in volume and the shapes were irregular, but collapse was never complete. The surface of these bubbles was smooth but distorted. It was noted that turbulent film coefficients at the surface of a rapidly growing bubble might be of one or more orders of magnitude higher than that for a bubble surrounded by a liquid in laminar radial flow, with no tangential components. They obtained film coefficients ranging from 13,000 to 316,000 Btu/hr-ft²°F and proposed empirical equations for the Nusselt number as a function of the Peclet and Strouhal numbers for different bubble groups.

The transfer coefficients reported by Bankoff compare favourably with those reported by Grassmann and Wyss (73), who studied heat transfer between steam bubbles and water.

It may be anticipated that the collapse rates of condensing bubbles, in a three-phase system with relatively low ΔT are similar to those encountered in two
phase systems, by being sufficiently slow as to be categorized as heat transfer controlled, but are faster than radially symmetrical collapses, due to translational effects. Unlike the single-component, the condensate in three-phase system remains within the confines of the bubble wall (103, 192, 193, 105). Sideman and Hirsch (192) have reported some low-speed cine camera studies of single isopentane bubbles condensing while rising in stagnant water. Two groups of bubble-size were observed; the first consisted of bubbles of an average initial diameter of 3.8 mm, which took on an ellipsoidal shape and fluctuated slightly; the second group with an average diameter of 5.5 mm underwent severe deformation into prolate spheroids, discs and cones. Also, the two groups studied by Sideman were similar in behaviour to the first and third group, described by Bankoff and Mason (8). In the latter study, collapse was complete only at large $\Delta T$. The appearance of the liquid layer was confined to runs of high temperature differences and was interpreted as an indication that condensation was incomplete at the more practical working range of 0.6 to $\theta^\circ$C.

The instantaneous overall heat transfer coefficient, $U^*_a$, was defined as:

$$U^* = \frac{\lambda_d}{\Delta T} \int \frac{P_{dv}}{A} \frac{dV}{dt} \ldots (2.70)$$
where $\frac{dV}{dt}$, the rate of change of vapour constituent, was assumed to be equal to the volumetric rate of change of the entire drop. When this coefficient was plotted against vapour content, two regions could be distinguished. In the first region, the coefficient was fairly constant and sometimes even increasing. The second region showed the coefficient to drop rapidly to less than 10 per cent of the average value of the first region. The instantaneous heat transfer coefficient was, in general, found to decrease with decreasing mass of the vapour constituent.

A plot of $U^*$ as a function of vapour content, revealed the expected sharper decrease in the coefficients for the larger bubbles, as compared to smaller ones. In many bubbles, the initial stages yielded different transfer rates. The authors attribute this to differences in formation time, degree of superheat, and liquid content of the bubbles. The dependence of the coefficients on temperature driving force could not be isolated in the small differences (0.5 to 3°C) employed. Additionally, some condensation was suspected to occur during the formation period, despite initial superheats as high as 10°C. For a given nozzle diameter, the initial and final bubble diameters were noted to be independent of temperature differences above 3.5°C. This was speculated to be due to relatively thicker, uniform
liquid film around the forming bubble wall, producing for this short duration a conduction controlled transfer rate.

The transfer coefficients obtained here were lower than those of Bankoff and Hasson (8), by one order of magnitude. This was attributed to the higher Reynolds numbers in the system of the latter and to mass diffusion of steam into water.

While comparing condensation and evaporation mechanisms in three phase systems (Fig. 2.1), Sideman and Hirsch (192) found the heat transfer coefficients in condensation, to be higher than those for evaporation. This, however, was attributed to the effect of drop diameter. In general, both processes consisted of a regime of low vapour content, where the transfer coefficient increased rapidly with increasing vapour content, followed by a region of high vapour content with relatively moderate changes in the coefficient. The characteristic 'plateau' of the latter regime was ascribed to the absence of internal resistance of the liquid drop. It was postulated that evaporation and condensation are similar mechanisms, implying that the heat is transferred at the liquid-liquid interface i.e. the condensate-continuous phase interface occupying the rear part of bubble surface just the same as in the vaporization of a drop (197) for
In addition, the condensation study found that in the low range of temperature differences of 0.5 to 3°C, the transfer coefficient was independent of temperature difference. By contrast, in evaporation (197), a marked increase in the coefficient was noted in this temperature range, which was attributed to the unaccounted for hydrostatic head. In the range of temperature differences of 4 to 15°C, the dependence of the heat transfer coefficient on the temperature, was found to be very weak. A careful study into the effect of the temperature driving force was suggested. Their expression for average Nusselt number has the form:

\[ \text{Nu}_c = 0.27 \ (\text{Pe})^{1/2} \quad \ldots \quad (2.71) \]

Later Isenberg and Sideman (100) reported their studies of single pentane and isopentane bubbles condensing while rising in stagnant water or aqueous-glycerol solutions. These authors presented a different model. They supposed a two-phase bubble to be covered with a thin film of condensate flowing downward along the surface into the puddle at the bottom, and presumed that heat transfer occurs exclusively at the interface of the bubble wall above the angle of separation of the boundary layer (i.e. frontal part of bubble surface
across the thin film of condensate) contrary to the former model of Sideman and Hirsch. These authors also introduced a Prandtl number-dependent velocity factor by which the average heat flux around a sphere in potential-flow field was "transformed" to yield the average heat flux in a viscous flow field. The governing equations were solved for collapse rate by numerical methods, requiring a different solution for different values of the governing parameters. These numerical solutions were evaluated at a constant value of angle of separation equal to 100°.

The corresponding heat transfer coefficients were then computed from the collapse rates obtained above.

A fairly good agreement between experiment and theory was reported for runs in water for all bubble sizes studies (1.5 < R_0 < 5 mm). However, for runs in aqueous glycerol, the analysis was reported to be valid only for R_0 < 2.3 mm, and for larger bubbles the experimental collapse rates were reported to be higher than the corresponding theoretical collapse rates. This was attributed to the onset of circulation within the bubble. The authors further concluded that collapse of translating bubbles in a two-component system was governed by four parameters: the Peclet, Jakob, and Prandtl numbers,
along with the amount of noncondensables present.

A simple, albeit approximate, solution for the transfer rate of a translating, collapsing bubble was suggested by Isenberg et al. (99) and Moalem and Sideman (138). In the absence of noncondensables, the instantaneous bubble radius, normalized with respect to the initial radius was given by (99):

\[ B = \frac{R}{R_0} = \left[ 1 - \left( \frac{3}{2} \right) \left( \frac{K_v}{\alpha} \right)^{3/2} \right] \mathcal{C}_0 \left\{ \frac{2}{3} \right\}^{2/3} \]

where

\[ \mathcal{C}_0 = J_a \frac{\alpha^{1/2}}{F_0} \]

and average Nusselt number was given by:

\[ Nu_c = \left( \frac{K_v}{\alpha} \right)^{1/2} \]

In the presence of noncondensables, the analytical quasi-steady-state solution, Eq. (2.72), was extended (138) to include the effect of inerts. The general solution reads

\[ \mathcal{C} = F_2(B_f) \left[ \mathcal{C}_0 + \mathcal{C}_1 \right] \]

where
\[ \mathcal{C}_1 = \left( \frac{\pi}{K_V} \right)^{\frac{1}{2}} \left( \frac{B_f - \frac{1}{m}}{3 \frac{B_f^{3/2}}{2}} \right) \ln \left\{ \frac{(1+3^{3/2}) (B_f^{3/2} + B_f^{3/2})}{(1+3^{3/2}) (B_f^{3/2} - B_f^{3/2})} \right\} \]

Here, \( B_f \) is the final radius, and \( F_2(B_f) \) is a modification function for the nonhomogeneity of the inerts in the bubble, \( F_2(B_f) = 1 \) for a homogeneous distribution. For a parabolic distribution of the inerts within the bubble, \( F_2(B_f) \) is given by

\[ F_2(B_f^p) = 1 - \frac{2}{5} (B_f^p)^3, \quad B_f^p = \left( \frac{5}{3+2B_f^H} \right)^{\frac{1}{3}} B_f^H \]

The superscript \( p \) denotes the parabolic profile and \( B_f = B_f^H \) for the homogeneous distribution of the inerts in the bubble. The latter is calculated by:

\[ B_f^H = \frac{R}{\lambda d (T^* - T_\infty)} + \frac{1}{m} \gamma^* \]

where \( y_0 \) is the initial molar concentration of the inerts. The authors observed that the concentration gradients were predominant in small bubbles, whereas larger bubbles (\( > 4 \text{ mm} \)) had a uniform distribution of inerts.

Jacobs et al. (103) investigated analytically the collapse of a bubble of saturated vapour rising through a cold continuous immiscible liquid. On comparing
their results with the experimental data (99), it was concluded that the internal resistance of the condensate film in a collapsing bubble would provide nearly one-third of the resistance to heat flow. These authors assumed a linear temperature profile for the condensate film and a quadratic profile to exist in the thermal boundary layer. The analytical quasi-steady-state solution for collapse rate thus obtained reads as:

\[
B = \frac{R}{R_0} = \zeta^{-1} 0.9009 \Phi Pe^{1/2} Ja Fo_0^{-7/3}
\]

\[
= \zeta^{-1} 0.9009 \Phi \zeta_0^{-7/3}
\]

where \( \Phi \) is a function of \( Ja \) and \( k_d/k_c \)

\[
\Phi = \sqrt{C^2 + 2C} - C
\]

where

\[
C = \frac{0.75 k_d}{Ja k_c}
\]

For isopentane-water system Eq.(2.78) yields:

\[
B = \frac{R}{R_0} = \zeta^{-1} 0.74 \zeta_0^{-7/3}
\]

However, their model overpredicted the experimental data of Isenberg et.al. (99).
Later, Jacobs and Major (105) used a quasi-steady-state integral boundary layer approach for heat transfer and the concentration profiles of non-condensables were determined by solving the transient diffusion equation. Numerical methods were used to solve the governing equations. The numerical solution thus obtained for collapse rate in the absence of non-condensables reads as:

$$B = \frac{R}{R_0} = \zeta \left(1 - 0.6049 \dfrac{C_P}{C_0} \zeta \right)^{\frac{2}{3}} \ldots (2.80)$$

Eq. (2.80) differs from the results of Jacob et al. (103) only in the constant of the second term in the brackets. This difference was due to truncation of the series expansion which earlier authors had used in their analysis.

For bubbles smaller than 4.5 mm in diameter, they found that the collapse was dominated by diffusion of the non-condensables; for larger bubbles, the early bubble distortion effectively kept the vapour gas mixture uniformly mixed as proposed earlier by Isenberg and Sideman (100).

Higeta, Mori and Komotori (94) have studied experimentally the effect upon the condensation patterns of the wettability of condensate on the continuous-phase
liquid. The two systems studied were, n-pentane-glycerol and water-silicone oil. It was observed that in the former system with higher wettability of condensate on continuous-phase liquid, a condensing vapour bubble turned into a stable two-phase bubble in which the condensate accumulated at the bottom. On the other hand, in the latter system with poor wettability, a number of tiny particles of the condensate detached from the vapor bubble.

The instantaneous overall heat transfer coefficient based on the total surface area of the bubble in case of pentane-glycerol system decreased monotonically with decrease in vapour content. The average overall condensation heat transfer was observed to be higher than vaporization coefficient in the same system (218). This was attributed to the presence of thin film of condensate at the frontal part of bubble in the condensation process.

The Nusselt numbers reported for water-silicon oil were lower than those in the pentane-glycerol system. This lowering of the Nusselt numbers was ascribed to the supercooling of the vapour in the bubble due to the poor wettability of the condensate on the continuous-phase liquid and to the lower thermal conductivity of the continuous-phase.
Later, these authors (25) reported a different pattern in case of methanol bubble condensing in silicon oil. In this system the condensate appeared at the rear part of the bubble surface as several blunt drops which turned into a conical tail through coalescence to each other and finally dropped out from the bubble.

The heat transfer characteristics of methanol-silicone oil system was intermediate between those of pentane-glycerol and water-silicone oil systems due to better wettability of the continuous-phase liquid.

More recent studies of bubble collapse deal with the condensation of vapour bubbles in an immiscible liquid that is lighter than the condensate. This causes the bubbles to decelerate after the condensation is near completion and fall backward somewhat like a "spouted-bed-device" (110,123).

An experimental and theoretical study of a pentane bubble condensing in water was recently reported by Kalman et.al (111). Their visualization studies of the pentane bubbles 4.4 mm diameter condensing in water have illustrated the existence of two zones of bubble motion viz. acceleration zone and deceleration zone. In the zone of acceleration a viscous boundary layer was
formed on the bubble and a wake was formed behind the bubble. The zone of deceleration was characterised by the envelopment of the bubble in its wake.

Based on the assumption that the velocity of the bubble is time dependent in both zones of collapse and a number of other simplifications, these authors used the model of Lee and Barrow (121) for predicting the external thermal resistance and suggested the following equations:

in the zone of acceleration:

\[ Nu_c = 0.898 \left( \frac{Re_c}{\mu} \right)^{0.52} \frac{\sin^4 \gamma}{\int \sin^7 \gamma d\gamma} \]  \hspace{1cm} (2.81)

in the zone of deceleration

\[ Nu_c = 0.0447 \left( Re_c \right)^{0.78} \left( Pr_c \right)^{0.78} \]  \hspace{1cm} (2.82)

Raina and Wanchoo (166) have recently reported a theoretical study of a collapsing bubble in an immiscible liquid. These authors have assumed Stoke's flow to be valid for the outside flow and considered total liquid-liquid interfacial area as the transfer area. Their expression for average Nusselt number can be written as:

\[ Nu_c = 0.4423 \left( \frac{Re_c}{\mu} \right)^{0.75} (Dc)^{0.75} \]  \hspace{1cm} (2.83)
In general, evaporation in three phase systems is complete at all ranges of temperature difference.

Conversely, condensation studies (8, 192, 100) showed bubble collapse to be complete only at high temperature differences. The heat transfer coefficients for condensation in comparison with those for evaporation show a steep decrease at high condensation ratios, caused mainly by inert gas. Towards the end of condensation process i.e. at low vaporization ratios, the fraction of inert gas attains very high values. This considerably increases the heat transfer resistance and, as a result, the condensation is often incomplete.

The degree of bubble distortion can be significantly influenced by liquid inertial forces within the bubble. During the deceleration of the collapsing bubble that accompanies loss of buoyancy, the liquid attempts to penetrate the forward wall. Profile drag is momentarily and abruptly reduced and the ensuing second mode, vibration augments the heat transfer rate. By contrast, in evaporation the bubble continuously accelerates, driving the liquid to the rear.

2.2.8 Important Side Effects

(a) Effect of surfactants

The importance of surfactants in heat and mass
transfer between a single drop or a bubble and the continuous phase has been established by many workers (6, 88, 206).

Garner and Hale (60) studied the effect of surface active agents in liquid extraction processes. They found that trace quantities of surfactants had a marked effect on the rate of mass transfer. The presence of adsorbed molecules at the interface reduced the rate of mass transfer from drops which do not circulate and also retarded internal circulation in circulating drops.

Elzinga and Banchero (47) claimed that surface active materials slow down internal circulation, cause the point of separation to remain near the equator of the drop and thus decrease the transfer rate.

Klipstein (115) studied the effects of surfactants on the heat transfer rate to an individual drop vaporizing in an immiscible continuous phase liquid. He concluded that surfactants act to lower the rate mainly by repressing circulation and interfacial rippling, and that they increase the rate by increasing the oscillation tendencies. Quantitatively he found a 30 percent gain in the rate for small drops ($D = 2.39$ mm), no change in the medium sized drops ($D = 3.01$ mm), and 20 percent rate loss in large drops ($D = 79$ mm), due to the presence of
(b) Effect of Drop Diameter

For predicting the effect of drop diameter on transfer rates and transfer coefficients, the drops have been generally divided into various size categories, such as small, medium and large (115, 213, 30). But it seems that no rigid classification can be made on this basis, since the exact transition value depends on the physical properties of the system involved.

Calderbank and Moo-Young (30) in gas-liquid dispersions, and Thorsen and Terjesen (213) during their work on liquid-liquid systems, reported abrupt change in heat and mass transfer coefficients in the transition zone (based on the drop diameter). This change was attributed to the onset of circulation in this region.

Calderbank and Moo-Young (30) also suggested different correlations for the small and large drop size ranges, and substantiated these with experimental data from various other workers. For small drop size (1-3 mm in diameter) they found

\[ \text{Nu}_C = 2 + 0.31 \text{Ra}^{1/3} \] ... (2.84)

For large drops (2.7 mm in diameter), they predicted;
\[ \text{Nu}_c = 0.42 \left( \frac{\Delta f_{L}}{v_f} \right)^{0.75} \text{Pr}_{c}^{0.75} \]

The above equations indicate that the heat transfer coefficients are independent of drop diameter (if \( \text{Nu}_c \gtrsim 2 \)), for heat transfer to non-evaporating drops.

Other investigators \((6, 197, 22)\) found that their experimental work with drops and bubbles in the size range 2-7 mm in diameter, satisfied an equation of the form

\[ \text{Nu} = \text{Constant} \, \text{Pe}^{0.5} \]

This model shows a small decrease in the heat transfer coefficient with increasing drop diameter. This also agrees with the Higbie's penetration theory.

Hammerton and Garner \((81)\) while working on water absorption of carbon dioxide and ethylene bubbles (2-3 mm in diameter), showed that the mass transfer coefficients were directly proportional to the bubble diameter. Leonard and Houghton \((122)\) also confirmed this result.

(c) Effect of Noncondensable Gases

As is well known, the presence of noncondensable gases in a condensing vapour, adversely affects
the heat transfer rate. This is so because the higher concentration of noncondensables at the interface than that existing in the bulk, reduces the partial pressure of the vapor at the interface with simultaneous reduction in the interface temperature and the heat transfer rate. Sparrow and Lin (201) studied the effects of noncondensables in condensation on a vertical plate. They envisioned the problem as consisting of interacting boundary layers. The inner layer is the well-known Churchill model, while the outer layer is a gas-vapor mixture. Initially, the noncondensables are transported from the bulk to the interface with the general convective flow. However, since the interface is impermeable to the noncondensables, and in order to preserve steady state conditions, these gases must be removed at the same rate at which they arrive. This is accomplished by means of a diffusive flow of the inert gases from the interface back to the bulk, which can only occur from a region of higher concentration. Conditions favouring a higher condensation rate will require an increase in convective flow, thus increasing the concentration of noncondensables at the interface. Similarly, an increase in the bulk concentration will reduce the heat transfer rate. However, Hasan et al. (90) found the decreasing heat transfer coefficient to level off at high air concentrations.
(d) End Effects

In mass transfer studies the importance of end effects is well known but in heat transfer the importance of end effects is doubtful. On the basis of various mass transfer studies (65, 109, 92), one may expect to find similar results during heat transfer, but in the absence of experimental verification the end effects are still uncertain.

McDowell and Myers (147), while studying the mechanism of heat transfer to individual liquid drops, found no evidence of end effects.

Garwin and Smith (67) during their work on heat transfer in a liquid-liquid spray tower, encountered a moderate end effect at the dispersed phase entry point when heat transfer was taking place from the continuous to the dispersed phase, but none when the reverse occurred.

Calderbank and Korchinski (29) during their study on heat transfer to or from bromobenzene drops falling in hot or cold glycerol-water solutions reported insignificant end effects.

(e) Multiparticle Interaction

The importance of multiparticle interaction
will determine the possibility of extending the results obtained from a single drop/bubble study, to similar multi drop/bubble phenomena.

Hoppel and Pfeffer (84) studied the motion of two spheres following one another or moving side by side at very low Reynolds numbers. For falling spheres following each other they predicted that the lower sphere will slow down due to inertial effects.

Howe and Henwood (177) studied the drag forces and particle interaction in a fluidised bed. They concluded that the drag force on a sphere depends on the arrangement of its neighbours. In a packed assembly, the drag on a sphere is increased by one or two orders of magnitude over that on a sphere at the same nominal velocity but in isolation. Adjacent particles repel one another and generally, it is only when spheres are in line that they reduce the drag on each other. With an increase in population density, the interaction between drops will also increase. All the drops except the first one will pass through a continuous phase in which some turbulence has been generated by the preceding drops. This will tend to increase the transfer rates. Pierce et.al. (154), however, while studying heat transfer in mercury-water spray columns, found lower heat transfer coefficients than those reported for stationary spheres. They also found no appreciable effect of drop diameter
on the heat transfer coefficient.

The independence of heat transfer coefficients with drop diameters was also supported by the results of Johnson and Minard (108) and Bowman and Johnson (22). Johnson and Minard found that the density of the dispersed phase in a spray column had little effect on the heat transfer coefficient over a wide range of operating conditions. Bowman and Johnson claimed that a tenfold increase in the rate of the dispersed phase caused only 35 per cent increase in the mass transfer coefficient.

Moalem et.al. (136) have developed a model for condensation of a bubble train based on single bubble condensation data.

Golafshani (69) improved on the model by using the more sophisticated model of Jacobs and Major (105) to account for the resistance of the condensate and to account for the effects of the inert gases. His results indicated that for no noncondensables present, the different models for single bubble collapse had small effects on the behaviour of the bubble train.

The literature cited indicates that the results from single drop/bubble studies can be extended to give a good approximation of a similar multidrop phenomenon.
2.3.0 CONCLUSIONS FROM LITERATURE REVIEW

This section summarises the findings of the literature review, and, thus, provides the basis of the work that is reported subsequently.

2.3.1 On Experimental Data

The vaporization of a drop in an immiscible liquid medium has been studied extensively, and lot of experimental data (115, 162, 198, 210, 217, 218) are available. However, very few studies on condensation of a bubble in an immiscible liquid have been carried out and the experimental data available is limited (100, 192, 94, 111).

The results of Sideman and Hirsch (192) on condensation of isopentane in water, indicate the existance of a maxima in $h_c$ vs $D/D_0$ plot, whereas, Isenberg and Sideman (99) have reported a monotonic decrease in the heat transfer coefficient with time. These authors have used bubbles (having some undetermined amount of noncondensables) with high degree of superheat ($\sim 10^0C$) and its effect on bubble collapse has been neglected.

Very few studies are available which report the dynamics of a collapsing bubble (94, 192), to allow
to draw a definite conclusion about the drag coefficient.

It is, therefore, desirable to study the collapse of different dispersed phase bubbles in various immiscible continuous phase liquids having different viscosities. The study should involve the use of pure bubbles (free from noncondensables) at low degree of superheat. The data collected from this study will be useful in determining the effect of various parameters on the bubble collapse.

2.3.2 On Models

No single model, reported in the literature (99,103,111,194) can explain the phenomenon of bubble collapse in three phase systems. From Figs. 1.1 to 1.3, the disagreement between the existing theory and the experiment is evident. These models either overpredict or underpredict the data and are dependent on experimentally determined Peclet number. Most of these authors (99,111,194) have neglected the condensate resistance to heat transfer except Jacobs et.al. (103) who have shown that the condensate film can offer 30 per cent of resistance to heat flow. However, their model is applicable to the early stages of condensation only.

It is, therefore, desirable to develop a suitable mathematical model for the collapse of a bubb-
le in three phase systems that can explain the pheno-
menon and is convenient to use.
FIG. 2.1: COMPARISON OF EVAPORATION AND CONDENSATION (189)