CHAPTER VI

UNSTEADY MHD MASS TRANSFER FLOW PAST A
SUDDENLY MOVING VERTICAL PLATE IN A POROUS
MEDIUM IN ROTATING SYSTEM

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6.1 INTRODUCTION

In recent years the analysis of hydromagnetic convective flow involving heat and mass transfer in porous medium has attached the attention of many scholars because of its possible applications in diverse fields of science and technology such as Soil-sciences, Astrophysics, Geophysics, nuclear power reactors etc. It is worth mentioning that MHD is now undergoing a state of great enlargement and differentiation of subject mater. These new problems draw the attention of the researchers due to their varied significance, in liquid metals, electrolytes and ionized gases etc. The MHD in the present form is due to contributions of several notable authors like Shercliff (1965), Ferraro and Plumton (1966) and Crammer and Pai (1973).

The heat and mass transfer effects on a flow along a vertical plate in the presence of magnetic field was investigated by Elbashbeshy (1997). The influence of combined natural convection from a vertical wavy surface due to thermal and mass diffusion was studied by Hossain and Ress (1999). Chen (2004a, 2004b) investigated the effects of heat and mass transfer in MHD free convection from a vertical surface. In addition, the applications of the effect of Hall current on the fluid flow with variable concentration have been seen in MHD power generators, Astrophysical and metrological studies as well as in plasma physics. The Hall effect is due merely to the sideways magnetic force on the drifting free charges. The electric field has to have a component transverse to the direction of the current density to balance this force. In many works on plasma physics, the Hall effect is disregarded. But if the strength of magnetic field is high and the number density of electrons is small, the Hall Effect can not be ignored as it has a significant effect on the flow pattern of an ionized gas. Hall Effect results in a development of an additional
potential difference between opposite surfaces of a conductor for which a current is induced perpendicular to both the electric and magnetic field. This current is termed as Hall current. Model studies on the effect of Hall current on MHD convection flows have been carried out by many authors due to application of such studies in the problems of MHD generators and Hall accelerators. Some of them are Aboeldhab (2001), Dutta et al. (1976), Acharya et al. (2001), and Ahmed et al. (2010, 2011a).

The rotating flow of an electrically conducting fluid in presence of a magnetic field is encountered in Geophysical fluid dynamics. It is also important in the solar physics dealing with the sunspot development, the solar cycle and the structure of rotating magnetic stars. It is well known that a number of astronomical bodies possess fluid interiors and magnetic fields. Changes that take place in the rate of rotation, suggest the possible importance of hydro magnetic spin-up. Many authors have studied this problem of spin-up in MHD under different conditions of whom the names of Debnath (1975), Singh (2001), Takhar et al. (2002), Seth et al. (2011), Ahmed et al. (2011) and Singh (1983).

In the above mentioned works the thermal diffusion (Soret) effect was not taken into account in the species continuity equation. The flux of mass caused due to temperature gradient is known as the Soret effect or thermal diffusion. The experimental investigation of the thermal diffusion effect on mass transfer related problems was first performed by Charles Soret in 1879. There after its effect is termed as Soret effect in the honour of his name. In general the Soret effect is of smaller order of magnitude than the effect is quite small, but the devices may be arranged to produce very sharp temperature gradient so that the separation of components in mixtures are affected. Eckert and Drake (1972) have emphasized that the Soret effect assumes significance in cases concerning
isotope separation and in mixtures between gases with very light molecular weight (H₂, He) and the medium molecular weight (N₂, air).

Based on the Eckert and Drake’s work (1972) many other investigators have carried out model studies on the Soret effect in different heat and mass transfer problems. Some of them are Dursunkaya et al. (1992), Kafoussias et al. (1995), Sattar and Alam (1994), Alam et al. (2005), Ahmed et al. (2011) and Raju et al. (2008).

The present work concerns with the study of an unsteady MHD mass transfer flow past a suddenly moving vertical plate embedded in a porous medium with Hall current and ramped plate temperature and concentration.

6.2 BASIC EQUATIONS
The equations governing the motion of an incompressible viscous electrically conducting fluid in a rotating system in presence of magnetic field are

Equation of continuity:

$$\vec{V}.\vec{q} = 0$$  \hspace{1cm} (6.2.1)

Momentum equation:

$$\rho \left[ \frac{\partial \vec{q}}{\partial t} + 2 \vec{\Omega} \times \vec{q} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) + (\vec{q} \vec{\nabla})\vec{q} \right] = -\vec{\nabla}p + \vec{J} \times \vec{B} + \rho \vec{g} + \mu \nabla^2 \vec{q} - \frac{\mu}{K_1} \vec{q}$$  \hspace{1cm} (6.2.2)

Energy equation:

$$\rho C_p \left[ \frac{\partial \vec{T}}{\partial t} + (\vec{q} \vec{\nabla})\vec{T} \right] = k \nabla^2 \vec{T} + \Phi + \frac{\vec{J}^2}{\sigma} + Q_0 (\vec{T}_\infty - \vec{T})$$  \hspace{1cm} (6.2.3)

Species continuity equation:
\[
\left(\bar{q}, \bar{V}\right) \bar{C} = D_M \nabla^2 \bar{C} + D_T \nabla^2 \bar{T}
\]  \hspace{1cm} (6.2.4)

Kirchhoff’s first law:

\[
\bar{V} \cdot \bar{J} = 0
\]  \hspace{1cm} (6.2.5)

General Ohm’s law:

\[
\bar{J} + \frac{\omega_e \tau_e}{B_0} (\bar{J} \times \bar{B}) = \sigma \left[ \bar{E} + \bar{q} \times \bar{B} + \frac{1}{\epsilon \eta_e} \bar{V} p_e \right]
\]  \hspace{1cm} (6.2.6)

Gauss’s law of magnetism:

\[
\bar{V} \cdot \bar{B} = 0
\]  \hspace{1cm} (6.2.7)

All physical quantities are defined in the Nomenclature.

Now we consider an unsteady hydromagnetic laminar natural convection boundary layer flow of a viscous incompressible electrically conducting and heat-absorbing fluid past an impulsively started infinite vertical plate embedded in a porous medium taking into account of Hall current in presence of a uniform magnetic field. Our investigation is restricted to the following assumptions.

i) All fluid properties except the density in the buoyancy force term are constants.

ii) The plate is electrically non-conducting.

iii) The entire system is rotating with a uniform angular velocity \( \tilde{\Omega} \) about the normal to the plate.

iv) The magnetic Reynolds number is so small that the induced magnetic field can be neglected. Also the electrical conductivity \( \sigma \) of the fluid is reasonably low and the Ohmic dissipation may be neglected.

v) The electric pressure \( p_e \) is constant.
vi) $\vec{E} = 0$ i.e., the electric field is negligible.

vii) $|\hat{\Omega}|$ is so small that $|\hat{\Omega} \times (\hat{\Omega} \times \vec{r})|$ i.e. the centrifugal force may be neglected.

![Flow configuration diagram](image)

**Figure 6.1: Flow configuration**

We introduce the coordinate system $(\vec{x}, \vec{y}, \vec{z})$ with $X$ axis along the plate in the upward direction, $Y$-axis normal to the plane of the plate in the fluid and $Z$-axis perpendicular to $XY$ plane. The fluid as well as the plate is in a state of rigid body rotation with a uniform angular velocity $\hat{\Omega}$ about $Y$-axis. Initially at time $\tau \leq 0$, both the fluid and the plate were at rest and at a uniform temperature $\bar{T}_\infty$ and concentration $\bar{C}_\infty$. At time $\tau > 0$, the plate starts moving in $X$ direction with uniform velocity $U_0$ and the temperature and concentration of the plate is raised or lowered to $\bar{T}_\infty + (\bar{T}_w - \bar{T}_\infty) \frac{\tau}{t_0}$ and
\( \bar{C}_n + \left( \bar{C}_w - \bar{C}_n \right) \frac{\bar{T}}{t_0} \) when \( \bar{T} \leq t_0 \) and there after it is maintained at uniform temperature \( \bar{T}_w \) and concentration \( \bar{C}_w \) when \( \bar{T} > t_0 \) (\( t_0 \) being the characteristic time). Let \( \bar{q} = \hat{i} \bar{u} + \hat{j} \bar{v} + \hat{k} \bar{w} \) be the fluid velocity, \( \bar{J} = \bar{J}_x \hat{i} + \bar{J}_y \hat{j} + \bar{J}_z \hat{k} \) denote the current density at point \( P(\bar{x}, \bar{y}, \bar{z}, \bar{T}) \) and \( \bar{B} = B_0 \hat{j} \) be the applied magnetic field, \( \hat{i}, \hat{j}, \hat{k} \) being the unit vectors along \( \bar{O}X \), \( \bar{O}Y \) and \( \bar{O}Z \) respectively. As the plate is infinite in \( X \) and \( Z \) direction, all physical quantities except possible the pressure are functions of \( \bar{y} \) and \( \bar{T} \) only and \( \bar{v} = 0 \)

The equation of continuity is trivially satisfied with

\[ \bar{u} = \bar{u}(\bar{y}, \bar{T}), \quad \bar{w} = \bar{w}(\bar{y}, \bar{T}) \]  

(6.2.8)

Therefore the velocity vector \( \bar{q} \) is given by

\[ \bar{q} = \hat{i} \bar{u} + \hat{k} \bar{w} \]  

(6.2.9)

The equation (6.2.7) is also satisfied by

\[ \bar{B} = B_0 \hat{j} \]  

(6.2.10)

The equation (6.2.5) reduces to

\[ \frac{\partial \bar{J}_y}{\partial \bar{y}} = 0 \]  

(6.2.11)

which shows that \( \bar{J}_y = 0 \)

(as the plate is electrically non conducting)

Hence the current density is given by

\[ \bar{J} = \bar{J}_x \hat{i} + \bar{J}_z \hat{j} \]  

(6.2.12)

Under the assumptions (v) and (vi), the equation (6.2.6) takes the form:
\[ \mathbf{J} + \frac{m}{B_0} (\mathbf{J} \times \mathbf{B}) = \mathbf{\sigma} (\mathbf{q} \times \mathbf{B}) \quad (6.2.13) \]

Where \( m = \omega \tau_c \) is the Hall parameter.

The equations (6.2.9), (6.2.10), (6.2.12) and (6.2.13) yield,

\[ \mathbf{J}_x = \frac{\mathbf{\sigma} B_0}{1 + m^2} (m \mathbf{u} - \mathbf{w}) \quad (6.2.14) \]
\[ \mathbf{J}_z = \frac{\mathbf{\sigma} B_0}{1 + m^2} (\mathbf{u} + m \mathbf{w}) \quad (6.2.15) \]

With the foregoing assumptions and under the usual boundary layer and Boussinesq approximations, the equations (6.2.2), (6.2.3) and (6.2.4) reduce to

\[ \frac{\partial \mathbf{u}}{\partial t} + 2 \Omega \mathbf{w} = \nu \frac{\partial^2 \mathbf{u}}{\partial y^2} + g \beta (\mathbf{T} - T_\infty) + g \beta (\mathbf{C} - C_\infty) - \frac{\nu \mathbf{u}}{K_i} - \frac{\mathbf{\sigma} B_0^2 (m \mathbf{u} - \mathbf{w})}{\rho (1 + m^2)} \quad (6.2.16) \]
\[ \frac{\partial \mathbf{w}}{\partial t} - 2 \Omega \mathbf{u} = \nu \frac{\partial^2 \mathbf{w}}{\partial y^2} + \frac{\mathbf{\sigma} B_0^2 (m \mathbf{u} - \mathbf{w})}{\rho (1 + m^2)} - \frac{\nu \mathbf{w}}{K_i} \quad (6.2.17) \]
\[ \frac{\partial \mathbf{T}}{\partial t} = \alpha \frac{\partial^2 \mathbf{T}}{\partial y^2} - \frac{Q_0}{\rho C_p} (\mathbf{T} - T_\infty) \quad (6.2.18) \]
\[ \frac{\partial \mathbf{C}}{\partial t} = D_M \frac{\partial^2 \mathbf{C}}{\partial y^2} + D_T \frac{\partial^2 \mathbf{T}}{\partial y^2} \quad (6.2.19) \]

The initial and boundary conditions for this fluid flow problem are:

\[ \mathbf{u} = \mathbf{w} = 0, \mathbf{T} = T_\infty, \mathbf{C} = C_\infty \text{ for } y \geq 0 \text{ and } \mathbf{T} \leq 0, \mathbf{u} = U_o, \mathbf{w} = 0 \text{ at } y = 0 \text{ for } \mathbf{T} > 0 \]
\[ \mathbf{T} = T_w + \left( T_\infty - T_w \right) \frac{t}{t_0}, \mathbf{C} = C_w + \left( C_\infty - C_w \right) \frac{t}{t_0} \text{ at } y = 0 \text{ for } 0 < \mathbf{T} \leq t_0 \]
\[ \mathbf{T} = T_w, \mathbf{C} = C_w \text{ at } y = 0 \text{ for } \mathbf{T} > t_0, \mathbf{u} \to 0, \mathbf{w} \to 0, \mathbf{T} \to T_\infty \text{ as } y \to \infty \text{ for } \mathbf{T} > 0 \quad (6.2.20) \]
In order to express the governing equations (6.2.16) to (6.2.19) and initial boundary conditions (6.2.20) in dimensionless form, the following non-dimensional quantities and parameters are introduced.

\[
\begin{align*}
y &= \frac{y}{U_0 t_0}, (u, w) = (\frac{\bar{u}}{U_0}, \frac{\bar{w}}{U_0}) \frac{1}{t_0}, T = \frac{T - \bar{T}_x}{\bar{T}_x - \bar{T}_w}, \phi = \frac{\bar{C} - \bar{C}_x}{\bar{C}_w - \bar{C}_x}, \\
K_i &= \frac{K_i U_0^2}{\nu^2}, Gr = \frac{g \beta \nu (\bar{T}_w - \bar{T}_x)}{U_0^3}, Gm = \frac{g \beta \nu (\bar{C}_w - \bar{C}_x)}{U_0^3}, \\
M &= \frac{\sigma B_0^2 \nu}{\rho U_0^2}, Pr = \frac{\nu}{\alpha}, Q = \frac{\nu Q_0}{\rho \alpha U_0^2}, Sr = \frac{D_T (\bar{T}_w - \bar{T}_x)}{\nu (\bar{C}_w - \bar{C}_x)}, K^2 = \frac{\bar{\Omega} \nu}{U_0^3}.
\end{align*}
\]

(6.2.21)

In view of (6.2.21) the equations (6.2.16) to (6.2.19) in non-dimensional form, reduce to

\[
\begin{align*}
\frac{\partial u}{\partial t} + 2K^2 w &= \frac{\partial^2 u}{\partial y^2} - \frac{M}{(1 + m^2)}(m w + u) - \frac{u}{K_i} + Gr T + Gm \phi \\
\frac{\partial w}{\partial t} - 2K^2 u &= \frac{\partial^2 w}{\partial y^2} - \frac{M}{(1 + m^2)}(m u - w) - \frac{w}{K_i} \\
\frac{\partial T}{\partial t} &= \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} - QT \\
\frac{\partial \phi}{\partial t} &= \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} + Sr \frac{\partial^2 T}{\partial y^2}
\end{align*}
\]

(6.2.22) to (6.2.25)

According to the above non-dimensionalization process, the characteristic time \( t_0 \) can be defined as

\[
t_0 = \frac{\nu}{U_0^3}
\]

(6.2.26)

Making use of (6.2.21) and (6.2.26) the initial boundary conditions (6.2.20), in non-dimensional form, become
6.3 METHOD OF SOLUTION

We introduce a new complex variable \( f \) defined by

\[ f = u + iw \] where \( i = \sqrt{-1} \)

The non dimensional form of the equations governing the flow can be rewritten as follows:

\[
\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial y^2} - \left[ \frac{M(1-im)}{(1+m^2)} - 2iK^2 + \frac{1}{K_1} \right] f + GrT + Gm\phi
\]

(6.3.1)

\[
\frac{\partial T}{\partial t} = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} - QT
\]

(6.3.2)

\[
\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} + Sr \frac{\partial^2 T}{\partial y^2}
\]

(6.3.3)

The initial and boundary condition in combined form are

\[
\begin{align*}
  f &= 0, \quad T = 0, \quad \phi = 0 \quad \text{for } y \geq 0 \text{ and } t \leq 0 \\
  f &= 1, T = 1 \quad \text{at } y = 0 \text{ for } t > 0 \\
  T &= t, \quad \phi = t \quad \text{at } y = 0 \text{ for } 0 < t \leq 1 \\
  T &= 1, \quad \phi = 1 \quad \text{at } y = 0 \text{ for } t > 1 \\
  f &\rightarrow 0, \quad T \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as } y \rightarrow \infty \text{ for } t > 1
\end{align*}
\]

(6.3.4)

On taking the Laplace Transforms of the equations (6.3.1), (6.3.2) and (6.3.3), the following differential equations are obtained
\[
\frac{d^2 f}{dy^2} - (\lambda + s) f = -Gr \bar{T} - Gm \phi
\]  \hspace{1cm} (6.3.5)

\[
\frac{d^2 \bar{T}}{dy^2} - Pr(s + Q) \bar{T} = 0
\]  \hspace{1cm} (6.3.6)

\[
\frac{d^2 \bar{\phi}}{dy^2} - sSc \bar{\phi} = -SrSc \frac{d^2 \bar{T}}{dy^2}
\]  \hspace{1cm} (6.3.7)

subject to the boundary conditions:

\[
\begin{align*}
\bar{f} &= \frac{1}{s} \quad , \quad \bar{T} = \frac{1}{s^2}(1 - e^{-s}) \quad , \quad \bar{\phi} = \frac{1}{s^2}(1 - e^{-s}) \quad \text{at} \quad y = 0 \\
\bar{f} &= 0 \quad , \quad \bar{T} = 0 \quad , \quad \bar{\phi} = 0 \quad \text{at} \quad y \to \infty
\end{align*}
\]  \hspace{1cm} (6.3.8)

The solution of the equations (6.3.5) to (6.3.7) under the conditions (6.3.8) are

\[
\bar{T} = \frac{1 - e^{-s}}{s^2} e^{-\sqrt{(s+Q)Pr} y}
\]  \hspace{1cm} (6.3.9)

\[
\bar{\phi} = \left[ \frac{1}{s^2} - a_1 \left( \frac{1}{s - \alpha} - \frac{1}{s} \right) - a_2 \left( \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s - \alpha} \right) \right] (1 - e^{-s}) e^{-sScy} +
\]

\[
\left[ a_1 \left( \frac{1}{s - \alpha} - \frac{1}{s} \right) + a_2 \left( \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s - \alpha} \right) \right] (1 - e^{-s}) e^{-\sqrt{(s+Q)Pr} y}
\]  \hspace{1cm} (6.3.10)

\[
\bar{f} = \frac{1}{s} \left( a_{14} \left( \frac{L}{s} + \frac{M}{s^2} + \frac{N}{s - \beta} \right) + a_{15} \left( \frac{D}{s} + \frac{E}{s^2} + \frac{F}{s - \gamma} \right) + a_{16} \left( \frac{1}{s} - \frac{1}{s - \gamma} \right) +
\]

\[
a_{17} \left( \frac{1}{s - \alpha} - \frac{1}{s - \gamma} \right) + a_{18} \left( \frac{1}{s - \alpha} - \frac{1}{s - \beta} \right) + a_{19} \left( \frac{1}{s - \beta} - \frac{1}{s} \right) \right] (1 - e^{-s}) e^{-\left(\lambda/s\right)y} +
\]

\[
a_3 \left( \frac{L}{s} + \frac{M}{s^2} + \frac{N}{s - \beta} \right) (1 - e^{-s}) e^{-\sqrt{(s+Q)Pr} y} + a_4 \left( \frac{D}{s} + \frac{E}{s^2} + \frac{F}{s - \gamma} \right) (1 - e^{-s}) e^{-sScy} +
\]

\[
a_5 \left( \frac{1}{s} - \frac{1}{s - \gamma} \right) (1 - e^{-s}) e^{-sScy} + a_6 \left( \frac{1}{s - \alpha} - \frac{1}{s - \gamma} \right) (1 - e^{-s}) e^{-sScy} +
\]

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\[
\begin{align*}
\left[ Aa_{7} \left( \frac{1}{s-\gamma} - \frac{1}{s} \right) + a_{2}a_{4}B \left( \frac{D}{s} + \frac{E}{s^2} + \frac{F}{s-\gamma} \right) + Ca_{8} \left( \frac{1}{s-\alpha} - \frac{1}{s-\gamma} \right) \right] (1-e^{-s}) e^{-\sqrt{sSC} y} +
\left[ a_{12}A \left( \frac{1}{s-\beta} - \frac{1}{s} \right) + a_{2}a_{9}B \left( \frac{L}{s} + \frac{M}{s^2} + \frac{N}{s-\beta} \right) \right] +
C \left( \frac{1}{s-\alpha} - \frac{1}{s-\beta} \right) a_{13} \right] (1-e^{-s}) e^{-\sqrt{(s+Q)Pr} y} \tag{6.3.11}
\end{align*}
\]

Taking inverse Laplace transforms of the equations (6.3.9), (6.3.10) and (6.3.11) we derive the fluid temperature, fluid concentration and fluid velocity are as follows:

\[
\begin{align*}
T (\eta, t) &= g_{1} - \bar{g}_{1} \tag{6.3.12} \\
\phi &= \psi_{1} - \bar{\psi}_{1} \tag{6.3.13} \\
f &= f_s - (\psi_{2} - \bar{\psi}_{2}) \tag{6.3.14}
\end{align*}
\]

6.4 COEFFICIENT OF SKIN FRICTION

The viscous drag at the plate per unit area in the direction of the plate velocity is given by the Newton’s law of viscosity in the form

\[
\tau' = -\mu \frac{\partial \bar{u}}{\partial y} = -\frac{\mu}{t_0} \frac{\partial u}{\partial y} \bigg|_{y=0}
\]

The co-efficient of skin-friction at the plate is given by

\[
\tau = \frac{\bar{\tau}}{\mu} = -\frac{\partial u}{\partial y} \bigg|_{y=0} = \tau_x + i \tau_z = -\left[ \omega_1 - \left( \Omega - \bar{\Omega} \right) \right]
\]
6.5 COEFFICIENT OF RATE OF HEAT TRANSFER

The heat flux $\bar{q}$ from the plate to the fluid is given by the Fourier law of conduction in the form

$$\bar{q} = -k \frac{\partial T}{\partial y} \bigg|_{y=0} = -\frac{k}{U_0 t_0} (T_w - T_x) \frac{\partial T}{\partial y} \bigg|_{y=0}$$

The co-efficient of the rate of heat transfer from the plate to the fluid in terms of Nusselt number is given by

$$Nu = \frac{\bar{q} U_0 t_0}{k (T_w - T_x)} = -\frac{\partial T}{\partial y} \bigg|_{y=0} \left( \psi_1 - \tilde{\psi}_1 \right)$$

6.6 COEFFICIENT OF RATE OF MASS TRANSFER

The co-efficient of the rate of mass transfer from the plate to the fluid in terms of Sherwood number is given by

$$Sh = -\frac{\partial \phi}{\partial y} \bigg|_{y=0} = (a_1 - a_2 A) \omega_7 - e^{a_1} (a_1 + a_2 C) \omega_{10} + e^{a_1} (a_1 + a_2 C) \omega_6 - (a_1 - a_2 A) \omega_5$$

6.7 RESULTS AND DISCUSSION

In order to get clear insight of the physical problem, numerical computations from the analytical solutions for the representative temperature field, concentration field, velocity field, the coefficient of skin-friction, the rate of heat transfer at the plate in terms of Nusselt number and the rate of mass transfer in terms of Sherwood number have been carried out by assigning some arbitrary chosen specific values to the physical parameters involved in
the problem, the normal coordinate y and time t. Throughout our investigation, the value of Pr (Prandtl number) has been kept fixed at .71 which corresponds physically to air, the values of Gr (Grashof number for heat transfer), Q (Heat sink parameter), $K_1$ (permeability parameter) and $K_2$ (rotation parameter) are fixed at 6, 1, .5 and 2 respectively as the numerical computations are concerned. The Schmidt number (Sc) are chosen to be $Sc = .22$ (Hydrogen), $Sc = .30$ (Helium), $Sc = .6$ (water vapor) and $Sc = .78$ (Ammonia) at $20^\circ$C temperature and one atmospheric pressure for air. The numerical results computed from the analytical solutions of the problem have been displayed in figures 6.2-6.31.

The velocity profiles under the influence of M (Hartmann number), Sc (Schmidt number), Gm (Grashof number for mass transfer), and Sr (Soret number) versus normal coordinate y are exhibited in figures 6.2-6.17 for ramped and isothermal temperature on both primary and secondary velocities.

It is inferred from figure 6.2 and 6.3 that for ramped temperature an increase in Hartmann number M has an inhibiting effect on the primary and secondary velocities. The primary and secondary velocities are continuously reduced with increasing M. In other words the imposition of the transverse magnetic field tends to retard the fluid flow irrespective of ramped plate temperature. This phenomenon has an excellent agreement with the physical fact that the Lorentz force generated in present flow model due to interaction of the transverse magnetic field and the fluid velocity acts as a resistive force to
the fluid flow which serves to decelerate the flow. As such the magnetic field is an effective regulatory mechanism for the flow regime.

Figures 6.4 and 6.5 illustrate the influence of Soret number Sr on both primary and secondary velocities for ramped temperature. Here it is seen that primary as well as secondary velocity increases with the increasing values of Sr.

It is depicted from figures 6.6 and 6.7 that primary and secondary velocity increase on increasing values of Gm, there is a comprehensive rise in both the velocities for ramped temperature.

The effect of Schmidt number Sc on primary and secondary velocity fields has been visualized in figures 6.8 and 6.9 for ramped temperature. These figures exhibit that an increase in Sc accelerates the fluid flow in both primary and secondary directions. In other words mass diffusivity causes the fluid motion to decrease.

Figures 6.10, 6.11, 6.12 and 6.13 have presented the variation of primary velocity u under the influence of Sc, M, Gm and Sr respectively for isothermal plate. It is inferred from these figure that primary velocity u remains negative for small and moderate values of y. That is the fluid flows in the downward vertical direction in the portion of the fluid region adjacent to the plate and far away from the plate the fluid has a tendency to move upward vertical direction and finally it vanishes as y → ∞ . It is also marked in these figures that for the fluid region adjacent to the plate, the magnitude of the fluid velocity is reduced due to increase in Sc but the effects of transverse magnetic field and mass Grashof number accelerate the flow near the plate and it has retarding influence on the flow far away from the plate. These results are clearly supported from the physical point of view. On the other
hand the effect of Soret number decelerates the flow near the plate as obtained in Figure 6.13 and it shows a reverse effect on the flow far away from the plate.

The profiles for the secondary velocity \( w \) are shown in figures 6.14, 6.15, 6.16 and 6.17 when the plate temperature is isothermal. It is observed from these figures that secondary velocity are reduced under the effect of transverse magnetic field and Soret number whereas it rises for the increasing values of Schmidt number and mass Grashof number.

Figures 6.18, 6.19, 6.20 and 6.21 demonstrate the variation of species concentration versus normal coordinate \( y \) for the cases \( 0 < t \leq 1 \) and \( t > 1 \) respectfully. It is observed from these figures that an increase in \( Sc \) or \( Sr \) causes the species concentration to increase in magnitude whereas this behavior takes a reverse tend from \( y =1 \) approx for increasing \( Sr \) for ramped temperature.

The profiles of the skin friction \( \tau_x \) at the plate due to primary velocity are displayed in figures 6.22, 6.23, 624 and 6.25. It is inferred from these figures that the viscous drag at the plate due to primary velocity rises under the effect of Hartmann number \( M \), Schmidt number \( Sc \) and Grashof number for mass transfer \( Gm \) whereas it falls under the effect of Soret number \( Sr \).

Figures 6.26, 6.27, 6.28 and 6.29 display the effect of skin friction \( \tau_z \) at the plate due to secondary velocity. It is observed from these figures that the magnitude of the secondary skin friction \( \tau_z \) decreases when the Soret number \( Sr \) is increased whereas \( |\tau_z| \) increases with Hartmann number \( M \) and Schmidt number \( Sc \).

These figures also predict that skin friction increases linearly with time \(( t \leq 1)\) and as time progresses \(( t > 1)\) it becomes almost stationary establishing the fact that co-efficient
of skin friction becomes uniform after time $t = 1$. This phenomenon physically states that the skin friction gets stabilized for time $t > 1$.

The effects of Schmidt number $Sc$ and Soret number $Sr$ on the rate of mass transfer from the plate to the fluid has been visualized in figures 30 and 31. Both the figures exhibit that the rate of mass transfer increases slightly between $0.2 \leq t \leq 1$ and increases steadily between $1 \leq t \leq 1.2$ and after this it starts diminishing. It is also observed that rate of mass transfer decreases for $t \leq 1.1$ and after that the rate of mass transfer increases with increasing Schmidt number $Sc$ and Soret number $Sr$.

**Comparison of Results**

To compare results of the present paper, the work of Ahmed et al. (2013) is considered. Our work is an extension to the work studied by Ahmed et al. (2013) to consider mass transfer with thermal diffusion effect on the flow and transport characteristics.

Comparing the figure 6.33 with figure 6.32 (the figure 4 of the work done by Ahmed et al. (2013)), we observe that the two figures uniquely indicate that the fluid motion is retarded due to the imposition of the transverse magnetic field irrespective of the plate temperature being ramped or isothermal. That is there is a good agreement between the results obtained by Ahmed et al. (2013) and the present authors.

**6.8 CONCLUSIONS**

i) Magnetic field tends to retard the fluid flow in both the primary and secondary flow directions for ramped temperature.
ii) Mass diffusivity causes the fluid motion to decrease in both directions for ramped temperature.

iii) Fluid flow in both directions is accelerated due to increasing Grashof number of mass transfer and Soret number for ramped temperature.

iv) The primary fluid flow in magnitude is retarded near the plate due to the effect of Schmidt number but the transverse magnetic field, Mass Grashof number accelerates the flow near the plate and it has retarding influence far away from the plate for isothermal plate.

v) The secondary velocity is reduced due to the effect of transverse magnetic field and Soret number whereas it rises for increasing Schmidt number and mass Grashof number for isothermal plate.

vi) An increase in Schmidt number and Soret number causes the species concentration to increase in magnitude but this behaviour takes a reserve trend from $y = 1$ approx. for Soret number for Ramped temperature.

vii) Transverse magnetic field, Schmidt number and Grashof number for mass transfer tends to increase primary skin friction whereas Schmidt number has tendency to reduce the magnitude of the secondary skin friction and transverse magnetic field.

viii) The rate of mass transfer decreases for $t \leq 1.1$ and after that the rate of mass transfer increases with increasing Schmidt number $Sc$ and Soret number $Sr$. 
Figure 6.2: Velocity distribution $u$ versus $y$ for $M$ for $Sc = .6$, $Sr = .5$, $Gm = 6$

Figure 6.3: Velocity distribution $w$ versus $y$ of $M$ for $Sc = .6$, $Sr = .5$, $Gm = 6$
Figure 6.4 Velocity distribution $u$ versus $y$ of Sr for $Sc = .6$, $M = 4$, $Gm = 6$

Figure 6.5 Velocity distribution $w$ versus $y$ of Sr for $Sc = .6$, $M = 4$, $Gm = 6$
Figure 6.6: Velocity distribution $u$ versus $y$ for of $Gm = 4, 5, 6$ for $Sc = .6, M = 4, Sr = .5$

Figure 6.7: Velocity distribution $w$ versus $y$ of $Gm = 4, 5, 6$ for $Sc = .6, M = 4, Sr = .5$
Figure 6.8: Velocity distribution $u$ versus $y$ for of $Sc$ for $Gm = 4$, $M = 4$, $Sr = .5$

Figure 6.9: Velocity distribution $w$ versus $y$ of $Sc$ for $Gm = 4$, $M = 4$, $Sr = .5$
Figure 6.10: Velocity distribution $u$ versus $y$ for of $Sc$ for $Gm = 4$, $M = 4$, $Sr = .5$

Figure 6.11: Velocity distribution $u$ versus $y$ of $M$ for $Gm = 4$, $Sc = .6$, $Sr = .5$
Figure 6.12: Velocity distribution $u$ versus $y$ of $G_m$ for $M = 4$, $Sc = .6$, $Sr = .5$

Figure 6.13: Velocity distribution $u$ versus $y$ of $Sr$ for $Sc = .6$, $M = 4$, $G_m = 6$
Figure 6.14 Velocity distribution $w$ versus $y$ of $M$ for $Gm = 4$, $Sc = .5$, $Sr = .5$

Figure 6.15 Velocity distribution $w$ versus $y$ of $Sr$ for $Gm = 4$, $Sc = .6$, $M = 4$
Figure 6.16 Velocity distribution $w$ versus $y$ of $Sc$ for $Gm = 4$, $M = 4$, $Sr = .5$

Figure 6.17: Velocity distribution $w$ versus $y$ of $Gm$ for $Sc = .6$, $M = 4$, $Sr = .5$
Figure 6.18: Concentration distribution versus $y$ of $Sc$ for $Sr = .5$

Figure 6.19: Concentration distribution versus $y$ of $Sc$ for $Sr = .5$
Figure 6.20: Concentration distribution versus y of Sr for Sc = .6

Figure 6.21: Concentration distribution versus y of Sr for Sc = .6
Figure 6.22: Co-efficient of skin friction $\tau_x$ versus $t$ of $M$ for $Sc = .6$, $Gm = 4$, $Sr = .5$

Figure 6.23: Co-efficient of skin friction $\tau_x$ versus $t$ of $Sc$ for $M = 4$, $Sr = .5$, $Gm = 4$
Figure 6.24: Co-efficient of skin friction $\tau_x$ versus $t$ of Gm for Sc = .6, M = 4, Sr = .5

Figure 6.25: Co-efficient of skin friction $\tau_x$ versus $t$ of Sc for M = 4, Sr = .6, Gm = 4
Figure 6.26: Co-efficient of skin friction $\tau_z$ versus $t$ of $M$ for $Sc = .6$, $Gm = 4$, $Sr = .5$

Figure 6.27: Co-efficient of skin friction $\tau_z$ versus $t$ of $Sc$ for $M = 4$, $Sr = .5$, $Gm = 4$
Figure 6.28: Co-efficient of skin friction $\tau_z$ versus $t$ of Gm for $M = 4$, $Sc = .6$, $Sr = .5$

Figure 6.29: Co-efficient of skin friction $\tau_z$ versus $t$ of Sr for $M = 4$, $Sc = .6$, $Gm = 4$
Figure 6.30: Sherwood number $Sh$ versus $t$ of $Sc$ for $Sr = .5$

Figure 6.31: Sherwood number $Sh$ versus $t$ of $Sr$ for $Sc = .6$
Figure 6.32 (Fig.4 of Ahmed et al.(2013)) velocity profile u when \( Q=1, K^2=2, K_1=.2, Gr = 4, t = .7, m = .5 \)

Figure 6.33: velocity profile u when \( Q=1, K^2=2, K_1=.2, t = .7, Gr = 4, m = .5, Sc = 0, Gm = 0, Gr = 4 \)
6.9 NOMENCLATURE

\( \vec{B} \) is the magnetic induction vector

\( B_0 \) is the strength of the applied magnetic field

\( C_p \) is the specific heat at constant pressure

\( \bar{C} \) is the species concentration

\( \bar{C}_w \) is the reference concentration

\( \bar{C}_\infty \) is the concentration far away from the plate

\( D_m \) is the coefficient of chemical molecular diffusivity

\( D_T \) is the coefficient of chemical thermal diffusivity

\( \vec{E} \) is the electric field

\( g \) is the acceleration due to gravity

\( Gr \) is the Grashof number for heat transfer

\( Gm \) is the Grashof number for mass transfer

\( \vec{J} \) is the current density vector

\( k \) is the thermal conductivity

\( \bar{K}_r \) is the permeability of porous medium

\( K_r \) is the non dimensional permeability parameter

\( K^2 \) non dimensional rotational parameter

\( m \) is the Hall parameter

\( M \) is the Hartmann number

\( Pr \) is the Prandtl number
p is the pressure

\( \vec{q} \) is the fluid velocity vector

\( Q_0 \) is the heat absorption coefficient

\( Q \) is the heat source parameter

\( Sc \) Schmidt number

\( Sr \) Soret number

\( \tau \) is the time

\( t_o \) is the characteristic time

\( \overline{T}_w \) is the reference temperature

\( \overline{T}_\infty \) is the temperature far away from the plate

\( \overline{T} \) is the fluid temperature

\( T \) is the non dimensional temperature

\( t \) is the non dimensional time

\( U_0 \) is the plate velocity

\( \overline{\alpha} \) is the fluid thermal diffusivity

\( \beta \) is the volumetric coefficient of thermal expansion

\( \overline{\beta} \) is the volumetric coefficient of mass expansion

\( \rho \) is the fluid density

\( \mu \) is the coefficient of viscosity

\( \sigma \) is the electrical conductivity

\( \nu \) is the kinametic viscosity
\( \phi \) is the non dimensional concentration

\( \omega_e \) Electron frequency

\( \tau_e \) Electron collision time

\( \eta_e \) Number density of electron

\( 2 \ddot{\Omega} \times \ddot{q} \) Corilis acceleration

\( \ddot{\Omega} \times (\ddot{\Omega} \times \ddot{r}) \) Centripetal acceleration

**APPENDIX**

\[
\lambda = \frac{M(1 - im)}{(1 + m^2)} - 2iK^2 + \frac{1}{K_1}, \quad \alpha = \frac{QPr}{Sc - Pr}, \quad A = -\frac{1}{\alpha^2}, \quad B = -\frac{1}{\alpha}, \quad C = \frac{1}{\alpha^2}, \quad a_1 = \frac{SrSc}{Q},
\]

\[
a_2 = SrSc \alpha, \quad a_3 = \frac{Gr}{1 - Pr}, \quad \beta = \frac{QPr - \lambda}{1 - Pr}, \quad L = -\frac{1}{\beta^2}, \quad M = -\frac{1}{\beta}, \quad N = \frac{1}{\beta^2}, \quad a_4 = \frac{Gm}{Sc - 1},
\]

\[
\gamma = \frac{\lambda}{Sc - 1}, \quad D = -\frac{1}{\gamma^2}, \quad E = -\frac{1}{\gamma}, \quad F = \frac{1}{\gamma^2}, \quad a_5 = \frac{a_1a_4}{\alpha - \beta}, \quad a_6 = \frac{a_1a_4}{\gamma}, \quad a_7 = \frac{a_2a_4}{\alpha - \gamma}, \quad a_8 = \frac{a_2a_4}{\alpha - \gamma},
\]

\[
a_9 = \frac{Gm}{1 - Pr}, \quad a_{10} = \frac{a_1a_9}{\alpha - \beta}, \quad a_{11} = \frac{a_1a_9}{\beta}, \quad a_{12} = \frac{a_2a_9}{\beta}, \quad a_{13} = \frac{a_2a_9}{\alpha - \beta}, \quad a_{14} = a_3 + a_2a_9B,
\]

\[
a_{15} = a_4 + a_2a_4B, \quad a_{16} = a_5 - a_7A, \quad a_{17} = a_6 + a_8C, \quad a_{18} = a_{10} + a_{13}C,
\]

\[
a_{19} = Aa_{12} - a_{11}, a_{20} = a_{14}L + a_{15}D + a_{16} - a_{19}, a_{21} = a_{14}M + a_{15}E, a_{22} = a_{14}N - a_{18} + a_{19},
\]

\[
a_{23} = a_{15}F - a_{16} - a_{17}, a_{24} = a_{17} + a_{18}, a_{25} = a_3M + a_2a_9BM,
\]

\[
a_{26} = a_3L + a_{11} - a_{12}A + a_2a_9BL, a_{27} = a_{10} + a_{13}C, a_{28} = a_4E + a_2a_4BE,
\]

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\[ a_{29} = a_3 N - a_{10} - a_{11} + a_2 a_9 B N - a_{13} C + A a_{12}, \]
\[ a_{30} = a_4 F - a_5 - a_6 + a_7 A + a_2 a_4 B F - a_8 C, a_{31} = a_6 + a_8 C, \]
\[ a_{32} = a_4 D + a_5 - a_7 A + a_2 a_4 B D \]
\[ \psi_1 = (1 - B a_2) \xi_1 + (a_1 - a_2 A) f_2 - e^{at} (a_1 + a_2 C) f_1 + e^{at} (a_1 + a_2 C) f_3 - \]
\[ (a_1 - a_2 A) f_4 + a_2 B g_1 \]
\[ \overline{\psi}_1 = (1 - B a_2) \overline{\xi}_1 + (a_1 - a_2 A) \overline{f}_2 - e^{at} (a_1 + a_2 C) \overline{f}_1 + e^{at} (a_1 + a_2 C) \overline{f}_3 - \]
\[ (a_1 - a_2 A) \overline{f}_4 + a_2 B \overline{g}_1 \]
\[ \psi_2 = a_{20} f_5 + a_{21} g_2 + a_{22} f_6 + a_{23} f_7 + a_{24} f_8 - a_{25} g_1 - a_{26} f_4 - a_{27} f_3 - a_{28} \xi_1 - a_{29} f_9 - a_{30} f_1 - a_{31} f_1 - a_{32} f_2 \]
\[ \psi_2 = a_{20} \overline{f}_5 + a_{21} \overline{g}_2 + a_{22} \overline{f}_6 + a_{23} \overline{f}_7 + a_{24} \overline{f}_8 - a_{25} \overline{g}_1 - a_{26} \overline{f}_4 - a_{27} \overline{f}_3 - a_{28} \overline{\xi}_1 - a_{29} \overline{f}_9 - a_{30} \overline{f}_1 - a_{31} \overline{f}_1 - a_{32} \overline{f}_2 \]
\[ g_1 = g(Pr, Q, y, t), \overline{g}_1 = g(Pr, Q, y, t-1) H(t-1) \]
\[ \xi_1 = \xi(Sc, y, t), \overline{\xi}_1 = \xi(Sc, y, t-1) H(t-1) \]
\[ f_1 = f(Sc, y, \alpha, t), \overline{f}_1 = e^{-\alpha} f(Sc, y, \alpha, t-1) H(t-1) \]
\[ f_2 = f(Sc, y, 0, t), \overline{f}_2 = f(Sc, y, 0, t-1) H(t-1) \]
\[ f_3 = f(Pr, y, Q + \alpha, t), \overline{f}_3 = e^{-\alpha} f(Pr, y, Q + \alpha, t-1) H(t-1) \]
\[ f_4 = f(Pr, y, Q, t), \overline{f}_4 = f(Pr, y, Q, t-1) H(t-1) \]
\[ f_5 = f(1, y, \lambda, t), \overline{f}_5 = f(1, y, \lambda, t-1) H(t-1) \]
\[ f_6 = f(1, y, \lambda + \beta, t), \overline{f}_6 = e^{-\beta} f(1, y, \lambda + \beta, t-1) H(t-1) \]
\[ f_7 = f(1,y,\lambda + \gamma,t), \quad \overline{f}_7 = e^{-\gamma} f(1,y,\lambda + \gamma,t-1)H(t-1) \]

\[ f_8 = f(1,y,\lambda + \alpha,t), \quad \overline{f}_8 = e^{-\alpha} f(1,y,\lambda + \alpha,t-1)H(t-1) \]

\[ f_9 = f(Pr,y,Q + \beta,t), \quad \overline{f}_9 = e^{-\beta} f(Pr,y,Q + \beta,t-1)H(t-1) \]

\[ f_{10} = f(Sc,y,\gamma,t), \quad \overline{f}_{10} = f(Sc,y,\gamma,t-1)H(t-1) \]

\[ g(x,y,z,t) = \frac{1}{2} \left[ \left( t + \frac{z}{2} \sqrt{\frac{x}{y}} \right) e^{\sqrt{xy}z} \text{erfc} \left( \frac{z}{2\sqrt{y}} + \sqrt{yt} \right) + \left( t - \frac{z}{2} \sqrt{\frac{x}{y}} \right) e^{-\sqrt{xy}z} \text{erfc} \left( \frac{z}{2\sqrt{y}} - \sqrt{yt} \right) \right] \]

\[ f(x,y,z,t) = \frac{1}{2} \left[ e^{\sqrt{xy}z} \text{erfc} \left( \frac{y}{2\sqrt{t}} + \sqrt{zt} \right) + e^{-\sqrt{xy}z} \text{erfc} \left( \frac{y}{2\sqrt{t}} - \sqrt{zt} \right) \right] \]

\[ \xi(x,y,t) = \left[ 1 + \frac{y^2x}{2t} \right] \text{erfc} \left( \frac{y}{2\sqrt{t}} \right) - y \sqrt{\frac{x}{\pi t}} e^{-\frac{y^2x}{4t}} \]

\[ \Omega = (a_{20} \omega_1 + a_{21} \varphi_1 + a_{22} e^{\beta t} \omega_2 + a_{23} e^{\gamma t} \omega_3 + a_{24} e^{\alpha t} \omega_4 - a_{25} \varphi_2 - a_{26} \omega_5 - a_{27} e^{\alpha t} \omega_6 - 2t a_{28} \omega_7 - a_{29} e^{\beta t} \omega_8 - a_{30} e^{\gamma t} \omega_9 - a_{31} e^{\alpha t} \omega_{10} - a_{32} \omega_7) \]

\[ \overline{\Omega} = (a_{20} \overline{\omega}_1 + a_{21} \overline{\varphi}_1 + a_{22} e^{\beta t} \overline{\omega}_2 + a_{23} e^{\gamma t} \overline{\omega}_3 + a_{24} e^{\alpha t} \overline{\omega}_4 - a_{25} \overline{\varphi}_2 - a_{26} \overline{\omega}_5 - a_{27} e^{\alpha t} \overline{\omega}_6 - 2t a_{28} \overline{\omega}_7 - a_{29} e^{\beta t} \overline{\omega}_8 - a_{30} e^{\gamma t} \overline{\omega}_9 - a_{31} e^{\alpha t} \overline{\omega}_{10} - a_{32} \overline{\omega}_7) \]

\[ \omega_1 = \omega(1,\lambda,t), \quad \omega_1 = \omega(1,\lambda,t-1)H(t-1), \]

\[ \omega_2 = \omega(1,\lambda + \beta,t), \quad \omega_2 = \omega(1,\lambda + \beta,t-1)H(t-1), \]

\[ \omega_3 = \omega(1,\lambda + \gamma,t), \quad \omega_3 = \omega(1,\lambda + \gamma,t-1)H(t-1), \]
\[ \omega_4 = \omega(1, \lambda + \alpha, t), \quad \bar{\omega}_4 = \omega(1, \lambda + \alpha, t-1)H(t-1), \]

\[ \omega_5 = \omega(Pr, Q, t), \quad \bar{\omega}_5 = \omega(Pr, Q, t-1)H(t-1), \]

\[ \omega_6 = \omega(Pr, Q + \alpha, t), \quad \bar{\omega}_6 = \omega(Pr, Q + \alpha, t-1)H(t-1), \]

\[ \omega_7 = \omega(Sc, 0, t), \quad \bar{\omega}_7 = \omega(Sc, 0, t-1)H(t-1), \]

\[ \omega_8 = \omega(Pr, Q + \beta, t), \quad \bar{\omega}_8 = \omega(Pr, Q + \beta, t-1)H(t-1), \]

\[ \omega_9 = \omega(Sc, \gamma, t), \quad \bar{\omega}_9 = \omega(Sc, \gamma, t-1)H(t-1), \]

\[ \omega_{10} = \omega(Sc, \alpha, t), \quad \bar{\omega}_{10} = \omega(Sc, \alpha, t-1)H(t-1), \]

\[ \varphi_1 = \varphi(1, \lambda, t), \quad \bar{\varphi}_1 = \varphi(1, \lambda, t-1)H(t-1), \]

\[ \varphi_2 = \varphi(Pr, Q, t), \quad \bar{\varphi}_2 = \varphi(Pr, Q, t-1)H(t-1), \]

\[ \omega(x, y, t) = \sqrt{xy} \text{erf} \left( \sqrt{yt} \right) + \frac{x}{\sqrt{\pi}t} e^{-yt} \]

\[ \varphi(x, y, t) = \frac{1}{2} \sqrt{\frac{x}{y}} \text{erf} \left( \sqrt{yt} \right) + t \left[ \sqrt{xy} \text{erf} \left( \sqrt{yt} \right) + \frac{x}{\sqrt{\pi}t} e^{-yt} \right] \]

\[ H(t-1) = \begin{cases} 0, & t < 1 \\ 1, & t > 1 \end{cases} \]