CHAPTER 3

HEART RATE VARIABILITY ANALYSES USING WINDOWS AND WAVELET TRANSFORM

3.1 INTRODUCTION

This chapter presents the analysis of heart rate variability using windowing techniques and wavelet transform. The activity of the autonomic nervous system is noninvasively studied by means of autoregressive (AR) frequency analysis of the heart-rate variability (HRV) signal. Spectral decomposition of the Heart Rate Variability was obtained, in order to assess the characteristic fluctuations in the heart rate and their spectral parameters under different conditions. Mental stress is related to changes in autonomic nervous system (ANS) activity. Heart rate variability (HRV) analysis is useful for assessing the activities of autonomic nervous system. The approach consists of 1) time and frequency response analysis of ECG signal using window functions 2) Signal processing using different wavelets to extract R-R intervals of HRV in time-frequency domain 3) Comparison of signal-to-noise ratio at different stages using various wavelets.
3.2 HEART RATE VARIABILITY

Spectral analysis, in the study of the cardiovascular signals, has demonstrated to be a fine non invasive tool for evaluating the state of the ANS, and it has been used in order to assess the HRV changes for smoker and nonsmoker. These variations exhibit characteristic oscillations which corresponds to variations of parasympathetic and sympathetic branches of the ANS. Wide band of the spectral components of HRV ranges from 0.003 Hz until 0.5 Hz, where the range between 0.003–0.04 Hz (Very Low Frequency component VLF) takes account of long-term regulation mechanisms, 0.04–0.15 Hz (Low Frequency Component LF) represents sympathetic activation, and finally the range between 0.15–0.5 Hz (High Frequency Component HF) corresponds to tone and it is highly influenced by respiration.

3.2.1 Interdependence of ANS and HRV

The nervous system is divided into two parts: Somatic nervous system which is in charge of voluntary control of organs which include mainly muscles. ANS also known as the visceral/automatic system regulates individual organ function and homeostasis, and for the most part is subject to involuntary control. Researchers have examined the effect of emotions on the autonomic nervous system by the analysis of heart rate variability, which serves as a dynamic comparison between autonomic function and balance. We know that in a healthy volunteer, the parasympathetic nervous system fibers and the sympathetic nervous system fibers richly innervate the Sino-atrial node. Parasympathetic innervations of the heart are mediated by the vagus nerve which causes a decrease in the SA node thereby decreasing the heat rate whereas stimulation by
the sympathetic fibers causes an increase in the heart rate. Thus the variability in
the heart rate is due to the action of balance and synergy between the two
branches of the Autonomic system, which is mainly enforced through neural
mechanical, humoral and other physiological mechanisms. It maintains
cardiocascular parameters in their most favorable ranges and permits suitable
reactions to change in external or internal stimuli. This balance between the
effect of the sympathetic nervous system and the parasympathetic nervous
systems is known as the Sympathovagal balance and is believed to be echoed in
the beat-to-beat changes of the cardiac cycle. The heart rate is defined by the
reciprocal of the RR interval with units of beats/min.

3.3 RELATION BETWEEN CARDIAC HEALTH AND ANS

HRV is reflective of the general state of well-being of the organism. It is
predominantly dependent on the extrinsic regulation of the Heart rate. HRV is
thought to reflect the heart’s ability to adapt to changing circumstances by
detecting and quickly responding to unpredictable stimuli. Recent experimental
confirmation for a connection between a tendency for fatal arrhythmias and
indications of either increased sympathetic or reduced parasympathetic activity
has motivated to the development of quantitative markers to adjudge the
autonomic activity. HRV represents one of these most potent markers. It is a
strong and independent forecaster of death following an acute myocardial
infarction.

It has been proved in research studies that during the period of a mental or
an emotional anxiety, an increase in the sympathetic activity and a simultaneous
decrease in the Parasympathetic activity were observed. This results in amplified
strain on the heart, immune as well as other important hormonal systems. The
increase of sympathetic activity is related to a reduced ventricular fibrillation threshold and thus an augmented threat of fibrillation, in contrast to an increase in parasympathetic activity, which protects the heart. HRV data mathematically is used to distinguish and measure the sympathetic and parasympathetic activity along with the activity of the autonomic nervous system, thus decomposing the HRV signal into its constituent frequency components and thus computing the relative power of the components.

The three main frequency bands of interest are referred to as:

- Very Low frequency (VLF) - 0.003-0.04 Hz.
- Low frequency band (LF) - 0.04 to 0.15 Hz.
- High frequency band (HF) -0.15 to 0.4 Hz.

The magnitude of the HF component provided an index of vagal activity and the magnitude of the LF component provided an index of sympathetic activity with vagal modulation. The LF/HF ratio was used as a marker of instant Sympathovagal balance. The distribution of the low frequency and high frequency power as well as their central frequencies may not be fixed but may vary with changes in autonomic modulations of the heart.

Measurements of VLF, LF and HF power components is usually made in absolute values of power (ms²), but LF and HF may also be measured in normalized units (n.u.), which represents the relative value of each power component in fraction to the total power excluding the VLF components. The representation of LF and HF in n.u. emphasizes the controlled and balanced behavior of the two branches of the ANS. The advantage of the n.u. units is that normalization tends to minimize the effect on the values of LF and HF components of the changes in total power.
3.4 WHY SHORT-TERM RECORDINGS ARE PREFERRED?

Problem of being stationary is frequently discussed with long-term recordings. If mechanisms responsible for heart period modulations of a certain frequency remain unchanged during the whole period of recording, the corresponding frequency components of HRV may be used as a measure of these modulations. If the modulations are not stable, interpretation of the results of frequency analysis is less well defined. In particular, physiological mechanisms of heart period modulations responsible for LF and HF power components cannot be considered stationary during the 24 hr period. Thus, spectral analysis performed in the entire 24 hr period as well as spectral results obtained from shorter segments (e.g. 5 min) averaged over the entire 24 hour period provide averages of the modulations attributable to the LF and HF components. Such averages obscure detailed information about autonomic modulation of RR intervals available in shorter recordings.

3.4.1 Requirements to obtain a reliable spectral estimation

1. Sampling Rate: Optimal range should be in the range of 250-500 MHz or higher.
2. Baseline and trend removal (if used) may affect the lower components in the spectrum, hence should be taken care of.
3. A stable and noise-independent fiducially point should be chosen.
4. Ectopic beats, arrhythmic events, missing data and noise effects may later the estimation of the PSD of HRV.
3.4.2 Applications of HRV in the detection of:

- Myocardial infarction
- Diabetic neuropathy
- Cardiac transplantation
- Myocardial dysfunction
- Tetraplegia

3.4.3 Influence of cigarette smoking on HRV

It has been observed that cigarette smoking increased sympathetic activity or decreased vagal cardiac activity and so has been recognized as a major mechanism for increased risk of coronary artery disease in smokers. Many recent studies have reported increased susceptibility to sudden coronary death and increased subsequent mortality after myocardial infarction in patients with decreased vagal cardiac control assessed by heart rate variability. The decrease in vagal cardiac activities may be present in smokers and may be a reason for the association between smoking and cardiac death generally is seen that smoking causes a significant:

1. Increase of the LF/HF ratio
2. Increase of the normalized values of the LF component
3. Decrease in absolute values of total power, LF and HF.
4. Significant decrease in the time-domain indices.

Spectral decomposition technique has shown consistent results: a vagal increase during NREM sleep stage measured by the HF component and increase
of sympathetic activity measure by the LF and LF/HF ratio during REM sleep stage. HRV signal is stationary in constant conditions, its first and second statistic moments through a time window of a few minutes do not change, and application of Fourier Transform or Autoregressive Batch analysis are adequate spectral decomposition techniques. However, when it is necessary to analyze sequences where transitory changes in the signal could happen, such as variations during Valsalva maneuver, tilt test or Obstructive Sleep Apnea events, these approaches don’t result to be the most suitable for the analysis, because the signal acquires non-stationary characteristics.

To overcome this inconvenient there are two possible ways. On one hand, techniques such as Short Time Fourier Transform, Discrete Wavelet Analysis, Time-Frequency Distributions and Time-Varying Analysis could be used. On other hand, it could be taken a short data sequences where fast changes are not present as some studies proposed, in different sleep stages. However, time-varying autoregressive models allow assessing, on a beat to beat basis, the spectral parameters of HRV signal in a fast and efficient way independently on the transitory events found through the whole night recording (provoked by arousals, body movements, and changes on sleep stages or apneas).

The aim of this work is to assess the spectral parameters by time-varying autoregressive spectral analysis of the HRV during whole night recordings, in a beat to beat basis. The study focus on the temporal evolution of LF, HF spectral powers and on the module and phase of a representative pole of the autoregressive model in HF band during REM and NREM sleep stages.
3.5 SIGNAL PROCESSING USING WINDOWS

In signal processing, a window function or finite weighting function is a mathematical function that is zero-valued outside of some chosen interval. For instance, a function that is constant inside the interval and zero elsewhere is called a rectangular window, which describes the shape of its graphical representation. When another function or a signal is multiplied by a window function, the product is also zero-valued outside the interval. The applications of window functions include spectral analysis, filter design, and beam forming. A more general definition of window functions does not require them to be identically zero outside an interval, as long as the product of the window multiplied by its argument is square integral, that is, that the function goes sufficiently rapidly toward zero. The Fourier transform of the function \( \cos \omega t \) is zero, except at frequency \( \pm \omega \). However, many other functions and data (that is, waveforms) do not have convenient closed form transforms. Alternatively, one might be interested in their spectral content only during a certain time period. In either case, the Fourier transform (or something similar) can be applied on one or more finite intervals of the waveform. In general, the transform is applied to the product of the waveform and a window function. Any window (including rectangular) affects the spectral estimate computed by this method. If the frame length of the Fourier analysis approaches infinity. However in general the ECG signal is non-stationary and very long windows are not applicable. The finite-length window effect causes the covariance matrix is generally non-diagonal. Therefore correlation exists among the frequency components. The coherence in phase corresponds to energy localization in the time domain, which can be modeled by a time envelope.
Signal processing is performed using windows technique and wavelet transform. The output obtained for signal processing using window functions are given. Used windows are Hanning, Hamming and Triangular. The signal in time domain and its frequency response are given below. In this section, we will discuss in brief about the different FIR windows in digital FIR filter system & their different characteristics like their time domain response & frequency domain response. The triangular window function is first used to find its frequency response. The variation in the frequency response is obtained through Hamming and Hanning windows. Figure 3.1 shows the time domain and frequency response of the given signal.

A finite length signal is rarely periodic at its boundaries and as a result, the power in the periodogram will appear at other frequencies. Leakage to nearby frequencies due to the finite interval over which the data is sampled takes the form of side-lobes in the periodogram. This spectral leakage is classically minimized through windowing or other mathematically equivalent techniques. Figure 3.2 describes the time domain and frequency response obtained for Hamming window.

![Figure 3.1.Triangular window frequency response](image-url)
In order to choose the necessary window size, we must balance the requirement of stationary versus the time required to resolve the information present. The European and North American Task force on standards in HRV suggested that the shortest time period over which HRV metrics should be assessed is 5 minutes. Figure 3.3 describes the time domain and frequency response obtained for Hanning window.

**Figure 3.2. Hamming window frequency response**

**Figure 3.3. Hanning window frequency response**
However, it would be incorrect to generate a time series at 7 Hz and then select physiologically plausible RR intervals. This would correspond to deriving RR intervals from an ECG that was sampled at only 7 Hz. It has been demonstrated that the inaccuracies of R-peak location due to finite sampling times can significantly affect the RR tachogram and any metrics derived from it. If a patient is suffering from low HRV (e.g. because they have recently undergone a heart transplant or are in a state of coma) then the sampling frequency of the ECG must be higher than normal. Abboud et al, (1995) show that for such patients a sampling rate of at least 1000Hz is required. For normal patients however, a sampling rate of 128Hz has been found to be accurate enough to locate the R-peaks and hence compute HRV.

3.6 ECG SIGNAL PROCESSING USING WAVELET TRANSFORM

Fourier analysis, using the Fourier transform, is a powerful tool for analyzing the components of a stationary signal (a stationary signal is a signal that repeats). For example, the Fourier transform is a powerful tool for processing signals that are composed of some combination of sine and cosine signals.

The Fourier transform is less useful in analyzing non-stationary data, where there is no repetition within the region sampled. Wavelet transforms (of which there are, at least formally, an infinite number) allow the components of a non-stationary signal to be analyzed. Wavelets also allow filters to be constructed for stationary and non-stationary signals.

Although Haar wavelets date back to the beginning of the twentieth century, wavelets as they are thought of today are new. Wavelet mathematics is less than a quarter of a century old. Some techniques, like the wavelet packet
transform are barely ten years old. This makes wavelet mathematics a new tool which is slowly moving from the realm of mathematics into engineering. For example, the JPEG 2000 standard is based on the wavelet lifting scheme.

The Fourier transform shows up in a remarkable number of areas outside of classic signal processing. Even taking this into account, we thought that it is safe to say that the mathematics of wavelets is much larger than that of the Fourier transform. In fact, the mathematics of wavelets encompasses the Fourier transform. The size of wavelet theory is matched by the size of the application area. Initial wavelet applications involved signal processing and filtering. However, wavelets have been applied in many other areas including non-linear regression and compression. An offshoot of wavelet compression allows the amount of determinism in a time series to be estimated.

Time–frequency signal analysis methods offer simultaneous interpretation of the signal in both time and frequency which allows local, transient or intermittent components to be elucidated. Such components are often obscured due to the averaging inherent within spectral only methods, i.e. the FFT. A number of time–frequency methods are currently available for the high resolution decomposition in the time–frequency plane useful for signal analysis, including the short time Fourier transform (STFT), Wigner–Ville transform (WVT), Choi–Williams distribution (CWD) and the continuous wavelet transform (CWT). Of these the continuous wavelet transform has emerged as the most favored tool by researchers as it does not contain the cross terms inherent in the WVT and CWD methods while possessing frequency-dependent windowing which allows for arbitrarily high resolution of the high frequency signal components (unlike the STFT).

Many of the ideas behind wavelet transforms have been in existence for a long time. However, wavelet transforms analysis as we now know it really began
in the mid 1980s where it was developed to interrogate seismic signals. Interest in wavelet analysis remained within a small, mainly mathematical community during the rest of the 1980s with only a handful of scientific papers coming out each year. The application of wavelet transform analysis in science and engineering really began to take off at the beginning of the 1990s, with a rapid growth in the numbers of researchers turning their attention to wavelet analysis during that decade. The last few years have each seen the publication of over 1000 refereed journal papers concerning application of the wavelet transform, and these covering all numerate disciplines.

![Complex wavelet function](image)

**Figure 3.4. Complex wavelet function**

A wavelet is simply a small wave which has energy concentrated in time to give a tool for the analysis of transient, non stationery or time-varying phenomena such as a wave shown in Figure 3.4. A signal as the function of f(t) shown in Figure 3.4 can often be better analyzed and expressed as a linear decomposition of the sums: products of the coefficient and function. In the Fourier series, one uses sine and cosine functions as orthogonal basis functions. But in the wavelet expansion, the two-parameter system is constructed such that one has a double sum and the coefficients with two indices. The set of coefficients are called the DWT of f (t). In its most common form, the DWT employs a dyadic grid (integer power of two scaling in a and b) and ortho normal wavelet basis functions and exhibits zero redundancy. A natural way to sample the parameters a and b is to use a logarithmic discretization of the a scale and
link this, in turn, to the size of steps taken between \( b \) locations. To link \( b \) to \( a \) we move in discrete steps to each location \( b \), which are proportional to the \( a \) scale. The wavelet transform has emerged over recent years as a key time–frequency analysis and coding tool for the ECG. As we have seen in this review, its ability to separate out pertinent signal components has led to a number of wavelet-based techniques which supersede those based on traditional Fourier methods. In its continuous form, the CWT allows a powerful analysis of non-stationary signals, making it ideally suited for the high-resolution interrogation of the ECG over a wide range of applications. In its discrete form, the DWT and its offshoots, the SWT and WPT, provide the basis of powerful methodologies for partitioning pertinent signal components which serve as a basis for potent compression strategies. It is interesting to note that researchers coming to the wavelet transform tend to take an either/or approach to their study: either concentrating on the DWT or the CWT relatively, few explore both in depth. The DWT has interesting mathematics and fits in with standard signal filtering and encoding methodologies. However, it exhibits non-stationary and coarse time–frequency resolution.

The CWT, on the other hand, allows arbitrarily high resolution of the signal in the time–frequency plane, which is a necessity for the accurate identification and partitioning of pertinent components. However, the discretization of the continuous wavelet transform, required for its practical implementation with discrete signals, involves a discrete approximation of the transform integral (i.e. a summation) computed on a discrete (but not dyadic) grid of \( a \) scales and \( b \) locations. The inverse continuous wavelet transform is also computed as a discrete approximation. How close an approximation to the original signal is recovered depends mainly on the resolution of the discretization used and, with care, usually a very good approximation can be recovered.
Wavelet transforms allow the components of a non-stationary signal to be analyzed. Wavelets also allow filters to be constructed for stationary and non-stationary signals. In signal analysis, the detection of discontinuities is useful for extracting various features. The processing of medical signal like electrocardiograms requires the discontinuities detection.

![Wavelet Real Part](image1.png)

![Wavelet Imaginary Part](image2.png)

![Wavelet Spectrum](image3.png)

**Figure 3.5 Complex Wavelet function and its frequency response**

Muscular contraction is associated with electrical changes known as depolarization. The electrocardiogram (ECG) is a measure of this electrical activity associated with the heart. The ECG is measured at the body surface and
results from electrical charges associated with activation first of the two small heart chambers, the atria, and then of the two larger heart chambers, the ventricles. The contraction of the atria manifests itself as the ‘P’ wave in the ECG and contraction of the ventricles produces the feature known as the ‘QRS’ complex. The subsequent return of the ventricular mass to a rest state depolarization produces the ‘T’ wave. Depolarization of the atria is, however, hidden within the dominant QRS complex. Analysis of the local morphology of the ECG signal and its time varying properties has produced a variety of clinical diagnostic tools. In this section we review the application of the wavelet transform to the analysis of the ECG signal.

The R-R intervals were used to produce an HR signal in beats per minute, i.e., HR (R-R interval). Figure 3.5 shows the analysis using the above defined complex wavelet function by plotting frequency response. The frequency response of the original ECG with noise provides less information. Producing an algorithm for the detection of the P wave, QRS complex and T wave in an ECG is a difficult problem due to the time varying morphology of the signal subject to physiological conditions and the presence of noise.

They used the method to determine timing intervals of the ECG signal including the widths of the QRS. Producing an algorithm for the detection of the P wave, QRS complex and T wave in an ECG is a difficult problem due to the time varying morphology of the signal subject to physiological conditions and the presence of noise. Recently, a number of wavelet-based techniques have been proposed to detect these features. Senhaji et al, (1997) et al compared the ability of three different wavelets transforms to recognize and describe isolated cardiac beats. Sahambi et al, (1997) employed a first-order derivative of the Gaussian function as the wavelet for the characterization of ECG waveforms. They then used modulus maxima-based wavelet analysis employing the dyadic wavelet
transform to detect and measure various parts of the signal, specifically the location of the onset and offset of the QRS complex and P and T waves. Sahambi et al, (1997) showed that the algorithm performed well in the presence of modeled baseline drift and high frequency noise added to the signal. They used the method to determine timing intervals of the ECG signal including the widths of the QRS complex, T and P waves, and PR, ST and QT intervals. The measurements of these intervals give the relative position of the components in the ECG which are important in delineating the electrical activity of the heart. Improvements to the technique are described in Sahambi et al, (1998). R wave detectors are extremely useful tools for the analysis of ECG signals. They are used both for finding the fiducially points employed in ensemble averaging analysis methods, and for computing the R–R time series from which a variety of heart rate variability (HRV) measures can be extracted. Both these techniques rely on the accurate determination of the temporal location of the R wave. There are currently a number of QRS detection algorithms available which use a variety of signal analysis methods.

The main ideas consists in isolating a block of wavelet coefficients and based upon the information collected about the entire set make a decision about decreasing or even entirely discard the group. This procedure will allow faster manipulation of the information and accelerated convergence rates. The recorded ECG signal is given in which is corrupted by noise and less informative. Hence, the preprocessing of ECG signal is a prerequisite for the mental stress assessment.

The R-R intervals were used to produce an HR signal in beats per minute, i.e., HR (R-R interval). Figure 3.6 shows the analysis using the above defined complex wavelet function by plotting frequency response. The frequency response of the original ECG with noise provides less information.
Thresholding approaches resorting to term-by-term modification on the wavelets coefficients attempt to balance variance and bias contribution to the mean squared error in the estimation of the underlying signal. However, it has been proven that such balance is not optimal. Term-by-term thresholding ends up removing too many terms leading to estimation prone to bias and with a slower convergence rate due to the number of operations involved.

Figure 3.6. Frequency response of noisy ECG

3.7 FEATURE EXTRACTION USING WAVELET TRANSFORM

Wavelet transform is generally divided into either a discrete and or continuous form. The continuous wavelet transform (CWT) of a signal s(t) is defined as the integral of the product between the signal s(t) and the daughter wavelets, which are the time translation and scale expansion/compression versions of a mother wavelet function w(t). Equivalent to a scalar production, this calculation generates continuous wavelet coefficients CWC (a,b), which determine the similarity between the signal and the daughter wavelets located at position ‘b’ (time shifting factor) and positive scale ‘a’.
\[
\text{CWC}(a,b) = \int_{-\infty}^{\infty} s(t) \frac{1}{\sqrt{a}} \psi^\ast \left( \frac{t-b}{a} \right) dt \quad \cdots (3.1)
\]

\[
F\{\text{CWC}(a,b)\} = \sqrt{a} \psi^\ast (a, \omega) s(\omega) \quad \cdots (3.2)
\]

Where \( \psi^\ast (a, \omega), s(\omega) \) stand for the Fourier transforms of the continuous wavelet coefficients CWC (a,b), the signal s(t), and the mother wavelet function w (t), respectively. Eq. (3.2) shows that a mother wavelet function is a band-pass filter in the frequency domain, and the use of CWC identifies the local features of the signal.

According to the theory of Fourier transform, the center frequency of the mother wavelet \( W(ax) \) is defined as \( F_0/a \), given that the center frequency of the \( W(x) \) is \( F_0 \). Consequently, extraction of frequency contents from the signal is possible in different scales. In the windowed Fourier transform, the frequency resolution is constant and depends on the width of window.

\[\text{Figure 3.7 Subband decomposition of discrete wavelet transforms implementation}\]

The procedure of multi rate solution decomposition of a signal \( x[n] \) is schematically shown in Figure 3.4. All wavelet transforms can be specified in terms of a low-pass filter \( h \), which satisfies the standard quadrature mirror filter condition:
\[ H(z)H(z^{-1}) + H(z)H(z^{-1}) = 1 \] .................................................. (3.3)

Where \( H(z) \) denotes the \( z \)-transform of the filter \( h \). Its complementary high-pass filter can be defined as

\[ G(z) = zH(-z^{-1}) \] .................................................. (3.4)

A sequence of filters with increasing length (indexed by \( i \)) can be obtained.

\[ H_{i+1}(z) = H(z^{2^i})H_i(z) \] .................................................. (3.5)

\[ G_{i+1}(z) = G(z^{2^i})H_i(z) \] .................................................. (3.6)

The two-scale relation in time domain with the initial condition \( H_0(z) = 1 \) are expressed as

\[ h_{i+1}(n) = [h]_{2^i} * h_i(n) \]
\[ g_{i+1}(n) = [g]_{2^i} * g_i(n) \] .................................................. (3.7)

Where the subscript \([.,.]_m\) indicates the up-sampling by a factor of \( m \) and \( k \) is the equally sampled discrete time.

The normalized wavelet and scale basis functions can be defined as

\[ \phi_{i,l}(k) = 2^{i/2} h_i(n - 2^i l) \]
\[ \psi_{i,l}(k) = 2^{i/2} h_i(n - 2^i l) \] .................................................. (3.8)

Where the factor 2\(^{i/2} 2i/2 \) is inner product normalization, \( i \) and \( l \) are the scale parameter and the translation parameter respectively. The discrete wavelet transform (DWT) decomposition can be described as

\[ a_{i}(l) = x(n) * \phi_{i,l}(k) \]
\[ d_{i}(l) = x(n) * \psi_{i,l}(k) \] .................................................. (3.9)

\( a_{i}(l) \) and \( d_{i}(l) \) are the approximation coefficients and the detail coefficients at resolution ‘\( i \)’ respectively.
3.8 HAAR, DAUBECHIES AND MEYER WAVELET

Haar wavelet is the simplest type of wavelet. In discrete form, Haar wavelets are related to a mathematical operation called the Haar transform. The Haar transform serves as a prototype for all other wavelet transforms. Like all wavelet transforms, the Haar transform decomposes a discrete signal into two sub signals of half its length. One sub signal is a running average or trend, the other sub signal is a running difference or fluctuation. The advantages of Haar wavelet transform are simple, fast and memory efficient. It is exactly reversible without the edge effects.

The Haar transform also has limitations, which can be a problem with for some applications. In generating each of averages for the next level and each set of coefficients, the Haar transform performs an average and difference on a pair of values. Then the algorithm shifts over by two values and calculates another average and difference on the next pair. The high frequency coefficient spectrum should reflect all high frequency changes. The Haar window is only two elements wide. If a big change takes place from an even value to an odd value, the change will not be reflected in the high frequency coefficients. So Haar wavelet transform is not useful in compression and noise removal of audio signal processing.

The Daubechies wavelet transforms are defined in the same way as the Haar wavelet transform by computing the running averages and differences via scalar products with scaling signals and wavelets the only difference between them consists in how these scaling signals and wavelets are defined. This wavelet type has balanced frequency responses but non-linear phase responses. Daubechies wavelets use overlapping windows, so the high frequency coefficient spectrum reflects all high frequency changes. Therefore Daubechies wavelets are
useful in compression and noise removal of audio signal processing. Daubechies 4-tap wavelet has been chosen for this implementation.

A useful resource to improve the quality of the aforementioned balanced is by using information of the set of data associated to a particular wavelet coefficient. In order to do so, a block strategy for threshold is proposed. The main idea consists in isolating a block of wavelet coefficients and based upon the information collected about the entire set make a decision about decreasing or even entirely discards the group. This procedure will allow faster manipulation of the information and accelerated convergence rates. The processed ECG signal using wavelet decomposition is given in Figure 3.8. The Figure 3.9 shows the recorded ECG and processed ECG using Daubechies wavelet. The Figure 3.10 and 3.11 shows the Meyer wavelet function and Denoised ECG signal using Meyer wavelet.

![Figure 3.8 Recorded ECG and processed ECG using Haar wavelet](image)

Figure 3.8 Recorded ECG and processed ECG using Haar wavelet
Figure 3.9 Recorded ECG and processed ECG using Daubechies wavelet

Figure 3.10: Meyer wavelet function
Figure 3.11: Denoised ECG signal using Meyer wavelet

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<th>Meyer</th>
<th>Bior1.1</th>
<th>Rbior</th>
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The Table 3.1 shows the Signal to Noise Ratio at different stages using various wavelets. From the table we conclude the signal to noise ratio for Db1 (Haar) is very high. The Figure 3.12 shows the comparison chart of SNR for different wavelet functions.
3.9 SUMMARY

The analysis of heart rate variability using windowing techniques and wavelet transform has been presented. Spectral decomposition of the Heart Rate Variability was obtained, in order to assess the characteristic fluctuations in the heart rate and their spectral parameters under different conditions. Heart rate variability (HRV) analysis is useful for assessing the activities of autonomic nervous system. Initially the time and frequency response analysis of ECG signal is performed using triangular, Hamming and Hanning window functions. R-R intervals are analyzed HRV in time-frequency domain Comparison of signal-to-noise ratio at different stages using various wavelets.