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A New Characterization of Paired Domination Number of a Graph

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Abstract. Paired domination is a relatively interesting concept introduced by Teresa W. Haynes [9] recently with the following application in mind. If we think of each vertex \( s \in S \), as the location of a guard capable of protecting each vertex dominated by \( S \), then for a paired domination the guards location must be selected as adjacent pairs of vertices so that each guard is assigned one other and they are designated as a backup for each other. A set \( S \subseteq V \) is a paired dominating set if \( S \) is a dominating set of \( G \) and the induced sub graph \( <S> \) has a perfect matching. The paired domination number \( \gamma_{pr}(G) \) is the minimum cardinality taken over all paired dominating sets in \( G \). The minimum number of colours required to colour all the vertices so that adjacent vertices do not receive the same colour and is denoted by \( \chi(G) \). In this paper we characterize the class of all graphs whose sum of paired domination number and chromatic number equals to \( 2n - 7 \), for any \( n \geq 4 \).

Keywords: Paired domination number, Chromatic number.

AMS subject Classification: 05C (primary).

1 Introduction

Throughout this paper, by a graph we mean a finite, simple, connected and undirected graph \( G(V, E) \). For notations and terminology, we follow [11]. The number of vertices in \( G \) is denoted by \( n \). Degree of a vertex \( v \) is denoted by \( \deg(v) \). We denote a cycle on \( n \) vertices by \( C_n \), a path of \( n \) vertices by \( P_n \), complete graph on \( n \) vertices by \( K_n \). If \( S \) is a subset of \( V \), then \( <S> \) denotes the vertex induced sub graph of \( G \) induced
A New Characterization of Paired Domination Number of a Graph

by S. A subset S of V is called a dominating set of G if every vertex in V-S is adjacent to at least one vertex in S. The domination number \( \gamma(G) \) of G is the minimum cardinality of all such dominating sets in G. A dominating set S is called a total dominating set if the induced subgraph \(<S>\) has no isolated vertices. The minimum cardinality taken over all total dominating sets in G is called the total domination number and is denoted by \( \gamma_t(G) \). One can get a comprehensive survey of results on various types of domination number of a graph in [10]. The chromatic number \( \chi(G) \) is defined as the minimum number of colors required to color all the vertices such that adjacent vertices receive the same color.

Recently many authors have introduced different types of domination parameters by imposing conditions on the dominating set and/or its complement. Teresa W. Haynes [9] introduced the concept of paired domination number of a graph. If we think of each vertex \( s \in S \), as the location of a guard capable of protecting each vertex dominated by S, then for domination a guard protects itself, and for total domination each guard must be protected by another guard. For a paired domination the guards location must be selected as adjacent pairs of vertices so that each guard is assigned one other and they are designated as a backup for each other. Thus a paired dominating set S with matching M is a dominating set \( S = \{v_1, v_2, v_3, \ldots, v_{2t}, v_{2t+1}\} \) with independent edge set \( M = \{e_1, e_2, e_3, \ldots, e_t\} \) where each edge \( e_i \) is incident to two vertices of \( S \), that is \( M \) is a perfect matching in \(<S>\). A set \( S \subseteq V \) is a paired dominating set if \( S \) is a dominating set of G and the induced subgraph \(<S>\) has a perfect matching. The paired domination number \( \gamma_{pr}(G) \) is the minimum cardinality taken over all paired dominating sets in G.

Several authors have studied the problem of obtaining an upper bound for the sum of a domination parameter and a graph theoretic parameter and characterized the corresponding extremal graphs. In [8], Paulraj Joseph J and Arumugam S proved that \( \gamma + \kappa \leq p \), where \( \kappa \) denotes the vertex connectivity of the graph. In [7], Paulraj Joseph J and Arumugam S proved that \( \gamma_\ell + \chi \leq p + 1 \) and characterized the corresponding extremal graphs. They also proved similar results for \( \gamma \) and \( \chi \). In [6], Mahadevan G Selvam A, Iravithul Basira A characterized the extremal of graphs for which the sum of the complementary connected domination number and chromatic number. In [3], Paulraj Joseph J and Mahadevan G proved that \( \gamma_{pr} + \chi \leq 2n - 1 \), and characterized the corresponding extremal graphs of order up to \( 2n - 6 \). Motivated by the above results, in this paper we characterize all graphs for which \( \gamma_{pr}(G) + \chi(G) = 2n - 7 \) for any \( n \geq 4 \).

2 Main Result

We use the following preliminary results and notations for our consequent characterization:

Theorem 2.1[4] For any connected graph G of order \( n \geq 3 \), \( \gamma_{pr}(G) \leq n - 1 \) and equality holds if and only if \( G = C_3, C_5 \) or subdivided star \( S(K_{1,n}) \).
**Notation 2.2.** $C_3(n_1 P_{m_1}, n_2 P_{m_2}, n_3 P_{m_3})$ is a graph obtained from $C_3$ by attaching $n_1$ times the pendant vertex of $P_{m_1}$ (Path on $m_1$ vertices) to a vertex $u_i$ of $C_3$ and attaching $n_2$ times the pendant vertex of $P_{m_2}$ (Path on $m_2$ vertices) to a vertex $u_j$ for $i \neq j$ of $C_3$ and attaching $n_3$ times the pendant vertex of $P_{m_3}$ (Path on $m_3$ vertices) to a vertex $u_k$ for $i \neq j \neq k$ of $C_3$.

**Notation 2.3.** $C_3(u(P_{m_1}, P_{m_2}))$ is a graph obtained from $C_3$ by attaching the pendant vertex of $P_{m_1}$ (Path on $m_1$ vertices) and the pendant vertex of $P_{m_2}$ (Paths on $m_2$ vertices) to any vertex $u$ of $C_3$.

**Notation 2.4.** $K_5(n_1 P_{m_1}, n_2 P_{m_2}, n_3 P_{m_3}, n_4 P_{m_4}, n_5 P_{m_5})$ is a graph obtained from $K_5$ by attaching $n_1$ times the pendant vertex of $P_{m_1}$ (Paths on $m_1$ vertices) to a vertex $u_i$ of $K_5$ and attaching $n_2$ times the pendant vertex of $P_{m_2}$ (Paths on $m_2$ vertices) to a vertex $u_j$ for $i \neq j$ of $K_5$ and attaching $n_3$ times the pendant vertex of $P_{m_3}$ (Paths on $m_3$ vertices) to a vertex $u_k$ for $i \neq j \neq k$ of $K_5$ and attaching $n_4$ times the pendant vertex of $P_{m_4}$ (Paths on $m_4$ vertices) to a vertex $u_l$ for $i \neq j \neq k \neq l$ of $K_5$ and attaching $n_5$ times the pendant vertex of $P_{m_5}$ (Paths on $m_5$ vertices) to a vertex $u_m$ for $i \neq j \neq k \neq l \neq m$ of $K_5$.

**Notation 2.5.** $C_3(P_n)$ is the graph obtained from $C_3$ by attaching the pendant edge of $P_n$ to any one vertices of $C_3$ and $K_n(P_m)$ is the graph obtained from $K_n$ by attaching the pendant edge of $P_m$ to any one vertices of $K_n$. For $n \leq p$, $K_p(n)$ is the graph obtained from $K_p$ by adding a new vertex and join it with $n$ vertices of $K_p$. $C_3(K_{1,n})$ is the graph obtained from $C_3$, by attaching the root vertex of $K_{1,n}$ to any one vertex of $C_3$.

**Theorem 2.6.** For any connected graph $G$ of order $n$ ($n \geq 3$), $y_p(G) + \chi(G) = 2n - 7$ if and only if $G \equiv C_3(P_3), C_5(K_{1,3}), C_5(2P_2, P_2), C_5(2P_2, P_2, 0), C_5(2P_2, 0, 0), C_5(3P_3, 0), C_5(P_3, P_2), C_5(P_3, P_2, 0), C_5(3P_3, P_3, 0), C_5(u(P_4, P_2)), C_5(u(P_2, P_2)), K_5(P_4), K_5(P_2), K_5(1), K_5(2), K_5(3), K_5(4), K_5(5), K_5(6), K_5(P_2, P_2, P_2, 0, 0), K_5(P_3, P_2, 0, 0, 0), K_5(P_2, P_2, 0, 0, 0), K_5$ or any one of the graphs shown in Fig. 1.
Fig. 1. (Continued)
Proof. If $G$ is any one of the graphs stated in the theorem, then it can be verified that $\gamma_{pr}(G) + \chi(G) = 9 = 2n - 7$. Conversely, let $\gamma_{pr}(G) + \chi(G) = 2n - 7$. Then the various possible cases are (i) $\gamma_{pr}(G) = n - 1$ and $\chi(G) = n - 6$ (ii) $\gamma_{pr}(G) = n - 2$ and $\chi(G) = n - 5$ (iii) $\gamma_{pr}(G) = n - 3$ and $\chi(G) = n - 4$ (iv) $\gamma_{pr}(G) = n - 4$ and $\chi(G) = n - 3$ (v) $\gamma_{pr}(G) = n - 5$ and $\chi(G) = n - 2$ (vi) $\gamma_{pr}(G) = n - 6$ and $\chi(G) = n - 1$ (vii) $\gamma_{pr}(G) = n - 7$ and $\chi(G) = n$. 
Case i. $\gamma_{pr} = n - 1$ and $\chi = n - 6$.

Since $\gamma_{pr} = n - 1$, by theorem, 2.1 $G \cong C_3, C_5$, or $S(K_{1,3})$. Then $\chi = 2$ or 3. If $\chi = 2$, $n = 8$, which is a contradiction. If $\chi = 3$, $n = 9$, which is a contradiction. Hence no graph exists.

Case ii. $\gamma_{pr} = n - 2$ and $\chi = n - 5$.

Since $\chi(G) = n - 5$, $G$ contains a clique $K$ on $n - 5$ vertices (or) does not contain a clique $K$ on $n - 5$ vertices. By all the various possible cases, it can be verified that no graph exists satisfying the hypothesis.

Case iii. $\gamma_{pr} = n - 3$ and $\chi = n - 4$.

Since $\chi = n - 4$, $G$ contains a clique $K$ on $n - 4$ vertices or does not contain a clique $K$ on $n - 4$ vertices. Let $G$ contain a clique $K$ on $n - 4$ vertices. Let $S = V(G) - V(K) = \{v_1, v_2, v_3, v_4\}$. Then the induced subgraph $<S>$ has the following possible cases: $<S> = K_4, \overline{K}_4, P_4, C_4, K_{1,3}, P_3 \cup K_2, K_2 \cup K_3, K_3 \cup K_2, K_2 \cup \overline{K}_2, K_4 - \{e\}, C_3(1,0,0)$. If $<S> = K_4$, then it can be verified that no graph exists.

Sub case i. $<S> = \overline{K}_4$. Let $\{v_1, v_2, v_3, v_4\}$ be the vertices of $\overline{K}_4$. By the various possible cases, the only graph exists in this case is $C_3(2P_2, P_2, P_2)$.

Sub case ii. $<S> = P_4$.

Let $\{v_1, v_2, v_3, v_4\}$ be the vertices of $P_4$. Since $G$ is connected, there exists a vertex $u_1$ in $K_{n-4}$ which is adjacent to $v_1$ (or $v_4$) (or) $v_2$ (or $v_3$). Let $u_1$ be adjacent to $v_1$, then $\{v_1, v_2, v_3, v_4\}$ forms a $\gamma_{pr}$ - set of $G$ so that $\gamma_{pr} = 4$ and $n = 7$. Hence $K = K_3 = \langle u_1, u_2, u_3 \rangle$. If $u_1$ is adjacent to $v_2$, then $\{u_1, u_2, v_2, v_3\}$ forms a $\gamma_{pr}$ - set of $G$ so that $\gamma_{pr} = 4$ and $n = 7$. Hence $K = K_3 = \langle u_1, u_2, u_3 \rangle$. Let $u_1$ be adjacent to $v_2$ and $u_2$ be adjacent to $v_3$. If $\deg(v_1) = \deg(v_4) = 1$, $\deg(v_2) = 3$, $\deg(v_3) = 2$, then $G \cong G_1$. Let $u_1$ be adjacent to $v_2$ and $u_3$ be adjacent to $v_3$. If $\deg(v_1) = \deg(v_4) = 1$, $\deg(v_2) = \deg(v_3) = 3$, then $G \cong G_3$. Let $u_1$ be adjacent to $v_2$ and $v_3$, and $u_3$ be adjacent to $v_3$. If $\deg(v_1) = \deg(v_4) = 1$, $\deg(v_2) = 3$, $\deg(v_3) = 4$, then $G \cong G_4$. All the remaining cases are not possible.

Sub case iii. $<S> = C_4$.

Let $\{v_1, v_2, v_3, v_4\}$ be the vertices of $C_4$. Since $G$ is connected, there exists a vertex $u_1$ in $K_{n-4}$ which is adjacent to $v_1$. Let $u_i$ for some $i$ in $K_{n-4}$, be adjacent to $v_1$ and $u_i$ for $i \neq j$, then $\{u_i, u_j, v_1, v_4\}$ forms a $\gamma_{pr}$ - set of $G$ so that $\gamma_{pr} = 4$ and $n = 7$. Hence $K = K_3 = \langle u_1, u_2, u_3 \rangle$. If $u_1$ is adjacent to $v_1$, then $\deg(v_1) = 3$, and so $G \cong G_5$. Let $u_1$ be adjacent to $v_1$ and $u_2$ be adjacent to $v_2$. If $\deg(v_1) = \deg(v_2) = 3$, then $G \cong G_6$. Let $u_1$ be adjacent to $v_1$ and $u_3$ be adjacent to $v_2$. If $\deg(v_1) = \deg(v_2) = 3$, then $G \cong G_7$. 
Sub case iv. \(<S> = K_{1,3}\).

Let \(v_1\) be the root vertex and \(v_2, v_3, v_4\) are adjacent to \(v_1\). Since \(G\) is connected, there exists a vertex \(u_i\) in \(K_{n-4}\) which is adjacent to \(v_1\) (or any one of \(\{v_2, v_3, v_4\}\) and \(v_4\). Let \(u_i\) for some \(i\) in \(K_{n-4}\) be the vertex adjacent to \(v_1\), then \(\{u_i, v_1\}\) is a \(\gamma_p\)-set of \(G\) so that \(\gamma_p = 2\) and \(n = 5\), which is a contradiction. Hence no such graph exists. Since \(G\) is connected, there exists a vertex \(u_i\) in \(K_{n-4}\) which is adjacent to any one of \(\{v_2, v_3, v_4\}\). Then \(u_i\) for some \(i\) is adjacent to \(v_2\). In this case, \(\{u_i, u_j, v_1, v_2\}\) is an \(\gamma_p\)-set of \(G\) so that \(\gamma_p = 4\) and \(n = 7\), and hence \(K = K_3 = \langle u_1, u_2, u_3 \rangle\). Let \(u_i\) be adjacent to \(v_2\). If \(\deg(v_1) = 3, \deg(v_2) = \deg(v_4) = 1, \deg(v_2) = 3\), then \(G \cong G_6\). Let \(u_1\) be adjacent to \(v_2\) and \(v_3\). If \(\deg(v_1) = 3, \deg(v_2) = \deg(v_3) = 2, \deg(v_4) = 1\), then \(G \cong G_9\). Let \(u_1\) be adjacent to \(v_2\) and \(u_2\) be adjacent to \(v_4\). If \(\deg(v_1) = 3, \deg(v_2) = \deg(v_4) = 2, \deg(v_3) = 1\), then \(G \cong G_{10}\).

Sub case v. \(<S> = K_3 \cup K_1\).

Let \(v_1, v_2\) and \(v_3\) be the vertices of \(K_3\) and \(v_4\) be the vertex of \(K_1\). Since \(G\) is connected, there exists a vertex \(u_i\) in \(K_{n-4}\) which is adjacent to any one of \(\{v_1, v_2, v_3\}\) and \(\{v_4\}\). In this case \(\{u_i, v_1\}\) is a \(\gamma_p\)-set of \(G\), so that \(\gamma_p = 2\) and \(n = 5\), which is a contradiction. Hence no graph exists. Since \(G\) is connected, there exists a vertex \(u_i\) in \(K_{n-4}\) which is adjacent to \(v_2\) and \(u_i\) for \(i \neq j\) is adjacent to \(v_4\). In this case, \(\{u_i, u_j, v_1, v_2\}\) is a \(\gamma_p\)-set of \(G\) so that \(\gamma_p = 4\) and \(n = 7\). Hence \(K = K_3 = \langle u_1, u_2, u_3 \rangle\). Let \(u_i\) be adjacent to \(v_2\) and \(u_1\) be adjacent to \(v_4\). If \(\deg(v_1) = \deg(v_2) = 3, \deg(v_3) = 1\), then \(G \cong G_{11}\). Since \(G\) is connected, there exists a vertex \(u_i\) in \(K_{n-4}\) which is adjacent to \(v_1\) and \(v_3\) and \(v_4\) is adjacent to \(v_4\). In this case, \(\{u_i, v_1, v_2, v_4\}\) is a \(\gamma_p\)-set of \(G\) so that \(\gamma_p = 4\) and \(n = 7\), and hence \(K = K_3 = \langle u_1, u_2, u_3 \rangle\). Let \(u_i\) be adjacent to \(v_1\) and \(u_1\) be adjacent to \(v_4\). If \(\deg(v_1) = \deg(v_2) = 2, \deg(v_3) = \deg(v_4) = 1\), then \(G \cong C_3(P_4, P_2)\). Since \(G\) is connected, there exists a vertex \(u_i\) in \(K_{n-4}\) which is adjacent to \(v_1\) and \(v_4\). In this case, \(\{u_i, v_1, v_2, v_4\}\) is a \(\gamma_p\)-set of \(G\) so that \(\gamma_p = 2\) and \(n = 5\), which is a contradiction. Hence no graph exists. Since \(G\) is connected, there exists a vertex \(u_i\) in \(K_{n-4}\) which is adjacent to \(v_2\) and \(u_j\) for \(i \neq j\) is adjacent to \(v_4\). In this case, \(\{u_i, u_j, v_2, v_4\}\) is a \(\gamma_p\)-set of \(G\) so that \(\gamma_p = 4\) and \(n = 7\), and hence \(K = K_3 = \langle u_1, u_2, u_3 \rangle\). Let \(u_i\) be adjacent to \(v_2\) and \(u_3\) be adjacent to \(v_4\). If \(\deg(v_1) = \deg(v_3) = \deg(v_4) = 1, \deg(v_2) = 3\), then \(G \cong G_{19}\). Let \(u_i\) be adjacent to \(v_2\) and \(u_3\) be adjacent to \(v_2\) and \(u_3\) be adjacent to \(v_4\). If \(\deg(v_1) = \deg(v_3) = \deg(v_4) = 1, \deg(v_2) = 4\), then \(G \cong G_{10}\).
Sub case vii. \( <S> = K_2 \cup K_2 \).

Let \( v_1 \) and \( v_2 \) be the vertices of \( K_2 \) and \( v_3, v_4 \) be the vertices of \( K_2 \). Since \( G \) is connected, there exists a vertex \( u_i \) in \( K_{n-4} \) which is adjacent to any one of \( \{v_1, v_2\} \) and any one of \( \{v_3, v_4\} \). Let \( u_i \) be adjacent to \( v_1 \) and \( v_3 \). In this case \( \{u_i, v_1, v_2, v_3\} \) forms a \( \gamma'_p \)-set of \( G \) so that \( \gamma'_p = 4 \) and \( n = 7 \). Hence \( K = K_3 = \langle u_1, u_2, u_3 \rangle \). Let \( u_1 \) be adjacent to \( v_1 \) and \( v_3 \). If \( \deg(v_1) = \deg(v_3) = 2 \), then \( G \cong C_6(2P_3, 0, 0) \). Let \( u_1 \) be adjacent to \( v_3 \) and \( v_4 \) and let \( u_2 \) be adjacent to \( v_1 \) and \( u_3 \) be adjacent to \( v_2 \). If \( \deg(v_1) = \deg(v_2) = \deg(v_3) = \deg(v_4) = 2 \), then \( G \cong G_6 \). Since \( G \) is connected, there exists a vertex \( u_i \) in \( K_{n-4} \) which is adjacent to \( v_1 \) and there exists \( u_j \) for \( i \neq j \), is adjacent to \( v_3 \). In this case, \( \{u_i, u_j, v_1, v_3\} \) is a \( \gamma'_p \)-set of \( G \) so that \( \gamma'_p = 4 \) and \( n = 7 \), and hence \( K = K_3 = \langle u_1, u_2, u_3 \rangle \). Let \( u_1 \) be adjacent to \( v_3 \) and \( u_2 \) be adjacent to \( v_1 \). If \( \deg(v_1) = \deg(v_3) = 2 \), then \( G \cong G_{15} \). Let \( u_1 \) be adjacent to \( v_3 \) and \( u_2 \) be adjacent to \( v_1 \) and \( v_3 \). If \( \deg(v_1) = 2, \deg(v_2) = \deg(v_4) = 3 \), then \( G \cong G_{16} \). Let \( u_1 \) be adjacent to \( v_3 \), \( u_2 \) be adjacent to \( v_1 \) and \( v_3 \), and let \( u_3 \) be adjacent to \( v_4 \). If \( \deg(v_1) = 2, \deg(v_2) = 1 \), then \( G \cong G_{17} \). Let \( u_1 \) be adjacent to \( v_3 \) and \( v_4 \), \( u_2 \) be adjacent to \( v_1 \) and let \( u_3 \) is adjacent to \( v_1 \). If \( \deg(v_1) = 3, \deg(v_2) = \deg(v_4) = 2 \), then \( G \cong G_{18} \). Let \( u_1 \) be adjacent to \( v_3 \) and \( u_2 \) be adjacent to \( v_1 \) and \( u_3 \) be adjacent to \( v_1 \) and \( v_2 \). If \( \deg(v_1) = 3, \deg(v_2) = \deg(v_3) = 2 \), then \( G \cong G_{19} \).

Sub case viii. \( <S> = K_2 \cup \overline{K}_2 \).

Let \( v_1 \) and \( v_2 \) be the vertices of \( \overline{K}_2 \) and \( v_3, v_4 \) be the vertices of \( K_2 \). Since \( G \) is connected, there exists a vertex \( u_i \) in \( K_{n-4} \) which is adjacent to \( v_1 \) and \( v_2 \) and any one of \( \{v_3, v_4\} \). Let \( u_i \) be adjacent to \( v_1 \), \( v_2 \), \( v_3 \). In this case \( \{u_i, v_3\} \) forms a \( \gamma'_p \)-set of \( G \), so that \( \gamma'_p = 2 \) and \( n = 5 \), which is a contradiction. Hence no graph exists. Since \( G \) is connected, there exists a vertex \( u_i \) in \( K_{n-4} \) which is adjacent to \( v_1 \) and there exists \( u_j \) for \( i \neq j \), is adjacent to \( v_2 \) and \( v_3 \). In this case, \( \{u_i, u_j, u_k, v_1\} \) for \( i \neq j \neq k \) forms a \( \gamma'_p \)-set of \( G \) so that \( \gamma'_p = 4 \) and \( n = 7 \). Hence \( K = K_3 = \langle u_1, u_2, u_3 \rangle \). Let \( u_1 \) be adjacent to \( v_2 \) and \( u_3 \) be adjacent to \( v_1 \). If \( \deg(v_1) = \deg(v_2) = \deg(v_4) = 2 \), then \( G \cong G_{20} \). Since \( G \) is connected, there exists a vertex \( u_i \) in \( K_{n-4} \) which is adjacent to \( v_1 \) and there exists \( u_j \) for \( i \neq j \), is adjacent to \( v_2 \) and \( u_k \) for \( i \neq j \neq k \) is adjacent to \( v_3 \). In this case, \( \{u_i, u_j, u_k, v_3\} \) for \( i \neq j \neq k \) forms a \( \gamma'_p \)-set of \( G \) so that \( \gamma'_p = 4 \) and \( n = 7 \). Hence \( K = K_3 = \langle u_1, u_2, u_3 \rangle \). Let \( u_1 \) be adjacent to \( v_2 \) and \( u_2 \) be adjacent to \( v_1 \) and \( u_3 \) be adjacent to \( v_2 \). If \( \deg(v_1) = \deg(v_2) = \deg(v_3) = 2 \), then \( G \cong G_{21} \). Let \( u_1 \) be adjacent to \( v_2 \) and \( v_3 \), \( u_2 \) be adjacent to \( v_1 \) and \( u_3 \) be adjacent to \( v_3 \). If \( \deg(v_1) = \deg(v_2) = \deg(v_4) = 1 \), then \( G \cong G_{22} \). Let \( u_1 \) be adjacent to \( v_2 \), \( u_2 \) be adjacent to \( v_1 \) and \( v_2 \) and let \( u_3 \) is adjacent to \( v_1 \) and \( v_3 \). If \( \deg(v_1) = 2, \deg(v_2) = \deg(v_3) = 2 \), then \( G \cong G_{23} \).
Sub case ix. \(<S> = K_4 - \{e\}\).

Let \(v_1, v_2, v_3, v_4\) be the vertices of \(K_4\). Let \(e\) be any one of the edges inside the cycle \(C_4\). Since \(G\) is connected, there exists a vertex \(u_1\) in \(K_{n-4}\) which is adjacent to \(v_1\). In this case \(\{u_1, v_1\}\) is a \(\gamma_{pr}\)-set of \(G\), so that \(\gamma_{pr} = 2\) and \(n = 5\), which is a contradiction. Hence no graph exists. Since \(G\) is connected, there exists a vertex \(u_1\) in \(K_{n-4}\) which is adjacent to \(v_2\). In this case, \(\{u_2, v_2\}\) is a \(\gamma_{pr}\)-set of \(G\) so that \(\gamma_{pr} = 4\) and \(n = 7\). Hence \(K = K_3 = \langle u_1, u_2, u_3 \rangle\). Let \(u_1\) be adjacent to \(v_2\). If \(\text{deg}(v_1) = \text{deg}(v_2) = \text{deg}(v_3) = 3\), then \(G \cong G_{24}\). Let \(u_1\) be adjacent to \(v_2\) and let \(u_2\) adjacent to \(v_4\). If \(\text{deg}(v_1) = 3\), \(\text{deg}(v_2) = 3\), \(\text{deg}(v_3) = 3\), \(\text{deg}(v_4) = 3\), then \(G \cong G_{25}\). Let \(u_1\) be adjacent to \(v_2\) and let \(u_3\) be adjacent to \(v_2\). If \(\text{deg}(v_1) = \text{deg}(v_3) = 3\), \(\text{deg}(v_2) = 4\), \(\text{deg}(v_4) = 2\), then \(G \cong G_{26}\).

Sub case x. \(<S> = C_3(1, 0, 0)\).

Let \(v_1, v_2, v_3\) be the vertices of \(C_3\) and let \(v_4\) be adjacent to \(v_1\). Since \(G\) is connected, there exists a vertex \(u_1\) in \(K_{n-4}\) which is adjacent to \(v_2\) (or) there exists a vertex \(u_1\) in \(K_{n-4}\) which is adjacent to \(v_1\) (or) there exists a vertex \(u_1\) in \(K_{n-4}\) which is adjacent to \(v_4\). In all the cases, by various arguments, it can be verified that \(G \cong G_{27}, G_{28}, G_{29}, G_{30}, \ldots\). If \(G\) does not contain a clique \(K\) on \(n - 4\) vertices, then it can be verified that no new graph exists.

Case iv. \(\gamma_{pr} = n - 4\) and \(\chi = n - 3\).

Since \(\chi = n - 3\), \(G\) contains a clique \(K\) on \(n - 3\) vertices or does not contain a clique \(K\) on \(n - 3\) vertices. Let \(S = V(G) - V(K) = \{v_1, v_2, v_3\}\). Then the induced subgraph \(<S>\) has the following possible cases. \(<S> = K_3, \overline{K}_3, P_3, K_2 \cup K_1\).

Sub case i. \(<S> = K_3\).

Let \(v_1, v_2, v_3\) be the vertices of \(K_3\). Since \(G\) is connected, there exists a vertex \(u_1\) in \(K_{n-3}\) which is adjacent to any one of \(\{v_1, v_2, v_3\}\). Let \(u_1\) be adjacent to \(v_2\), then \(\{u_1, v_2\}\) is a \(\gamma_{pr}\)-set of \(G\), so that \(\gamma_{pr} = 2\) and \(n = 6\). Hence \(K = K_3 = \langle u_1, u_2, u_3 \rangle\). Let \(u_1\) be adjacent to \(v_2\) and let \(u_2\) adjacent to \(v_3\). If \(\text{deg}(v_1) = \text{deg}(v_3) = 2\), \(\text{deg}(v_2) = 3\), then \(G \cong G_{32}\). Let \(u_1\) be adjacent to \(v_1\) and \(v_2\). If \(\text{deg}(v_1) = \text{deg}(v_2) = 3\), \(\text{deg}(v_3) = 2\), then \(G \cong G_{33}\). Let \(u_1\) be adjacent to \(v_2\), \(u_2\) be adjacent to \(v_1\) and let \(u_3\) be adjacent to \(v_3\). If \(\text{deg}(v_1) = \text{deg}(v_2) = \text{deg}(v_3) = 3\), then \(G \cong G_{34}\). Let \(u_1\) be adjacent to both the vertices \(v_1, v_2, u_2\) be adjacent to \(v_1\) and let \(u_3\) be adjacent to \(v_3\). If \(\text{deg}(v_1) = 4\), \(\text{deg}(v_2) = \text{deg}(v_3) = 3\), then \(G \cong G_{35}\).

Sub case ii. \(<S> = \overline{K}_3\).

Let \(v_1, v_2, v_3\) be the vertices of \(\overline{K}_3\). Since \(G\) is connected, one of the vertices of \(K_{n-3}\) say \(u_1\) is adjacent to all the vertices of \(S\) (or) \(u_1\) be adjacent to \(v_1, v_2\) and \(u_1\) be adjacent to \(v_3\) for \(i \neq j\) (or) \(u_1\) be adjacent to \(v_1\) and \(u_1\) be adjacent to \(v_2\) and \(u_1\) be adjacent to \(v_3\) for \(i \neq j \neq k\). If \(u_1\) for some \(i\), \(v\) for some \(v\) in \(K_{n-3}\), is a \(\gamma_{pr}\)-set of \(G\), so that \(\gamma_{pr} = 2\) and \(n = 6\). Hence \(K = K_3 = \langle u_1, u_2, u_3 \rangle\). If \(u_1\) is adjacent to all the vertices \(v_1, v_2, v_3\), then \(G \cong C_3(K_{1, 3})\). Since \(G\) is connected, there
exists a vertex \( u_i \) in \( K_{n-3} \) is adjacent to \( v_1 \) and \( u_j \) for \( i \neq j \) is adjacent to \( v_2 \) and \( v_3 \), then \( \{u_i, u_j\} \) is a \( \gamma_{pr} \) set of \( G \), so that \( \gamma_{pr} = 2 \) and \( n = 6 \). Hence \( K = K_3 = <u_1, u_2, u_3> \). Let \( u_1 \) be adjacent to \( v_1 \) and \( v_2 \) and let \( u_3 \) be adjacent to \( v_3 \). If \( \deg(v_1) = \deg(v_2) = \deg(v_3) = 1 \) then \( G \cong C_3(2P_2, P_2, 0) \). Since \( G \) is connected, there exists a vertex \( u_i \) in \( K_{n-3} \) is adjacent to \( v_1 \) and \( u_j \) for \( i \neq j \) is adjacent to \( v_2 \) and \( u_k \) for \( i \neq j \neq k \) in \( K_{n-3} \) is adjacent to \( v_3 \), then \( \{u_i, u_j, u_k, v\} \) for some \( v \) in \( K_{n-3} \) is a \( \gamma_{pr} \) set of \( G \), so that \( \gamma_{pr} = 4 \) and \( n = 8 \). Hence \( K = K_4 = <u_1, u_2, u_3, u_4, u_5> \). Let \( u_1 \) be adjacent to \( v_2 \), \( u_3 \) be adjacent to \( v_3 \) and let \( u_5 \) be adjacent to \( v_1 \). If \( \deg(v_1) = \deg(v_2) = \deg(v_3) = 1 \) then \( G \cong K_5(P_2, P_2, P_2, 0, 0) \).

**Sub case 3.** \(<S> = P_3 \).

Let \( v_1, v_2, v_3 \) be the vertices of \( P_3 \). Since \( G \) is connected, there exists a vertex \( u_i \) in \( K_{n-3} \) which is adjacent to \( v_1 \) (or equivalently \( v_3 \)) or \( v_2 \). If \( u_i \) is adjacent to \( v_2 \), then \( \{u_i, v_2\} \) is a \( \gamma_{pr} \) set of \( G \), so that \( \gamma_{pr} = 2 \) and \( n = 6 \). Hence \( K = K_3 = <u_1, u_2, u_3> \). Let \( u_1 \) be adjacent to \( v_2 \). If \( \deg(v_2) = 3, \deg(v_1) = \deg(v_3) = 1 \), then \( G \cong G_{36} \). By increasing the degrees of the vertices, it can be verified that \( G \cong G_{37}, G_{38} \). Since \( G \) is connected, there exists a vertex \( u_i \) in \( K_{n-3} \) is adjacent to \( v_1 \) then \( \{u_i, u_j, v_1, v_2\} \) for \( i \neq j \) is a \( \gamma_{pr} \) set of \( G \), so that \( \gamma_{pr} = 4 \) and \( n = 8 \). Hence \( K = K_5 = <u_1, u_2, u_3, u_4, u_5> \). Let \( u_1 \) be adjacent to \( v_1 \). If \( \deg(v_1) = \deg(v_2) = 2, \deg(v_3) = 1 \), then \( G \cong K_5(P_2) \). By increasing the degrees of the vertices, it can be verified that \( G \cong G_{39}, G_{40} \).

**Sub case 4.** \(<S> = K_2 \cup K_1 \).

Let \( v_1, v_2 \) be the vertices of \( K_2 \) and \( u \) the vertex of \( K_1 \). Since \( G \) is connected, there exists a vertex \( u_i \) in \( K_{n-3} \) which is adjacent to any one of \( \{v_1, v_2\}\) and \( \{v_3\}\) (or \( u_i \) is adjacent to any one of \( \{v_1, v_2\}\) and \( u_j \) for \( i \neq j \) is adjacent to \( v_3 \)). In this case, \( \{u_i, u_j, u_k, v_1\} \) for \( u_i, u_k \) in \( K_{n-3} \) for \( i \neq j \neq k \) forms a \( \gamma_{pr} \) set of \( G \), so that \( \gamma_{pr} = 4 \) and \( n = 8 \). Hence \( K = K_5 = <u_1, u_2, u_3, u_4, u_5> \). Let \( u_1 \) be adjacent to \( v_1 \) and let \( u_2 \) be adjacent to \( v_3 \). If \( \deg(v_1) = 2, \deg(v_2) = \deg(v_3) = 1 \), then \( G \cong K_5(P_3, P_2, 0, 0, 0) \). By increasing the degrees of the vertices, it can be verified that \( G \cong G_{41}, G_{42}, G_{43} \). Since \( G \) is connected, there exists a vertex \( u_i \) in \( K_{n-3} \) which is adjacent to \( v_1 \), \( v_3 \), so that \( \{u_i, v\} \) is a \( \gamma_{pr} \) set of \( G \). Hence \( \gamma_{pr} = 2 \) and \( n = 6 \), so that \( K = K_3 = <u_1, u_2, u_3> \). Let \( u_1 \) be adjacent to \( v_1 \). If \( \deg(v_1) = 2, \deg(v_2) = \deg(v_3) = 1 \), then \( G \cong C_3(u(P_3, P_2)) \). By increasing the degrees of the vertices, it can be verified that \( G \cong G_{44}, G_{45} \). If \( G \) does not contain a clique \( K \) on \( n-3 \) vertices, then it can be verified that no new graph exists.

**Case v.** \( \gamma_{pr} = n-5 \) and \( \chi = n-2 \).

Since \( \chi = n-2 \), \( G \) contains a clique \( K \) on \( n-2 \) vertices or does not contains a clique \( K \) on \( n-2 \) vertices. Let \( G \) contains a clique \( K \) on \( n-2 \) vertices. Let \( S = V(G) - V(K) = \{v_1, v_2\} \). Then the induced sub graph \(<S>\) has the following possible cases. \(<S> = K_2, \mathbb{R}_2 \). In both cases, by the various possible arguments, it can be verified that \( G \cong K_5(P_3), G_{46}, G_{47}, G_{48}, G_{49}, G_{50}, G_{51}, K_5(P_2, P_2, 0, 0, 0), G_{52}, G_{53}, G_{54}, G_{55}, G_{56} \). If \( G \) does not contain a clique \( K \) on \( n-2 \) vertices, then it can be verified that no new graph exists.
Case vi. $\gamma_{pr} = n - 6$ and $\chi = n - 1$.

Since $\chi = n - 1$, G contains a clique $K_n$ on $n - 1$ vertices. By various arguments it can be verified that $G \cong K_7(1), K_7(2), K_7(3), K_7(4), K_7(5),$ or $K_7(6)$.

Case vii. $\gamma_{pr} = n - 7$ and $\chi = n$.

Since $\chi = n$, G $\cong K_n$. But for $K_n$, $\gamma_{pr} = 2$ so that $n = 9$. Hence $G \cong K_9$.

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