CHAPTER 5

FEATURE EXTRACTION BY TEXTURE ANALYSIS

5.1 INTRODUCTION:

Recently, Image analysis or pattern recognition has been used in diversified applications like optical character recognition, speech recognition, seismic analysis, industrial inspection and content based image retrieval [31,38]. Pattern recognition can be defined as the process of learning to distinguish and classify pattern.

An image feature is a distinguishing characteristic of an image. One of the prerequisite of pattern recognition is feature extraction. Image features are of importance in the isolation of regions within an image and subsequent labeling and classification. Therefore, extraction of feature is the important step in pattern recognition. The goal of feature extractor is to characterize an object to be recognized, by measurement of those attributes of image which can is used for differentiation of various categories[21,26]. Generally, feature space has large dimensions and therefore, feature selection is used for dimensionality reduction.
The features commonly used for pattern recognition are color, texture, shape etc. Features generally come from a number of information domain as specified below [51]:

1. **Spectral Domain**: Visible and infrared spectra may be used to generate independent object features. The visible spectra are based on the visible appearance of an object and the infrared spectrum is based on its temperature.

2. **Spatial Domain**: The spatial distribution of a pattern of intensities over a region is used as a regional feature to characterize object types. Texture is one of the examples of the spatial feature.

3. **Temporal Domain**: The change or motion of image attributes is a dynamic or time varying feature. It provides a method to discriminate stationary and moving scenes.

Segmentation of MR images is based on sets of features that can be extracted from the images, such as pixel intensities which can be used to calculate other features such as edges and texture. Instead of using all the features simultaneously, feature extraction and selection breaks down the problem of segmentation to the grouping of feature vectors[21]. Practical pattern classification and knowledge discovery problems require selection of a subset features to represent the patterns to be classified. Selection of good features is the key to successful segmentation [28].

Feature extraction can be extracted from various classes mentioned below:

(i) Pixel intensity based features
(ii) Edges and based features
(iii)Texture based features

(i) **Pixel intensity-based features**: Many segmentation approaches reported in the literature simply use the gray scale values of the pixels. The pixel intensity input can come from a single image or a multispectral data set of the same anatomical location. Transformations can be applied to any known vectors to generate new features.

(ii) **Edges based features**: Some researchers have operated on edges of MR gray scale image. Due to image artifacts and noise in MR images, a global threshold cannot segment MRI data and additional features are required. Some researchers have used
various edge detection methods. But it is concluded that methods based on edge
detection do not give optimum classification.

(iii) Texture based features: Another feature reported in the literature on MR image
segmentation is texture. Similar to the reported use of calculated parameters, texture
features are mainly applied for classification, rather than delineation or segmentation of
regions. Texture is a statistical feature necessarily derived from a large number of
pixels.

5.2 TEXTURE

Recent research in various medical applications shows that texture analysis in
medical imaging has been used as supportive tool in diagnosis of Digital
Mammography, Prostate Ultrasound Diagnosis. Studies of intracranial tumors have
demonstrated that MR image texture may be used to determine the tumor type
[24,39,40].

In literature, texture is defined in many ways. IEEE Standard 610.4 for image
processing & pattern recognition defines texture as an attribute representing the spatial
arrangement of the gray levels of the pixels in a region. R.C. Gonzalez and R.E. Woods
defines texture descriptor as a measure of properties such as smoothness, coarseness
and regularity[27]. Texture is a tool that provides information in the spatial arrangement
of colours or intensities in an image and cannot be defined for a point. It is
characterized by the spatial distribution of intensity levels in a neighbourhood[26]. It is
a repeating pattern of local variations in image intensity.

In search for meaningful features for describing pictorial information, spectral,
textural, and contextual features are three fundamental pattern elements used in human
interpretation[41,42]. Spectral features describe the average tonal variations in various
bands of the visible and/or infrared portion of an electromagnetic spectrum, whereas
textural features contain information about the spatial distribution of tonal variations
within a band. Contextual features contain information derived from blocks of pictorial
data surrounding the area being analyzed.
To characterize texture, it is necessary to characterize the tonal primitive properties as well as the spatial inter-relationships between them. This implies that Texture-tone is a two-layered structure, the first layer specifies the local properties which manifest themselves in tonal primitives and the second layer specifies the organization among the tonal primitives [21,29]. One important property of tone-texture is the spatial pattern of the resolution cells composing each discrete tonal feature.

Various texture analysis approaches are reported in the literature. They can be broadly classified as mentioned below:

(i) Structural approach
(ii) Statistical approach
(iii) Transform /Spectral approach

(i) **Structural approach**
It represents texture by well defined primitives (micro texture) and a hierarchy of spatial arrangements of those primitives. The advantage of this approach is that it provides a good symbolic description of the image but this method is more useful in synthesis than analysis of image.

(ii) **Statistical approach**
It represents the texture by non-deterministic properties that govern the distribution and relationships between the grey levels of an image. Methods based on second order statistics i.e. statistics given by pairs of pixels have given better texture differentiation than other methods. The most popular second order statistical features for texture analysis are derived from co-occurrence matrix defined by Haralick. It has potential of extracting features for effective texture discrimination in biomedical images. After undergoing detailed literature survey, this methodology has been adopted in the presented work.
(iii) Transform based approach

This approach interprets image features in frequency domain using various transforms like Fourier, Gabor and Wavelet transform. These transforms represent an image in a space whose coordinate system has an interpretation closely related to characteristics of a texture such as frequency. Fourier transforms are not efficient due to lack of spatial localization. Gabor filter provide better spatial resolution but are not recommended due to computational complexity [43]. Comparatively, wavelet transform based feature extraction has several advantages. Variable spatial resolution represents textures at suitable scale and there is a wide range of choices for the wavelet basis function.

After undergoing exhaustive literature survey, considering merits and demerits of various approaches and after performing lot of experimentation, the methodology used in this present work is the combination of statistical approach and transform domain approach. Gray Level Dependence Matrix (GLDM) in spatial domain and standard Discrete Wavelet Transform (DWT) and Dual Tree Complex Wavelet Transform (DT-CWT) in transform domain have been used for estimating texture features of tumor tissue and normal brain tissue in this presented work.

5.3 TEXTURE ANALYSIS USING STATISTICAL APPROACH

The statistical method of texture classification makes use of features that measure the coarseness and the directionality of the measures in terms of averages on a window of the image. A number of texture descriptors for statistical analysis discussed in the literature are autocorrelation function, structural element, Gray level co occurrence matrix and Run lengths algorithms.

Studies of intracranial tumors have demonstrated that MR image texture may be used to determine the tumor type. MR Image texture analysis is proved to be useful in the detection of Alzheimer’s disease [24]. An efficient, integrated image textural analysis and classification of transrectal prostate ultrasound images into clusters potentially representing cancerous or normal tissue areas is based on Haralick’s textural features using Gray Level Co-occurrence Matrix (GLCM) [42,44]. Thus, the literature
indicates the use of co-occurrence matrix for various medical image analysis [24, 39, 45].

Experimentation performed in this study is focused upon assessing the statistical parameters of MR image as a measure of texture, used as a tool to classify the type of the tissue using Gray Level Co-occurrence matrix. The following figure 5.1 gives the basic model for feature extraction.

![Basic model for feature extraction](image_url)

**Fig. 5.1 Basic model for feature extraction**

**5.3.1 Spatial Gray Level Dependence Matrix:**

A gray tone spatial dependence matrix or Gray Level Co-occurrence Matrix (GLCM) approach [42,44], is a well-known statistical method for extracting second order texture information, is used in the presented algorithm. In research work
presented by Haralick, Gray tone spatial dependence matrix is defined and a set of 28 textural features are suggested which can be extracted from each of the gray-tone spatial dependence matrices.

In the method of Co-occurrence matrix, texture is defined by the statistical distribution of the spatial dependence of gray level properties. The gray level spatial dependence approach characterizes texture by the co-occurrence of its gray levels. Coarse textures are those for which the distribution changes only slightly with distance and fine textures are those for which the distribution changes rapidly with distance. Image texture is characterized by a given pixel and the pattern in a local area around the pixel.

A means of analyzing texture within an image involves the creation of Gray Level Co-occurrence Matrix (GLCM), which is an indication of how different combinations of gray levels exist in a portion of the image. GLCM is generated for a small square window of the image. Within the window, unordered pairs of pixels are examined that are separated by a given distance and are oriented to each other by a given angle. The window size can be between 3 x 3 and 21 x 21 pixels, and angles of 0°, 45°, 90°, or 135° are used. An entire image can be analyzed by moving the window across the image in an overlapping manner, advancing one pixel column to the right, then one pixel row downward at a time.

Thus, computation of spatial Grey Level Dependence Matrix (GLDM) involves the estimation of the discrete second order probability function, \( C(i, j / d, \theta) \) where \( \theta = 0, 45, 90 \) and 135 degrees. Each \( C(i, j / d, \theta) \) is the probability of going from gray level \( i \) to gray level \( j \), given that the inter sample spacing is \( d \) and the direction is given by the angle \( \theta \). This is referred to as a Co-occurrence matrix. Such matrices of spatial gray level dependence frequencies are symmetric and a function of the angular relationship between the neighboring resolution cells as well as a function of the distance between them. Each element \((i, j)\) in GLCM specifies the number of times that the pixel with value \( i \) occurred horizontally adjacent to a pixel with value \( j \).

The following figure 5.2 shows how gray co-occurrence matrix calculates several values in the GLCM of the 4-by-5 image I. Element (1,1) in the GLCM contains the value 1 because there is only one instance in the image where two, horizontally adjacent pixels have the values 1 and 1. Element (1, 2) in the GLCM contains the value
2 because there are two instances in the image where two, horizontally adjacent pixels have the values 1 and 2. Gray co occurrence matrix continues this processing to fill in all the values in the GLCM. The co-occurrence matrix $C_d$ has dimension $N \times N$, where $N$ is the number of gray levels in the image.

![Digital Image Co-occurrence Matrix]

**Fig. 5.2 Calculation of GLCM**

### 5.3.2 Estimation of statistical parameters

The statistical texture parameters calculated from GLCM are Contrast, Correlation, Energy, Homogeneity, Standard Deviation, Mean, Angular Second Moment, Sum Average, Difference Average, Variance, Sum Variance, Difference Variance, Entropy, Sum Entropy and Difference Entropy.

A co-occurrence matrix is a two-dimensional array in which both the rows and the columns represent a set of possible image values. GLCM $C[i, j]$ is defined by first specifying a displacement vector $d$ and counting all pairs of pixels separated by $d$ having gray levels $i$ and $j$. The algorithm for calculation of GLCM is as explained below:

**GLCM Algorithm:**

i) Count all pairs of pixels in which the first pixel has a value $i$, and its matching pair displaced from the first pixel by $d$ has a value of $j$. 
ii) This count is entered in the ith row and jth column of the matrix C[ i, j ].

iii) Note that C[ i, j ] is not symmetric, since the number of pairs of pixels having gray levels [i,j] does not necessarily equal the number of pixel pairs having gray levels [j,i].

iv) The elements of C[ i, j ] can be normalized by dividing each entry by the total number of pixel pairs. Normalized GLCM N[ i, j ], defined by

\[ N[ i, j ] = \frac{C[ i, j ]}{\sum_i \sum_j C[ i, j ]} \] ..........................(5.1)

which normalizes the co-occurrence values to lie between 0 and 1, and allows them to be thought of as probabilities.

v) Calculate statistical texture parameters using equations 5.2 to 5.11 given below.

Let N be the number of distinct gray levels,

\[ C_x (i) = \sum_{j=1}^{N} C(i, j) \] ..........................(5.2)

\[ C_y (i) = \sum_{i=1}^{N} C(i, j) \] ..........................(5.3)

\[ C_{x+y}(k) = \sum_{i=1}^{N} \sum_{j=1}^{N} C(i, j) \quad i+j=k \quad k = 2,3,...,2N \] ..........................(5.4)

\[ C_{x-y}(k) = \sum_{i=1}^{N} \sum_{j=1}^{N} C(i, j) \quad i-j=k \quad k = 0,1,...,N-1 \] ..........................(5.5)

1. **Contrast** is a measure of the local variations present in an image.

\[ f^1 = \sum_{n=0}^{N-1} n^2 \left\{ \sum_{i=1}^{N} \sum_{j=1}^{N} C(i, j) \right\} \]

\[ |i - j| = n \] ..........................(5.6)

If there is a large amount of variation in an image the C[ i, j ]’s will be concentrated away from the main diagonal and contrast will be high.
2. **Homogeneity**: A homogeneous image will result in a co-occurrence matrix with a combination of high and low \( C[i, j] \)'s.

\[
C_h = \sum_i \sum_j \frac{C[i, j]}{\sum_j C[i, j]} \quad \text{..................(5.7)}
\]

Where the range of gray levels is small the \( C[i, j] \) will tend to be clustered around the main diagonal. A heterogeneous image will result in an even spread of \( C[i, j] \)'s.

3. **Entropy**: It is a measure of information content. It measures the randomness of intensity distribution. Entropy is highest when all entries in \( C[i, j] \) are of similar magnitude, and small when the entries in \( C[i, j] \) are unequal.

\[
f_2 = -\sum_{i=1}^{N} \sum_{j=1}^{N} C(i, j) \log(C(i, j)) \quad \text{..................(5.8)}
\]

4. **Correlation**: It is a measure of image linearity. Correlation will be high if an image contains a considerable amount of linear structure.

\[
C = \frac{\sum_i \sum_j [ijC[i, j]] - \mu_i \mu_j}{\sigma_i \sigma_j} \quad \text{..................(5.9)}
\]

Where \( \mu \) is mean and \( \sigma \) is variance given by

\[
\mu_i = \sum_i iC[i, j], \quad \sigma_i^2 = \sum_i i^2C[i, j] - \mu_i^2
\]

5. **Energy or Angular Second Moment**: The texture energy measure is computed by summing the absolute values in a local neighborhood:

\[
f_3 = \sum_{i=1}^{N} \sum_{j=1}^{N} \{ C(i, j) \}^2 \quad \text{..................(5.10)}
\]
6. Variance: Another texture estimator is the variance or sum of difference between intensity of the central pixel and its neighborhood. It is a measure of the dispersion of the gray-level differences at a certain distance, d.

\[
f_4 = \sum_{i=2}^{2N} (i - f)2C_x + y(i) \quad \ldots \ldots(5.11)
\]

\[
f = \sum_{i=2}^{2N} iC_x + y(i) \quad \ldots \ldots(5.12)
\]

7. Sum Entropy

\[
f_5 = -\sum_{i=2}^{2N} C_x + y(i) \log(C_x + y(i)) \quad \ldots \ldots(5.13)
\]

8. Difference Entropy

\[
f_6 = -\sum_{i=0}^{N-1} C_x - y(i) \log(C_x - y(i)) \quad \ldots \ldots(5.14)
\]

In this experimentation, T1 and T2 images are selected and preprocessed by implementing image resizing and median filtering. Tumor is segmented by implementing algorithm explained in the previous chapter. Using equation 5.1 to 5.14, the set of twelve statistical parameters is calculated for tumor tissue from segmented image. These parameters are Contrast, Correlation, Energy, Homogeneity, Standard Deviation, Mean, Angular Second Moment, Sum Average, Variance, Sum Variance,
Difference Variance, Entropy, Sum Entropy and Difference Entropy, for the pixel offset distances of 1, 2, and 4 and for angles $\theta = 0, 45, 90$ and 135 degrees. A large range of distances is chosen so that the derived measures may be sensitive to texture differences at many scales. The estimation of these parameters is repeated for normal brain tissue, for which a region of interest, ROI, is selected in the original image by mouse driven user interface. This procedure is carried out for 75 samples of MR Images of brain tumor.

The following observations are drawn from the above set of readings.

(i) Total twelve statistical parameters are calculated for tumor and normal brain tissue. It is observed that not all statistical parameters show substantial discrimination for tumor and normal brain tissue. Entropy, Contrast, Energy and Variance are the parameters which indicate precise discrimination in parameter values for tumor tissue and normal brain tissue. Therefore, only these parameters are included in the feature vector.

(ii) The parameters do not vary substantially if the inter pixel distance is incremented beyond 2. Therefore, feature vector comprising of these parameters is calculated for above four angles and inter pixel distance $d = 1$ and 2.

Thus, feature vector created for tumor tissue and normal brain tissue consists of statistical parameters Entropy, Contrast, Energy and Variance for inter pixel distance, $d = 1, 2$. Data structure containing the spatial information of neighboring pixels in the specified regions and the corresponding intensity vectors is created and stored in a file. The created file is then used to train the neural network.

5.4 TEXTURE ANALYSIS IN TRANSFORM DOMAIN:

5.4.1 Introduction

Statistical texture analysis algorithms focus only on the local coupling between image pixels. In transform based approach for texture analysis, time / frequency analytical tools such as Fourier, Gabor and Wavelet transforms can efficiently
characterize the coupling of different scales in textures and help to overcome this difficulty [46, 47].

The Fourier transforms analyzes a signal in the time domain for its frequency content. The transform works by first translating a function in the time domain into a function in the frequency domain. The Fourier coefficients of the transformed function represent the contribution of each sine and cosine function at each frequency. Fourier and wavelet analysis have very strong correlation.

The discrete Fourier transform (DFT) estimates the Fourier transform of a function from a finite number of its sampled points. Disadvantages of FT is that it has only frequency resolution and no time resolution and therefore, does not allow analysis of non-stationary signals. Disadvantages of FT are overcome by Short Time Fourier Transforms (STFT)[50].

Short Time Fourier Transforms are used for better representation of the non-periodic signal. STFT segments the signal into narrow time intervals, narrow enough to be considered stationary, and then take the Fourier transform of each segment.

The disadvantage of STFT is that wide analysis window provides good frequency resolution but results in poor time resolution. Narrow analysis window gives good time resolution but results in poor frequency resolution. Once the window is chosen, the resolution is set for both time and frequency. Time resolution defines how well two spikes in time can be separated from each other in the transform domain, whereas frequency resolution defines how well two spectral components can be separated from each other. Both time and frequency resolutions cannot be arbitrarily high. It is not possible to precisely know at what time instance a frequency component is located. It can only indicate what interval of frequencies is present in which time intervals[51,52].

5.4.2 Wavelet Transform

The resolution problem of the STFT is overcome by using a variable length window in wavelet transform. Here, analysis windows of variable lengths are used for different frequencies i.e. for analysis of high frequencies narrower windows are used for better time resolution whereas for analysis of low frequencies wider windows are used for better frequency resolution. The function used to window the signal is called the
The most important dissimilarity between Fourier and wavelet transforms is that individual wavelet functions are localized in space whereas Fourier sine and cosine functions are not.

Wavelets are functions defined over a finite interval and having an average value of zero. The basic idea of the wavelet transform is to represent any arbitrary function $f(t)$ as a superposition of a set of such wavelets or basis functions. These basis functions or baby wavelets are obtained from a single prototype wavelet called the mother wavelet, by dilations or contractions (scaling) and translations (shifts).

Wavelet is a small wave which has its energy concentrated in time. It has an oscillating wavelike characteristic but also has the ability to allow simultaneous time and frequency analysis and it is a suitable tool for transient, non-stationary or time-varying phenomena [49,52].

The important characteristic of wavelets is that they can serve as deterministic or non-deterministic basis for generation and analysis of the most natural signals to provide better time-frequency representation, which is not possible with waves using conventional Fourier analysis [60].

Wavelet transforms do not have a single set of basis functions like the Fourier transform, which utilizes only sine and cosine functions. Instead, wavelet transforms have an infinite set of possible basis functions. The different wavelet families make different trade-offs between how compactly the basis functions are localized in space and how smooth they are. The Daubechies wavelet family is one example of wavelet basis function family. Within each family of wavelets, wavelet subclasses are distinguished by the number of coefficients and by the level of iteration. In Figure 5.4, illustrate several different wavelet families.
Heisenberg's uncertainty principle, in signal processing terms, states that it is impossible to know the exact frequency and the exact time of occurrence of this frequency in a signal. In other words, a signal can simply not be represented as a point in the time-frequency space. In wavelet analysis, the use of a fully scalable modulated window solves the signal-cutting problem. The window is shifted along the signal and for every position the spectrum is calculated. Then this process is repeated many times with a slightly shorter (or longer) window for every new cycle. In the end the result will be a collection of time-frequency representations of the signal, all with different resolutions. Because of this collection of representations, it is called as multiresolution analysis. In the case of wavelets it is normally not referred to as time-frequency representations but as time-scale representations, scale being in a way the opposite of frequency, because the term frequency is reserved for the Fourier transform [60].

Continuous wavelet transform of the signal \( x(t) \) using the analysis wavelet \( \psi \) is given by equation given below:

\[
CWT_x^\psi (\tau, s) = \Psi_x^\psi (\tau, s) = \frac{1}{\sqrt{|s|}} \int_{\mathbb{R}} x(t) \psi^\ast \left( \frac{t - \tau}{s} \right) dt \quad \text{……(5.15)}
\]

where,
CWT = Continuous wavelet transform of the signal \( x(t) \) using the analysis wavelet \( \psi(\cdot) \).

\[ \tau = \text{Translation parameter, measure of time} \]

\[ S = \text{Scale parameter, measure of frequency} \]

\[ \frac{1}{\sqrt{|S|}} = \text{A normalization constant} \]

\[ X(t) = \text{Signal to be analyzed} \]

\[ \psi^* = \text{Mother wavelet} \]

The CWT has the drawbacks of redundancy and impracticability with digital computers. As all scale and shift parameters take continuous values, the resulting CWT is a very redundant representation, and impracticability is the result of redundancy. Therefore, the scale and shift parameters are evaluated on a discrete grid of time-scale plane leading to a discrete set of continuous basis functions [46].

Wavelet transform can be considered as the decomposition of a signal with a family of real orthonormal bases \( \psi(x) \) obtained through translation and dilation of a kernel function \( \psi(x) \) known as the mother wavelet, i.e. Continuous wavelet transform of the signal CWT in continuous time-frequency plane can be discretized by sampling the time-frequency plane on a dyadic (octave) grid and represented by equation given below,

\[ \psi_{m,n}(x) = 2^{-m/2} \psi(2^{-m} x - n) \] ........................(5.16)

5.4.3 Implementation of DWT using filter bank

It is convenient to view the decomposition of a given signal as passing a signal through a pair of filters \( h_0(n) \) and \( h_1(n) \) and down sampling the filtered signals by two (dropping every other sample)[50,51,52]. The procedure of performing DWT of an image is as shown in figure 5.4.
Fig. 5.4 Single level analysis filter bank for 2-D DWT

The pair of filters $h_0(n)$ and $h_1(n)$ correspond to the half band low pass and high pass filters, respectively, and are called the quadrature mirror filters in the signal processing literature. A low pass filter and a high pass filter are chosen, such that they exactly halve the frequency range between themselves. This filter pair is called the Analysis Filter pair. First, the low pass filter is applied for each row of data, thereby getting the low frequency components of the row. But since the LPF is a half band filter, the output data contains frequencies only in the first half of the original frequency range. So, by Shannon's Sampling Theorem, they can be sub sampled by two, so that the output data now contains only half the original number of samples. Now, the high pass filter is applied for the same row of data, and similarly the high pass components are separated, and placed by the side of the low pass components. This procedure is done for all rows. Next, the filtering is done for each column of the intermediate data. The resulting two-dimensional array of coefficients contains four bands of data, each labeled as LL (low-low), HL (high-low), LH (low-high) and HH (high-high). The LL band gives approximate details, LH band gives horizontal texture information, HL band gives vertical texture information and HH band gives diagonal texture features. The LL band can be decomposed once again in the same manner, thereby producing even more sub bands. This can be done up to any level, thereby resulting in a pyramidal decomposition as shown in figure 5.5.
As mentioned above, the LL band at the highest level can be classified as most important, and the other 'detail' bands can be classified as of lesser importance, with the degree of importance decreasing from the top of the pyramid to the bands at the bottom. The energy of LL sub band can be calculated using equation given below,

\[
e = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} |x(m, n)|.
\]

\[\text{.........(5.17)}\]

where \(x(m, n)\) is decomposed LL band, with \(1 \leq m \leq M\) and \(1 \leq n \leq N\).

Thus, it can be concluded that standard Discrete Wavelet Transform (DWT) provides texture information in three directions i.e. horizontal (0 degree), vertical (90 degree) and diagonal (45 degree).

**5.4.4 Estimation of texture feature using standard DWT**

The pyramid structured wavelet transform provides multiresolution analysis filter bank decomposition with a set of frequency channels which have narrower bandwidths in the lower frequency regions. DWT decomposition of the segmented image of tumor tissue is performed using algorithm as explained below:
Algorithm I: Texture analysis using Pyramid structured wavelet transforms:

1. Decompose a given textured image with 2-D Discrete wavelet transform into four sub images or sub bands, namely, LL, LH, HL, HH.

2. Calculate the energy of each decomposed image. That is, if the decomposed image is \( x(m, n) \), with \( 1 \leq m \leq M \) and \( 1 \leq n \leq N \), the energy is

\[
e = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} |x(m, n)|.
\]

3. The energies of four sub bands of first level decomposition will be added to the feature vector which is used as input to train the Artificial Neural Network.

4. The LL sub band obtained from the first level decomposition is further decomposed by DWT decomposition using above equation.

5. The energies of these four sub bands of second level decomposition will be further added to the feature vector.

6. The procedure of decomposition can be repeated for desired number of decompositions.

In the following figure 5.6 (a) shows the original MR Image of brain with tumor, (b) shows segmented tumor, (c) and (d) shows the sub bands or sub images obtained after one and two levels of decompositions respectively.

![Fig. 5.6: (a) T1 weighted MRI (b) Segmented Tumor](image-url)
Fig. 5.7 Four subbands after (a) first level (b) second level decomposition

Fig. 5.7 (c) Third level decomposition

5.4.5 Limitations of DWT

Although the discrete wavelet transform (DWT) is a powerful signal-processing tool, it has three disadvantages that undermine its usage in many applications.

1. **Shift Sensitivity**: It is shift sensitive because input-signal shifts generate unpredictable changes in DWT coefficients. Shift sensitivity is an undesirable property because it implies that DWT coefficients fail to distinguish between input signal shifts.

2. **Poor Directionality**: The DWT suffers from poor directionality because DWT coefficients reveal only three spatial orientations. As shown in figure 5.7, 2-D DWT can resolve only three spatial-domain feature orientations i.e. horizontal (HL), vertical (LH)
and diagonal (HH). Input images may contain number of smooth regions and edges with random orientations; hence poor directionality affects the optimal representation of natural images with of the separable standard 2-D DWT.

3. Absence of Phase Information: DWT analysis lacks the phase information that accurately describes non-stationary signal behavior. DWT implementations (including standard DWT, WP and SWT) use separable filtering with real coefficient filters associated with real wavelets resulting in real-valued approximations and details. Such DWT implementations cannot provide the local phase information. All natural signal are basically real-valued, hence to avail the local phase information, complex-valued filtering is required.

5.5 DUAL-TREE COMPLEX WAVELET TRANSFORMS

To overcome these problems, Kingsbury created the dual-tree wavelet transform (DTWT), a redundant, complex wavelet transform with excellent directionality, reduced shift sensitivity and explicit phase information. The DTWT is redundant because it consists of a pair of filter banks that simultaneously operate on the input signal and provide two wavelet decompositions. The wavelets associated with the filter banks are a Hilbert pair. This property is critical since it provides the advantages of reduced shift sensitivity, improved directionality and explicit phase information [52,53,54].

The DT–CWT is a spatial frequency transform that uses spatial filters to decompose an image region into dyadic sub bands. Shift invariance can be achieved in a dyadic wavelet transform by doubling the sampling rate. This is effected in the DT-CWT by eliminating the down sampling by 2 after the first level of filtering. Two parallel fully decimated trees are constructed by placing the down sampled outputs of first level filters of tree one sample offset from the outputs of the other. To get uniform intervals between the two trees’ samples, the subsequent filters in one tree must have delays that are displaced by one half samples. For linear phase, this is enforced if the filters in one tree are of even length and the filters in the others are of odd length. Additionally, better symmetry is achieved if each tree uses odd and even filters alternatively from level to level. The filters are biorthogonal and the impulse responses
can be considered as the real and the imaginary parts of the complex wavelet. Thus the complex wavelet decomposition is able to separate the given signal or an image into six sub bands oriented at ±15, ±45, ±75 degrees. Thus DT-DWT provides texture information in six directions as compared to DWT that provides texture information in three directions i.e. horizontal, vertical.

Figure 5.8 shows the implementation of dual tree complex wavelet transform using filterbanks.

Algorithm for texture analysis using Dual Tree Complex wavelet transforms is explained below:

1. Decompose a given textured image with Dual Tree Complex wavelet transform into six sub bands, oriented at ±15, ±45, ±75 degrees

2. Calculate the energy of each decomposed sub band image. That is, if the decomposed image is \( x(m, n) \), with \( 1 \leq m \leq M \) and \( 1 \leq n \leq N \), the energy is given by the equation,
3. One of the sub band having maximum energy, is decomposed further for second level of decomposition.

4. Thus, the maximum energy at each level of decomposition will be added to the texture feature for the given image.

5. The procedure of decomposition and estimation of maximum energy is performed for segmented Tumor tissue and normal brain tissue for desired number of decompositions. Thus, the texture feature is created which may be further used for training of Artificial Neural Network.

Figure 5.9 shows the Sub bands after two levels of decomposition.

Fig. 5.9 Sub bands after two levels of decomposition.
The following observations are drawn from the set of readings.

1. In case of feature extraction using standard DWT, the wavelet basis functions Haar, Biortho-2.2 and Debauchees DB1 are used for two level decomposition of image and calculation of LL sub band.

2. It is observed that decomposition using basis function Biortho 2.2 gives optimum classification performance.

3. In case of feature extraction using DTCWT, the first level of decomposition splits the image into six sub bands. The energy of sub band having maximum energy is included in the feature vector. This highest energy sub band is decomposed further.

**SUMMARY**

Pattern recognition can be defined as the process of learning to distinguish and classify pattern. An image feature is a distinguishing characteristic of an image. One of the prerequisite of pattern recognition is feature extraction. Image features are of importance in the isolation of regions within an image and subsequent labeling and classification. Therefore, extraction of feature is the important step in pattern recognition.

Feature extraction can be extracted from various classes as Pixel intensity based features, Edge based feature and Texture based features. Various texture analysis approaches are broadly classified as Structural approach, Statistical approach and Transform /Spectral approach.

Statistical approach represents the texture by non-deterministic properties that govern the distribution and relationships between the grey levels of an image. Methods based on second order statistics i.e. statistics given by pairs of pixels have given better texture differentiation than other methods. The most popular second order statistical features for texture analysis are derived from co-occurrence matrix. It has potential of extracting features for effective texture discrimination in biomedical images.
Transform based approach interprets image features in frequency domain using various transforms like Fourier, Gabor and Wavelet transform. These transforms represent an image in a space whose coordinates system has an interpretation closely related to characteristics of a texture such as frequency. Wavelet transform based feature extraction has several advantages. Variable spatial resolution represents textures at suitable scale and there is a wide range of choices for the wavelet basis function.

The following observations are drawn from spatial and transform domain analysis.

(i) Total twelve statistical parameters are calculated for tumor and normal brain tissue. It is observed that not all statistical parameters show substantial discrimination for tumor and normal brain tissue. Entropy, Contrast, Energy and Variance are the parameters which indicate precise discrimination in parameter values for tumor tissue and normal brain tissue. Therefore, only these parameters are included in the feature vector.

(ii) The parameters do not vary substantially if the inter pixel distance is incremented beyond 2. Therefore, feature vector comprising of these parameters is calculated for above four angles and inter pixel distance $d = 1$ and 2.

(iii) In case of feature extraction using standard DWT, the wavelet basis functions Haar, Biortho-2.2 and Debauchees DB1 are used for two level decomposition of image and calculation of LL sub band.

(iv) It is observed that decomposition using basis function Biortho 2.2 gives optimum classification performance.

(v) In case of feature extraction using DTCWT, the first level of decomposition splits the image into six sub bands. The energy of sub band having maximum energy is included in the feature vector. This highest energy sub band is decomposed further.